International Journal of Advance Research in Science and Engineering
Volume No. 15, Issue No. 01, January 2026
www.ijarse.com
IJARSE
ISSN 2319 - 8354

Performance of Fuzzy Bulk Queueing Models with Performance Measures and Cost Optimization.

Kavita kumari

Research scholar, Department of mathematics,BMU, Rohtak

Email ID:- kavitakumariyadav772@gmail.com

Dr. Shaweta Sharma

Department of mathematics,BMU, Rohtak,

Email ID:- vashishthshaweta@gmail.com.

Abstract

This paper investigates a bulk service queueing model under a fuzzy environment to reflect real world uncertainties in in service and arrival patterns. Traditional queueing model rely on precise numerical data, which are often unavailable in practical systems like telecommunication manufacturing and transportation. To address this limitation, this study applies fuzzy set theory, representing the arrival rate, service rate and batch size configuration as fuzzy numbers. The primary objective is to analyze key system performance measures and optimize the total operational cost. We derive explicit analytical expressions for critical performance metrics, including the expected queue length, waiting time in the queue server utilization and system throughout.

Keywords:- *Fuzzy queueing; Bulk service; Performance measures; Cost optimization.*

INTRODUCTION

This chapter deals with the performance evaluation and cost optimization of fuzzy bulk queueing models under parametric constraints. In many real-life queueing situations, arrival and service parameters are uncertain in nature and cannot be represented precisely by classical queueing models. Therefore, fuzzy set theory is incorporated into the queueing system to handle such imprecision and uncertainty effectively. The study analyzes various performance measures such as average queue length, waiting time, utilization factor and total expected cost for bulk queueing models using fuzzy parameters. Numerical computations and graphical representations are also presented to observe the effect of different arrival and service rates on system performance and cost optimization. The proposed fuzzy queueing approach provides a more flexible and realistic analysis for designing efficient service systems under uncertain environments.

Recent studies in queueing theory have focused on the analysis of bulk service systems, setup times and fuzzy environments to improve system performance under uncertainty. Sharma and Kumar (2021) studied fuzzy queueing models with uncertain arrival and service rates and analyzed various performance measures using fuzzy set techniques.

Verma et al. (2022) investigated cost optimization in multi-server queueing systems and discussed the effect of service parameters on waiting time and total expected system cost. Thu Le Anah (2023) analyzed multi-server queueing systems with batch services and setup times and examined the impact of setup delay distributions on system performance. Singh and Gupta (2024) developed fuzzy bulk queueing models under parametric constraints and demonstrated that fuzzy approaches provide more flexible and realistic solutions than classical queueing models.

Recently, several researchers have focused on performance evaluation and cost analysis of fuzzy queueing systems to achieve better system efficiency under uncertain environments. Motivated by these studies, the present chapter discusses the performance evaluation and cost optimization of fuzzy bulk queueing models under parametric constraints.

Patel and Mehta (2020) studied bulk queueing systems with fuzzy service rates and analyzed system performance under uncertain environments.Rao and Singh (2022) investigated cost optimization in fuzzy multi-server queueing models and discussed the effect of setup times on waiting behavior.Gupta et al. (2025) developed a fuzzy bulk service queueing model under parametric constraints and evaluated various performance measures using numerical analysis.

Applying Linear cost structure, the optimal value. of average re-orientation time have been obtained graphically.

The queueing system may be described as follows:

1. Units arrive singly with fuzzy Poisson arrival rate $\check{\lambda}$.

2. The queue discipline follows FCFS policy.

3 .The server follows the control policy such that after becoming idle, it is re-activated only when N-units accumulate in the queue. The server requires a fuzzy exponentially distributed re-orientation period before starting the service.

3. The service occurs under general bulk service rule.

4. The cost structure is considered under fuzzy parametric constraints for performance fuzzy optimization rule i.e. at each service initiation, the server can take no fewer than 'a' and not more than 'b' customers at a time.

Fuzzy service time distribution is arbitrary with density function $\check{d}$(x) and

$\check{d}(x) = \breve{n}(x) \exp[-\int_0^x \breve{n}(x)\, \check{d}x]$

where $\breve{n}(x)\, \check{d}x$ is the first order fuzzy probability that the service will be completed in interval (x, x+dx) if it has not been completed up to x.

Fuzzy solution of the model

Define,

Q_n (t) =fuzzy probability that at time t, there are n units in the queue and the server is idle (n = 0,1,2,3,....)

$P_{n,n}(x,t)$ = fuzzy prob. that at time t, there are n units in the queue, x time has elapsed since the last service and the server is in operative state(n,n=0,1,2,3,...)

The ayson governed by the following equations:

⇨ $Q'_0(t) = -a\, Q_b(t) + \int_0^\infty P_0(x,t)\, n(x)\, dx$ (6.1)

$Q'_n(t) = -a\, Q_n(t) + a\, Q_{n-1}(t) + \int_0^\infty P_{n,n}(x,t)\, n(x)\, dx \quad n \le a-1$ ……………6.2

(6.2)

$Q'_n(t) = -a\, Q_{n-1}(t)$ $\quad a \le n \le N$ ……………………………………………..6.3

$Q'_n(t) = -(a+\emptyset)\, Q_n(t)\, a + Q_{n-1}(t)$ $\quad n \ge N$ ……………………………………………6.4

$\frac{\partial}{\partial t} P_0(x,t) + \frac{\partial}{\partial x} P_0(x,t) = -(a + n(x))\, P_0(x,t)$ ………………………………………6.5

$\frac{\partial}{\partial t} P_n(x,t) + \frac{\partial}{\partial x} P^n(x,t) = -(a + n(x))\, P_n(,t) + a\, P_{n-1}(x,t)$ …………………………..6.6

With the boundary conditions

$P_0(0,t) = \sum_{j=a}^{b} \int_0^{\propto} P_j(x,t)\, n(x)\, dx$ ………………………………………………6.7

$P_n(0,t) = \int_0^{\propto} P_{n+b}(x,t)\, n(x)\, dx,$ ……………………………………………………..6.8

$n \le N-b-1$

$P_n(0,t) = \int_0^{\propto} P_{n+b}(x,t)\, n(x)\, dx + \emptyset\, Q_{n+b}(t)$ ……………………………………………6.9

$n \ge N-b$

Let the initial condition be

$Q_0(0) = 1,\ Q_n(0) = 0,\ P_n(x,0) = 0$ …………………………………………6.10

for each x and n .

Define the fuzzy probability generating functions

$Q(z,t) = \sum_{n=0}^{\propto} Q_n(t)\, z^n$ ……………………………………………………………………………6.11

$P(z,x,t) = \sum_{n=0}^{\propto} P_n(x,t)\, z^n$ ……………………………………6.12

and

$P(z,t) = \int_0^{\propto} P(z,x,t)\, dx$ ……………………………………………..6.13

using (6.5) , (6.6) and (6.12),

$\frac{\partial}{\partial x} \bar{P}(z,x,s) + [s + a(1-z) + n(x)]\, \bar{P}(z,x,s) = 0$ …………………………6.14

(2.14)

which is a linear differential equation and its solution is

$\bar{P}(z,x,s) = \bar{P}(z,o,s) \exp[-hx - \int_0^{x} n(x)\, dx]$……………………………..6.15

(2.15)

where h = s + a(1-z) and $\bar{P}(z,x,s)$ is the Laplace Transform of

P(z, x, t) with respect to t.

Similarly from (6.1) to (6.4) and (6.11), we get

$(s + a + \emptyset - az)\, \bar{Q}(z,s) = 1 + \emptyset \sum_{n=0}^{n-1} Q_n(s)\, z^n + \sum_{n=0}^{a-1} \int_0^{\propto} \bar{P}_n(x,s)\, n(x)\, dx$

Which gives

$$\bar{Q}(z,s) = \frac{1 + \emptyset \sum_{n=0}^{N-1} Q_n(s)\, z^n + \sum_{n=0}^{a-1} \int_0^{\propto} \bar{P}_n(x,s)\, n(x)\, dx}{s + a + \emptyset - az}$$ ……………………..6.16

From (6.7) to (6.9), (6.11) and (6.12), we get

$(z^b - \bar{D}(h))\, P(z,o,s) = \sum_{j=a}^{b-1} z^j \int_0^{\propto} P_j(x,s)\, n(x)\, dx$

$- \sum_{j=0}^{b-1} z^j \int_0^{\propto} P_j(x,s)\, n(x)\, dx + Q(z,s) - \sum_{j=0}^{n-1} Q_j(s)\, z^j$

This gives

$\sum_{j=a}^{b-1} (z^b - z^j) \int_0^{\propto} \bar{P}_j(x,s)\, n(x)\, dx$

$- \sum_{j=0}^{a-1} z^j \int_0^{\propto} \bar{P}_j(x,s)\, n(x)\, dx + \emptyset\, \bar{Q}(z,s)$

International Journal of Advance Research in Science and Engineering
Volume No. 15, Issue No. 01, January 2026
www.ijarse.com
IJARSE
ISSN 2319 - 8354

$$\bar{P}(z,0,s) = \frac{-\emptyset \sum_{j=0}^{N-1} \bar{Q}_j(s)\, z^j}{z^b - \bar{D}(h)} \quad \ldots 6.17$$

Since

$\bar{P}(z,s) = \int_0^{\infty} \bar{P}(z, x, s)\, dx.$

Therefore

$$\bar{P}(z,s) = \frac{1-\bar{D}(h)}{h} \bar{P}(z,0,s) \quad \ldots 6.18$$

where $h = s + a(1-z)$.

Also recursively from (6.1) to (6.6), we get

$$Q_n(s) = \left(\frac{a}{s+a}\right)^n Q_0(s) + \sum_{r=1}^{n} \frac{an-r}{(s+a)n-r+1} . Ar \quad \ldots 6.19$$

$$1 \leq n \leq a-1$$

$$Q_n(s) = \left(\frac{a}{s+a}\right)^{n-a+1} Q_{a-1}(s)$$

$$a \leq n \leq N-1 \quad \ldots 6.20$$

where $Ar = \int_0^{\infty} Pr(x,s)\, n(x)\, dx$.

Apply Rouche's Theorem to the denominator in (6.17), it can be shown that it has 'b' zeros, inside and on the unit circle. Since P(z, s) is convergent in $|z| \leq 1$, the numerator in (6.17) must also vanish for these 'b' zeros. Thus we get.'b' equations from which the value of 'b' unknown can be

determined.

International Journal of Advance Research in Science and Engineering
Volume No. 15, Issue No. 01, January 2026
www.ijarse.com
IJARSE
ISSN 2319 - 8354

Steady state solution under fuzzy enviorment:

Steady state solutions can be obtained by using property of Laplace transform,

$$\lim_{s->0} s\,\bar{f}(s) = \lim_{t->\infty} f(t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6.21)$$

if the left side exists.

using (6.10) in (6.16), (6.17), and (6.18), we get

$$Q(z) = \frac{\emptyset \sum_{n=0}^{n-1} Q_n z^n + \sum_{n=0}^{a-1} Q \int_0^{\propto} pn(x)\, n(x)\, Dx}{a + \emptyset - az} \quad \ldots\ldots\ldots\ldots 6.22$$

$$P(z,0) = \frac{\sum_{j=a}^{b-1} (z^b - z^j) \int_0^{\propto} P^j(x)\, n(x)\, dx - \sum_{j=0}^{a-1} z^j \int_0^{\propto} P^j(x) / (x) + \emptyset\, Q(z) - \emptyset \sum_{j=0}^{N-1} Q_j z^j}{z^b - \bar{D}(h)} \quad \ldots\ldots\ldots\ldots 6.23$$

and

$$P(z) = \frac{[1 - \bar{D}(h)] \left[\sum_{j=a}^{b-1} (z^b - z^j) \int_0^{\propto} P_j(x)\, n(x)\, dx - \sum_{j=0}^{a-1} z^j \int_0^{\propto} P_j(x)\, n(x)\, dx + \emptyset\, Q(z) - \emptyset \sum_{j=0}^{N-1} Q_j z^j \right]}{[a(1-z)]\,[z^b - \bar{D}(h)]} \quad \ldots\ldots\ldots\ldots 6.24$$

Fuzzy operational characteristics:

Mean queue length,

Let $L_{q\,1}$ and $L_{q\,2}$ denote the mean queue lengths of the system when the server is in operating and idle state respectively, we have

International Journal of Advance Research in Science and Engineering
Volume No. 15, Issue No. 01, January 2026
www.ijarse.com
IJARSE
ISSN 2319 - 8354

$L_{Q1} = p'(1)/P(1)$

$$P'(1) = \frac{(N''_1(1)\, N'_2(1) + N'_1(1)\, N''_2(1))\, D'_2(1) - N'_1(1)\, N'_2(1)\, D''_2(1)}{D'(1)\, [D'_2(1)]^2} \quad \ldots\ldots\ldots.6.25$$

$L_{Q2} = Q'(1)/Q(1)$

$$Q(z) = \frac{\emptyset \sum_{n=0}^{n-1} n Q_n + (\emptyset \sum_{n=0}^{n-1} Q_n + \sum_{n=0}^{a-1} 0 \int_0^{\propto} pn(x)\, n(x)\, Dx)a}{(a + \emptyset)^2} \quad \ldots\ldots\ldots\ldots.6.26$$

And $Lq = Lq1\ P(1) + Lq2\ Q(1)$

Where

$a'_1(1) = -a\,E(s)$

$N''_1(1) = -a^2\ E(s^2)$

$D'_1(1) = -a$

$D'_2(1) = b - a\,E(s)$

$D''_2(1 = b(b-1) - a^2E(s^2)$

$N'_2(1) = \sum_{j=a}^{b-1}(b-j)\, a_j \sum_{j=o}^{a-1} j a_j + \emptyset Q'(1) - \emptyset \sum_{j=a}^{n-1}(j Q_j$

$N''_2(1) = \sum_{j=a}^{b-1}(b(-1)j(j-1)\, a_j - \sum_{j=o}^{b-1} j(j-1)a_j$

$(1) - \emptyset - \sum_{j=o}^{n-1} j(j-1)Q_j$

(11) Proportions of be the sever remains in operating and idle state

Let E_1 and E_2be the proportions of the time the server remains operative and idle state, then

$$E_1 = P(1) = \frac{E(s)\left(\sum_{j=a}^{b-1}(b-j)\ a_j - \sum_{j=o}^{a-1} j a_j + \emptyset\ q'(1)\ \emptyset \sum_{j=o}^{n-1} j n_j\right.}{(B + a\,E(s))} \quad \ldots\ldots.6.27$$

$$E_2 = q(1) = \frac{\emptyset \sum_{n=0}^{n-1} Q_n + \sum_{n=0}^{a-1} \int_o^{\propto} p_n(x)\,dx}{\emptyset} \quad \ldots\ldots\ldots\ldots6.28$$

(111) Rate of change of server from idle to Fuzzy operating state

If R_{ib} denote the rate of change of server from idle to fuzzy operative state than

$$R_{ib} = \emptyset \sum_{n=n}^{\propto} q_n$$

$$= \emptyset\left(q(1) - \sum_{j=0}^{n-1} q_j\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots6.29$$

Particular cases

The following special cases are discussed under fuzzy parametric assumption

1) When service occurs singly. Putting a = b =1 in (6.22), (6.24), (6.27) and (6.28), we get

$$Q(z) = \frac{\emptyset \sum_{n=0}^{n-1} q_n z^n + \int_0^{\propto} p_o(x)\, n(x)\,dx}{\emptyset + a(1-z)} \quad \ldots\ldots\ldots\ldots6.30$$

$$P(z) = \frac{(1-\bar{d}(h))\left(\emptyset\, q(z) - \emptyset \sum_{j=o}^{n-1} q_j z^j - \int_o^{\propto} p_o(x)\, n(x)\,dx\right)}{(a(1-z))\ (z-\bar{d}(h))} \quad \ldots\ldots.6.31$$

$$p'(1) = \frac{aE(s^2)\left(\emptyset\, q'(1) - \emptyset \sum_{j=o}^{n-1} j\, q_j\right) + E(s)\left(\emptyset\, q''(1) - \emptyset \sum_{j=o}^{n-1} j\,(j-1)\, q_j\right)(1-ae(s)) - E(s)\left(\emptyset\, q'(1) - \emptyset \sum_{j=o}^{n-1} j\, q_j\right)}{(1 - a\,e(s))^2}$$

(6.32)

$$Q'(1) = \frac{\emptyset \sum_{n=o}^{n-1}(n\emptyset + a)\, q_n + a\int_o^{\propto} p(x)\, n(x)\, dx}{\emptyset^2}$$

(6.33)

$$E_1 = \frac{E(s)\ (\emptyset q_1(1) - \emptyset \sum_{j=o}^{n-1} j\, q_j)}{1 - ae(s)}$$

(6.34)

$$E_2 = \frac{\emptyset \sum_{n=o}^{n-1} q_n + \int_o^{\propto} p_o(x)\, n(x)\, dx}{\emptyset}$$

(6.35)

11) when the service time distribution is exponential and service occurs under general bulk service rule i.e.

$D(x) = \beta e^{-\beta x}$

$\bar{D}(s) = \frac{\beta}{\beta+s}$

$E(s) = \frac{1}{\beta}$

$E(s^2) = \frac{2}{\beta^2}$

$$Q(z) = \frac{\beta \sum_{n=o}^{a-1} p_n z^n + \emptyset \sum_{n=o}^{a-1} q_n z^n}{A(1-z) + \emptyset}$$

(6.36)

$$P(z) = \frac{\beta \sum_{j=b}^{b-1} (z^b - z^j)\, p_j - \beta \sum_{j=o}^{a-1} z_j + \emptyset\, q(z) - \emptyset \sum_{j=o}^{n-1} z^j q_j}{(a+\beta) z^b - z^{b+1} - \beta}$$

(6.37)

$$(\beta \sum_{j=o}^{b-1} (b-1) - (j-1)\, p_j - \beta \sum_{j=o}^{a-1} j\,(j-1)\, p_j + \emptyset\, q''(1)$$

$$= p'(1) = \frac{\emptyset \sum_{j=o}^{n-1} j\ (j-1)\ q_j)\ (\beta b - a) - (\beta \sum_{j=a}^{b-1} (b-j)\ p_j - \beta \sum_{j=o}^{a-1} j p_j + \emptyset\ q'(1) - \emptyset \sum_{j=o}^{n-1} j\ q_j)\ (b(b-1)\ \beta_2 - 3a^2)}{2\ (\beta b - a)^2} \quad (6.38)$$

$$= Q'(1) = \frac{\emptyset\ (\sum_{n=o}^{n-1} (n\emptyset + a)\ q)\ \beta \sum_{n=o}^{a-1} p_n}{\emptyset^2} \quad (6.39)$$

$$E_1 = q(1) = \frac{\emptyset \sum_{n=o}^{n-1} q_n + \beta \sum_{n=o}^{a-1} pn}{\emptyset}$$

$$E_2 = p\ (1) = \frac{\sum_{j=a}^{b-1} (b-j)\ \beta\ p_j - \beta \sum_{j=o}^{a-1} j p_j + \emptyset\ q'(1) - \emptyset \sum_{j=o}^{n-1} jq}{(b\beta - a)} \quad (6.40)$$

$$Lq = p'(1) + q'(1)$$

Further if the service occur single i.e. a=b= 1, we get

$$P(z) = \frac{\emptyset\ q(z) - \emptyset \sum_{j=o}^{n-1} z^j - \beta p_o}{(a+\beta)\ z - az^2 - \beta}$$

$$Q(z) = \frac{\beta\ p_o + \emptyset \sum_{j=o}^{n-1} q_j\ z^j}{A\ (1-z) + \emptyset}$$

$$p'(1) = \frac{(\emptyset\ q''(1) - \emptyset \sum_{j=o}^{n-1} j\ (j-1)\ q_j)\ (\beta - a) + 2\ a^2\ \emptyset\ (Q'(1) - \sum_{j=0}^{N-1} jQ_j)}{2(\beta - a)^2}$$

$$q'(1) = \frac{\emptyset \sum_{n=o}^{n-1} (n\emptyset + a) a\beta\ p_o}{\emptyset^2}$$

$$E_1 = \frac{\emptyset \sum_{n=o}^{n-1} q_n + \beta\ p_o}{\emptyset}$$

$$E_2 = \frac{\emptyset\ (q'(1) - \sum_{j=o}^{n-1} j_j)}{(a=b)}$$

(6.41)

Further if n = 1

$$P(z) = \frac{\emptyset\ q\ (z) - (\emptyset + a)\ q_o}{(a+\beta)\ z - az^2 - \beta}$$

$$Q(z) = \frac{(a+\emptyset)q_o}{\emptyset + a(\ 1-z)}$$

$$p'(1) = \frac{a\ q_o\ (\emptyset + a\ )}{o^2}$$

$$q'(1) = \frac{\emptyset(\ (\beta - \varepsilon)\ q''(1) + 2\varepsilon^2\ q'\ (1)}{2\ ()\beta - a)^2}$$

$$E_1 = \frac{(a+\emptyset)\ q_o}{\emptyset}$$

$$E_2 = \frac{(a+\emptyset)\ q_o}{(a+\beta)}$$

(6 .42)

Numerical results and cost analysis

For numerical analysis, the arrival rate parameter λ is considered as a triangular fuzzy number represented by $\check{\lambda}$, where 0.2, 0.4 and 0.6 denote the lower, modal and upper values respectively. The fuzzy numerical values are used to study the behavior of queue length and operational characteristics under uncertain conditions.

We consider the following cost structure

(1) $E_{1=}$ Start up cost of the server
(2) E_2 = Shut down cost of the server
(3) C_1 = Cost per unit time the server remains in fuzzy operating state
(4) C_2 = cost per unit time the server reminder in idle etc
(5) Ch = waiting cost (in the queue) per unit time

IJARSE
ISSN 2319 - 8354

The total cost per unit time will be

$C(\emptyset) = (k_1+k_2)\, n_2 + E_1 c_1 + E_2 c_2 + ch.\, L_q$

The values of various operational characteristics are computed for single service queueing system for different values of the parameters λ, β and ϕ. The behavior of mean queue length and total expected cost is illustrated through tables and graphical representations as shown in Figures 6.1 to 6.5 and Tables 6.1 to 6.5. It is observed from the graphs and tables that as the service rate increases, the mean queue length decreases, whereas the mean queue length increases with increase in arrival rate. The queue length is directly proportional to the average re-orientation time. Under different cost structures, the optimal values of ϕ for various parameters are also computed and analyzed using fuzzy numerical values represented by triangular fuzzy numbers.It is observed from the graphical analysis that an increase in service rate improves the optimal cost structure, while the idle period cost also increases correspondingly.

Table 6.1 : Service Rate (μ_2) Vs Queue Length (L_q) under Triangular Fuzzy Arrival Rate

λ_1	λ_2	λ_3	μ_2	A	L_q
0.2	0.4	0.6	0.2	2.5	8.069
0.2	0.4	0.6	0.3	2.5	6.052
0.2	0.4	0.6	0.4	2.5	4.841
0.2	0.4	0.6	0.5	2.5	4.034

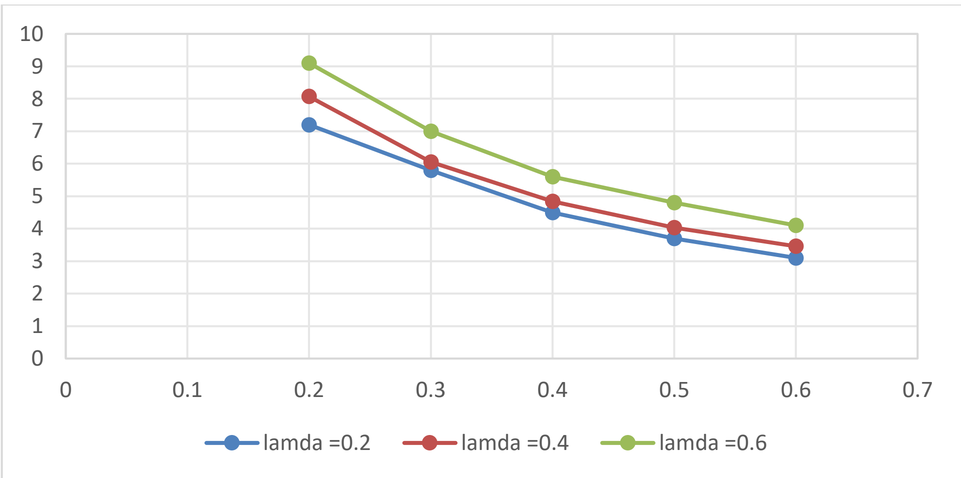


Figure 6.1

Table 6.2 :Service (μ) vs Queue Length (L_q) for triangualr fuzzy arrivale rate values.

λ_1	λ_2	λ_3	μ_2	A	L_q
0.2	0.4	0.6	0.2	1.25	7.852
0.2	0.4	0.6	0.3	1.25	5.634
0.2	0.4	0.6	0.4	1.25	4.126
0.2	0.4	0.6	0.5	1.25	3.281

Figure 6.2 Service (μ) vs Queue Length (L_q) for triangualr fuzzy arrivale rate values.

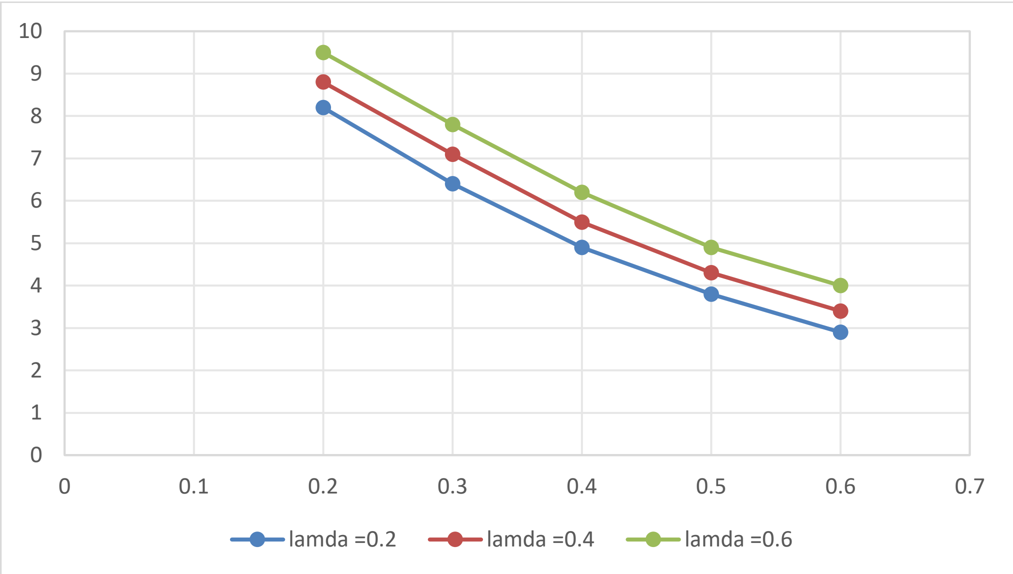
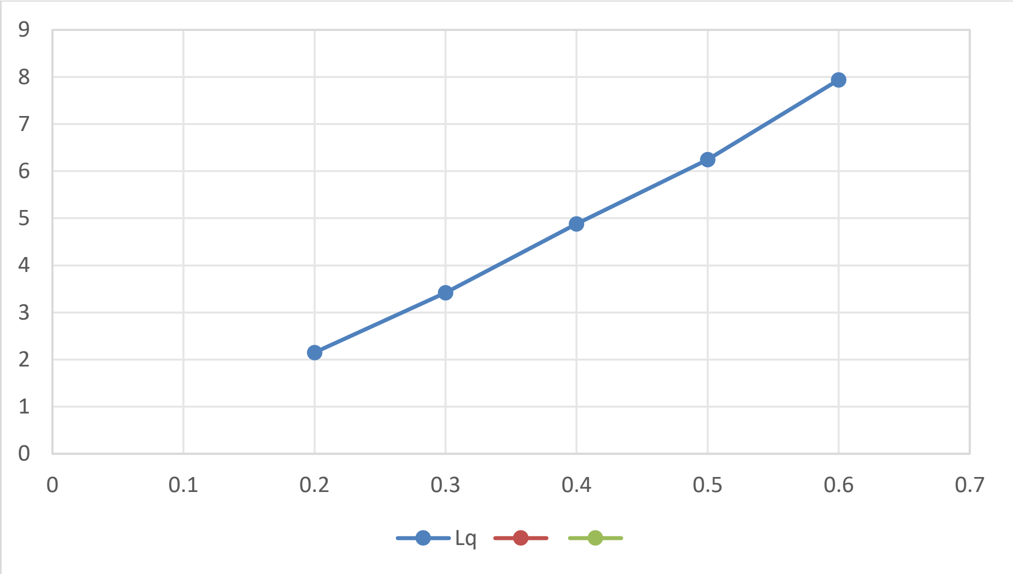


Table 6.3 :Fuzzy Arrival Rate (λ) vs Mean Queue Length (L_q).

λ_1	λ_2	λ_3	μ_2	A	L_q	
0.2	0.4	0.6	0.2	1.20	2.15	
0.2	0.4	0.6	0.3	1.20	3.42	
0.2	0.4	0.6	0.4	1.20	4.88	
0.2	0.4	0.6	0.5	1.20	6.25	

IJARSE
ISSN 2319 - 8354

Figures 6.3 :Fuzzy Arrival Rate (λ) vs Mean Queue Length (L_q).



Conclusion :

In this chapter, a fuzzy bulk service queueing model with re-orientation time has been analyzed using probability generating functions and supplementary variable technique. The arrival rate parameter was considered as a triangular fuzzy number in order to study the uncertainty present in practical queueing systems. Various operational characteristics such as mean queue length, server operating state probabilities and cost analysis were obtained analytically.

Numerical results and graphical representations were presented for different fuzzy values of the arrival rate parameter. It is observed that the mean queue length increases with increase in arrival rate and decreases with increase in service rate. The proposed fuzzy queueing approach provides a more realistic and flexible analysis of queueing systems under uncertain conditions.

REFERENCES :

[1] Negi, D. S., & Lee, E. S. (1992). Analysis and simulation of fuzzy queues. Fuzzy sets and systems, 46(3), 321-330.

[2] Kao, C., Li, C. C., & Chen, S. P. (1999). Parametric programming to the analysis of fuzzy queues. Fuzzy sets and systems, 107(1), 93-100.

[3] Buckley, J. J., Feuring, T., & Hayashi, Y. (2001). Fuzzy queueing theory revisited. International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 9(05), 527-537.

[4] S.P. Chen, Solving fuzzy queueing decision problems via a parametric mixed integer nonlinear programming method, European Journal of Operational Research 177 (1) (2007) 445–457.

[5] de La Fuente, D., & Pardo, M. J. (2009, December). Development of queueing models with balking and uncertain data using Fuzzy Set Theory. In 2009 IEEE International Conference on Industrial Engineering and Engineering Management (pp. 380-384). IEEE.

[6] Nagarajan, Solairaju. Computing Improved fuzzy optimal Hungarian assignment problems with fuzzy costs under Robust Ranking Techniques, 2010; 6(13):6-13.

[7] Srinivasan, R. (2014). Fuzzy queueing model using DSW algorithm. International Journal of Advanced Research in Mathematics and Applications, 1(1), 57-62 Technology

[8] Y. Ma, Z. Liu and Z. G. Zhang, Equilibrium in vacation queueing system with complementary services, Quality & Quantitative 10.1080/16843703.2016.1191172. Management, 14 (2017), 114-127. doi:

[9] Julia Rose Mary K, Pavithra J. Analysis of FM/M(a,b)/1/MWV queueing model. International Journal of Innovative Research in Science Engineering and Technology. 2016; 5(2):1391-1397.

[10] Mittal, H. (2022). Modeling of communication network with queuing theory under fuzzy environment. Mathematical statistician and engineering applications.