Queuing Analysis of Machine Interference Systems with Fuzzy Parameters

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Abstract:

Machine interference problems occur in manufacturing systems where several machines are maintained by a limited number of repair persons. When a machine fails, it must wait for service, which creates a queue. In real-life situations, system parameters such as machine failure rate, repair rate, and service time are often uncertain and cannot be represented accurately using exact numerical values. To handle this uncertainty, fuzzy logic can be used. In this study, a machine interference queuing model under a fuzzy environment is developed and analyzed. Fuzzy Triangular numbers are used to represent uncertain system parameters, making the model more realistic and practical. The proposed model evaluates important performance measures such as the average number of failed machines, average waiting time, system utilization, and repairman idle time under fuzzy conditions. The results show that the fuzzy queuing approach provides a better understanding of system behavior when precise data is not available. This model helps decision-makers analyze machine repair systems more effectively and support improved planning and maintenance strategies in industrial environments.

Keywords: *Queuing model, Fuzzy logic, Machine repair system, Performance analysis, Triangular Fuzzy number.*

1.1 Introduction: Queuing models are used widely in both service organizations and manufacturing companies, where different customers are served by different types of servers according to specific rules. In traditional queuing theory, the times between arrivals and the service times are assumed to follow known probability distributions. But in real-life situations, these values are often described in vague or linguistic terms like "fast," "slow,   or" "moderate," which are better represented using fuzzy sets. By applying the parametric method to fuzzy queue models, this research aims to offer a simple, flexible, and realistic" framework for studying uncertain queuing situations. The results can help decision-makers better understand how uncertainty affects system performance and plan more efficient service strategies in real-life environments. Machine interference

queue models are very important in many real-life systems such as manufacturing units, textile industries, grinding workshops, service centers, and client–server computing systems. In these systems, machines may stop working due to breakdowns and require repair services. Important performance measures of such systems include the machine breakdown rate, service rate, loss of production, and repair rate. In practical situations, these parameters cannot always be measured accurately because of uncertain and uncontrollable factors such as machine condition, operator skill, and environmental effects. Due to this uncertainty, the system parameters behave in a fuzzy manner rather than having fixed values. To handle this uncer38tainty, this chapter proposes a method in which the breakdown rate and service rate are represented as triangular fuzzy numbers. The α-cut technique and fuzzy arithmetic operations are used to obtain the membership functions of various system performance measures. The proposed fuzzy model is simple and useful for practitioners.

1.2 Problem Description:

The machine interference problem is an important application of queuing models with a finite calling population, such as the M/M/W: N/FCFS model, where N is the number of machines and W is the number of servers, N > W. In this model, one or more maintenance workers or servers are responsible for repairing machines when they break down. If workers work independently on different machines, they are treated as separate servers in the queuing system. However, if all repairmen work together as a team on one machine at a time, the entire group is considered as a single server.

Most researchers have derived system performance measures for machine interference problems and their variants by assuming that the system parameters are stochastic but precisely known. In such studies, parameters like breakdown rate, service rate, and loss of production rate are treated as random variables with exact probability distributions. For instance, Zadeh [1965, 1978] introduced the concept of fuzziness. Gross and Harris [1981] analyzed a machine repair model with a heterogeneous population under a stochastic environment. Li & lee [1989] investigated the analytical result for two special queues M/F/1/∞ and FM/FM/1/∞ where F denotes the fuzzy time and FM denotes the Fuzzified exponential distribution. Negi and Li [1992] proposed a procedure using α- cut. Chen S.H. et. al [1996] introduced functional principle in fuzzy inventory model.

1.3. FUZZY SET THEORY:

The concept of fuzzy sets was first introduced by Lotfi A. Zadeh in 1965. Unlike classical (crisp) set theory, where an element either belongs to a set or does not belong to it, fuzzy set theory allows partial membership. This means that an element can belong to a set with a certain degree of membership ranging between 0 and 1.

1**.3.1. Definition of a Fuzzy Set**

Let X is a universe of discourse. A fuzzy set $\tilde{A}$ in X is defined as:

$\tilde{A}$= {(x, $\mu_{\tilde{A}}(x)$), x ∈ X}

Where, $\mu_{\tilde{A}}(x)$ is the membership function of the fuzzy set $\tilde{A}$, and $0 \leq \mu_{\tilde{A}}(x) \leq 1$

The membership function represents the degree to which an element belongs to the fuzzy set.

1.3.2. Membership Functions

A membership function describes how each element in the universe of discourse is mapped to a membership value between 0 and 1. Commonly used membership functions include:

- Triangular fuzzy numbers
- Trapezoidal fuzzy numbers
- Gaussian fuzzy numbers
- Bell shaped

In queuing and machine interference problems, triangular and trapezoidal fuzzy numbers are widely used because of their simplicity and ease of computation. These functions provide flexibility in modeling uncertain parameters such as arrival rate, service rate, or failure rate in queuing and reliability systems. In this paper, we mainly use triangular fuzzy number to solve the problem.

1.3.3. Triangular Fuzzy Number (TFN)

A triangular fuzzy number $\tilde{A} = (a, b, c)$ is defined by the fuzzy number:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0, & x > c \end{cases}$$

Here,

- a = lower bound,
- b = most probable value,
- c = upper bound

1.3.4. α-Cut of a Fuzzy Set

An α-cut of a fuzzy set $\tilde{A}$ is a crisp interval containing all elements whose membership degree is at least α:

$A^{(\alpha)} = \{x \in X, \quad \mu_{\tilde{A}}(x) \geq \alpha\}$, for $0 \leq \alpha \leq 1$

α-cuts are very useful for performing arithmetic operations on fuzzy numbers and are widely used in fuzzy queuing models.

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1.3.5. Multiple Identical Servers (M/M/c-type fuzzy system)

- Classical Case: Multiple parallel servers, each with the same service rate.
- Fuzzy Case: Service capacity (number of servers) or the rate per server could be fuzzy. Load balancing decisions can be fuzzy if the system dynamically chooses which server gets the next customer.
- Server condition: If some servers are unreliable, the failure rate or mean time to repair can be expressed using fuzzy numbers. This impacts effective capacity.

1.4. Model Description: Finite Source Machine Repair Model

Consider a system consisting of N machines serviced by W workers. Each machine operates independently and may break down randomly. When a machine breaks down, it   immediately joins the repair facility, resulting in a loss of one unit of production. A machine under repair cannot generate further breakdown calls.

Thus, the system represents a finite calling source model with a maximum of N potential customers (machines). After repair, the machine returns to a good operating condition and again becomes susceptible to breakdown.

1.4.1. Fuzzy Assumptions

- $\tilde{\lambda}$: The breakdown rate per machine is denoted by $\tilde{\lambda}$ (arrival rate) and is represented as a triangular fuzzy number.
- $\tilde{\mu}$: The repair rate per repairman is denoted by $\tilde{\mu}$ (service rate) and is also represented as a triangular fuzzy number.
- The operating (up-time) and repair (down-time) durations are assumed to follow exponential distributions with fuzzy rates $\tilde{\lambda} and \tilde{\mu}$, respectively.
- $\rho = \frac{\tilde{\lambda}}{\tilde{\mu}}$ (Fuzzy busy time of the server)
- 'k' be the number of failed machines (customers) in the system at time 't'.

1.4.2. Effective Arrival Rates (State-Dependent)

The fuzzy arrival rate in state 'k' is

$$\tilde{\lambda}_k \ = \ (\text{N-k}) \ \tilde{\lambda} \ , \qquad \text{for,} \quad \text{k} = 0,1,2,\ldots,\text{N}$$

Product of Arrival Rates up to State k is:

$$\prod_{i=0}^{k-1} \tilde{\lambda}_i = \prod_{i=0}^{k-1}(N - i) \ \tilde{\lambda}$$

This represents the cumulative effect of breakdowns as the system moves from

state 0 to state k.

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Expanding:

$$\prod_{i=0}^{k-1}(N-i)\,\tilde{\lambda} = \tilde{\lambda}^k \text{ (N) (N-1) (N-2) } \ldots\ldots \text{ (N – K + 1)}$$

1.4.3. Service (Repair) Rate

The effective service rate depends on the number of available repairmen:

$$\widetilde{\mu}_k = \begin{cases} k.\widetilde{\mu}\ , & 1 \leq\ k{<}W \\ W.\widetilde{\mu}\ , & k\ >W \end{cases}$$

1.4.4. Transient State Analysis:

Let P_k(t) denote the possibility measure that there are 'k' failed machines in the system at time 't'. The system behavior is governed by the following fuzzy differential–difference equations.

Case 1:   k = 0

$$\frac{dP_0(t)}{dt} =\ -\tilde{\lambda}_0 P_0(t) +\ \widetilde{\mu}_1 P_1(t) \qquad\qquad\qquad \ldots\ldots\ldots\ldots(1.1)$$

Case 2: $1 \leq k\ \leq W-1$

$$\frac{dP_k(t)}{dt} = \text{(N-k+1) } \tilde{\lambda}_{k-1}P_{k-1}(t) - [\ (\ N-k)\tilde{\lambda}_k + k\ \widetilde{\mu}_k]P_k(t) + \text{(k+1) } \widetilde{\mu}_{k+1}P_{k+1}(t) \quad \ldots(1.2)$$

Case 3: $W \leq k\ \leq N-1$

$$\frac{dP_k(t)}{dt} = \text{(N-k+1) } \tilde{\lambda}_{k-1}P_{k-1}(t) - [\ (\ N-k)\widetilde{\tilde{\lambda}_k} + W\ \widetilde{\mu}_k]\ P_k(t) + \text{W } \widetilde{\mu}_{k+1}P_{k+1}(t)$$
$$\ldots\ldots\ldots\ldots\ldots(1.3)$$

Case 4: $k = N$

$$\frac{dP_N(t)}{dt} = \tilde{\lambda}_{N-1}P_{N-1}(t)-\ W\widetilde{\mu}_N P_N(t) \qquad\qquad\qquad \ldots\ldots\ldots\ldots(1.4)$$

Initial Condition:

At time, t = 0, the system is assumed to be empty:

$$P_0(0) = 1\ \ and\ \ \ P_k(0) = 0\ , for\ k \geq 1 \qquad\qquad\qquad \ldots\ldots\ldots\ldots(1.5)$$

1.4.5 IN THE STEADY STATE, the system probabilities become independent of time. Hence,

$$\frac{dP_k(t)}{dt} = 0 \, , k = 0,1,2, \dots \dots . N \qquad \qquad \dots\dots\dots\dots\dots..(1.6)$$

Let the steady-state possibility measure be denoted by:

$$P_k \; = \; \lim_{t\to\infty} P_k(t) \qquad\qquad \dots\dots\dots\dots..(1.7)$$

1.4.6. Steady-State Balance Equations:

Case 1: k = 0

$$\tilde{\lambda}_0 P_0 = \tilde{\mu}_1 P_1 \qquad\qquad \dots\dots\dots\dots(1.8)$$

$$\tilde{\lambda}_{k-1} P_{k-1} = \tilde{\mu}_k P_k \qquad \text{for k = 1,2,3…….N}$$

Case 2: $1 \le k \le W - 1$

$$(N - k + 1) \, \tilde{\lambda}_{k-1} P_{k-1} = [(N - k) \, \tilde{\lambda}_k + k \, \tilde{\mu}_k] \, P_k - (k +1) \, \tilde{\mu}_{k+1} P_{k+1} \qquad \dots\dots\dots\dots.(1.9)$$

Case 3: $W \le k \le N$

$$(N - k + 1) \, \tilde{\lambda}_{k-1} P_{k-1} = [(N - k) \, \tilde{\lambda}_k + W \, \tilde{\mu}_k] \, P_k - W \, \tilde{\mu}_{k+1} P_{k+1} \qquad \dots\dots\dots\dots..(1.10)$$

Solution of these equations (1.8), (1.9), (1.10) is:

$$P_k = \begin{cases} \frac{(N)_k \rho^k}{k!} P_0 \, , & 0 \le k < W \\ \frac{(N)_k \rho^k}{W! \, W^{k-W}} P_0, & W \, \le k \, \le N \end{cases} \qquad \dots\dots\dots\dots.(1.11)$$

Where, $(N)_k =$ N (N-1) (N-2) .... (N-k+1)

The normalization condition is: $\sum_{k=0}^{N} P_k \; = 1$, which determines P_0 ….…….(1.12)

1.4.7 SYSTEM PERFORMANCE MEASURES:

- Expected Number of Failed Machines (out of order) is L_q

    $$L_q = \sum_{k=0}^{N} k P_k$$

- Average number of machine waiting for service is W_q
- Expected Number of Operating Machines

$$\dot{M} = N - \dot{L}_q$$

- Machine Availability

$$\tilde{A} = \frac{N - \dot{L}_q}{N}$$

- $\widetilde{\lambda_{ef}}$ is the effective arrival rate

1.5. Numerical Illustration:

To demonstrate the applicability of the proposed finite-source fuzzy machine repair model, a numerical example is presented in this section.

Consider a production system consisting of four identical machines operated simultaneously. Each machine is subject to random breakdowns. When a machine fails, it immediately enters the repair facility and remains non-operational until repair completion. A failed machine does not generate further breakdown calls. After repair, the machine returns to service and becomes susceptible to breakdown again.

The repair facility consists of two servicemen, and each serviceman can repair only one machine at a time. When more than two machines are out of order, the excess machines must wait in a queue until a serviceman becomes available. Thus, the system represents a finite calling source machine repair model.

Given that:

- Total number of machines: N = 4
- Number of servicemen: W = 2
- Fuzzy breakdown rate per machine (triangular fuzzy number): $\tilde{\lambda} = (1, 2.5, 4)$

- Fuzzy repair rate per serviceman (triangular fuzzy number): $\tilde{\mu} = (5, 6.5, 8)$

Fuzzy traffic intensity: $\tilde{\rho} = \frac{\tilde{\lambda}}{W \tilde{\mu}} = \frac{(1,2.5,4)}{2 (5,6.5,8)} = (0.0625, 0.1923, 0.4)$

1.5.1. Steady-State probability distribution:

Let k = number of machines out of order

$$P_k = P_0 \prod_{i=1}^{k} \frac{(N-i+1)\tilde{\lambda}}{\min(i,W)\tilde{\mu}} \qquad \text{................(1.13)}$$

$$P_0 = \left[1 + \sum_{k=1}^{4} \prod_{i=1}^{k} \frac{(N-i+1)\tilde{\lambda}}{\min(i,2)\tilde{\mu}}\right]^{-1} \qquad \text{.................(1.14)}$$

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Using (1.13), (1.14), we get,

$$\left.\begin{array}{l} P_0 = (0.62, 0.51, 0.18) \\ P_1 = (0.24, 0.26, 0.28) \\ P_2 = (0.09, 0.15, 0.22) \\ P_3 = (0.03, 0.06, 0.12) \\ P_4 = (0.01, 0.02, 0.08) \end{array}\right\} \qquad \ldots\ldots\ldots\ldots\ldots(1.15)$$

1.5.2. Average number of machine out of order is:

$$L_q = \sum_{k=0}^{4} k\, P_k \qquad\qquad \ldots\ldots\ldots\ldots..(1.16)$$

$$L_q = 1\, P_1 + 2\, P_2 + 3\, P_3 + 4\, P_4 \qquad\qquad \ldots\ldots\ldots\ldots\ldots(1.17)$$

Using (1.p), (1.q), we get,

$$L_q = (0.84, 1.12, 1.54) \qquad\qquad \ldots\ldots\ldots\ldots..(1.18)$$

1.5.3. Average waiting time before repair start is:

$$W_q = \frac{L_q}{\tilde{\lambda}_{ef}}$$

When only 2 machines wait, $L_q = P_3 + 2 P_4 = (0.05, 0.10, 0.28)$ $\qquad$ ....…….…(1.19)

And $\tilde{\lambda}_{ef} = (1, 2.5, 4)$

Hence, $W_q = (0.0125, 0.04, 0.28)$

1.5.4. α-cuts of Input Parameters

(1.5.4)(a) Breakdown rate

$\tilde{\lambda} = (1, 2.5, 4)$
$\tilde{\lambda}^{(\alpha)} = [\, 1 + 1.5\alpha,\ 4 - 1.5\alpha\,]$

(1.5.4)(b) Repair rate

$\tilde{\mu} = (5, 6.5, 8)$
$\tilde{\mu}^{(\alpha)} = [5 + 1.5\alpha,\ 8 - 1.5\alpha\,]$

(1.5.4)(c) Fuzzy traffic intensity

$\rho = (0.0625, 0.1923, 0.4)$

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$\rho^{(\alpha)} = [0.0625 + 0.1298\alpha, \ 0.4 - 0.2077\alpha]$

1.5.5. α-cuts of Steady-State Probabilities

$P_0 = (0.62, 0.51, 0.18)$

$$\left.\begin{aligned}
P_0^{(\alpha)} &= [0.62 - 0.11\alpha, \ 0.18 + 0.33\alpha] \\
P_1^{(\alpha)} &= [0.24 + 0.02\alpha, \ 0.28 - 0.02\alpha] \\
P_2^{(\alpha)} &= [0.09 + 0.06\alpha, 0.22 - 0.07\alpha] \\
P_3^{(\alpha)} &= [0.03 + 0.03\alpha, 0.12 - 0.06\alpha] \\
P_4^{(\alpha)} &= [0.01 + 0.01\alpha, 0.08 - 0.06\alpha]
\end{aligned}\right\} \qquad \text{………(1.20)}$$

1.5.6. α-cut of Average Number of Machines Out of Order

$L_q = (0.84, 1.12, 1.54)$

$L_q^{(\alpha)} = [0.84 + 0.28\alpha, \ 1.54 - 0.42\alpha]$

1.5.7 α-cut of Average Waiting Queue Length (when more than two machines are down)

$L_{q,w} = (0.05, 0.10, 0.28)$

$L_{q,w}^{(\alpha)} = [0.05 + 0.05\alpha, \ 0.28 - 0.18\alpha]$

1.5.8. α-cut of Average Waiting Time before Repair

$W_q = (0.0125, 0.04, 0.28)$

$W_q^{(\alpha)} = [0.0125 + 0.0275\alpha, \ 0.28 - 0.24\alpha]$

The numerical illustration clearly demonstrates that the proposed finite-source fuzzy machine repair model effectively captures the uncertainty inherent in breakdown and repair processes. The fuzzy performance measures provide a range of possible system behaviors rather than single crisp values, offering greater flexibility and realism in decision-making for production and maintenance planning. This highlights the practical usefulness of the proposed model in real-world manufacturing environments where precise parameter estimation is often difficult.
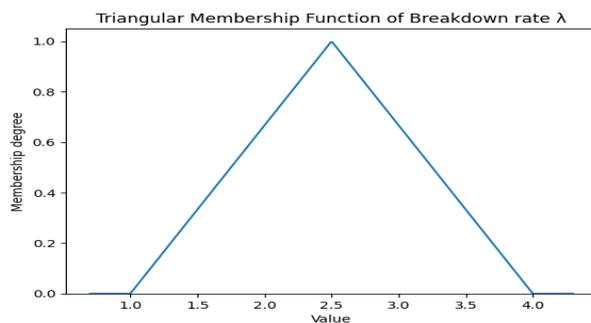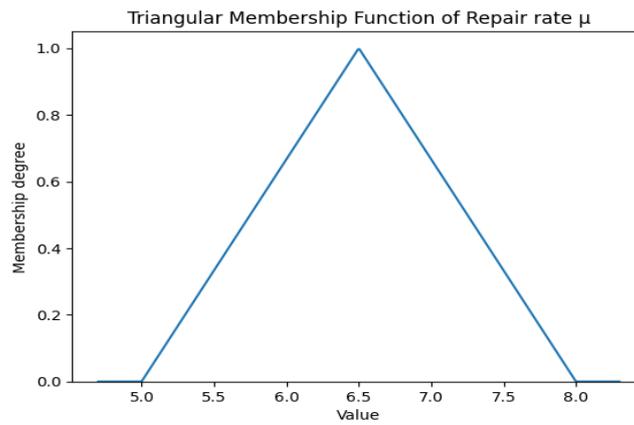

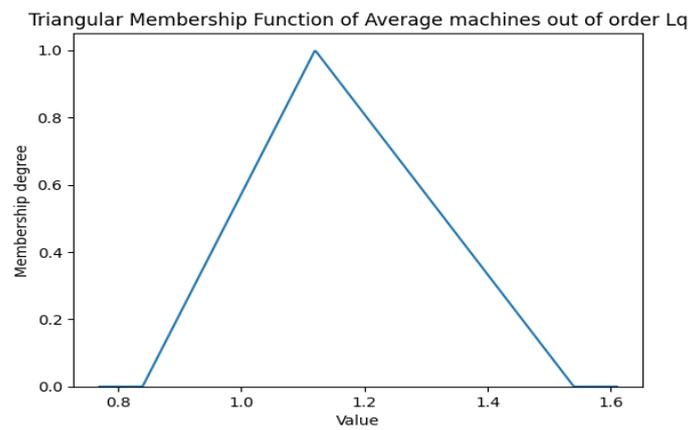Figure: (1.5.4)(a)

Figure: (1.5.4)(b)

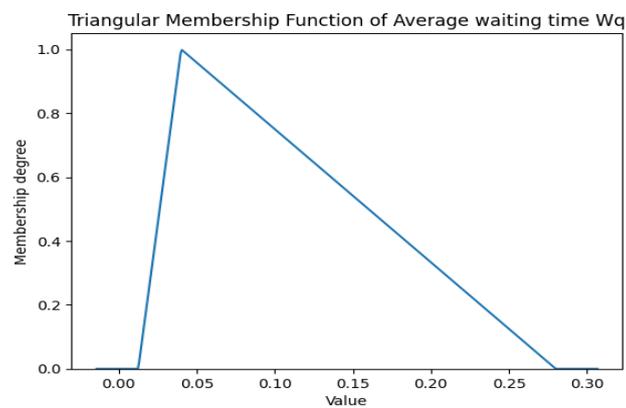Figure: 1.5.2 (Average Machine out of order, L_q)

Figure: 1.5.8 (Average Waiting Time W_q)

1.6. Conclusion: This paper developed a finite-source fuzzy machine repair model supported by a complete analytical framework. The system behavior was characterized through well-defined steady-state balance equations, from which fuzzy state probabilities were derived using normalization equations. Performance measures such as the average number of machines out of order, average waiting queue length, and average waiting time were explicitly obtained using summation-based expectation equations. Furthermore, α-cut formulations were systematically applied to all input parameters and output measures, converting fuzzy numbers into interval-valued equations for sensitivity analysis. Hence, the proposed equation-based fuzzy modeling approach is mathematically sound and practically valuable for maintenance planning under uncertainty.

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