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Availability Optimization of Complex Systems with Correlated Maintenance and Failure Durations

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Abstract:

In this research paper we investigates the availability optimization of complex repairable systems subject to statistically correlated maintenance and failure durations. Unlike classical reliability models that assume independence between breakdown and repair processes, the proposed framework incorporates dependence structures to better represent real-world operational behavior. A stochastic modeling approach based on semi-Markov processes and copula-based joint distributions is developed to capture the correlation between failure and maintenance times. Steady-state availability expressions are derived, and key performance indices are formulated under various maintenance strategies, including preventive and corrective policies. An optimization model is constructed to determine optimal maintenance scheduling and resource allocation that maximize system availability while minimizing downtime and operational costs. Numerical experiments demonstrate the significant impact of correlation on system performance and highlight the potential losses arising from independence assumptions. The results provide practical insights for decision-makers in manufacturing, power, and industrial systems seeking to enhance reliability and operational efficiency under different conditions.

Keywords: System Availability, Standby System, Imperfect Switching, Sequential Repair, Reliability Modeling, Backup Failure, Two-Stage Repair.

2.1. INTRODUCTION

A huge amount of literature is available in the field of reliability theory on the analysis of two unit priority system models. Various authors including [28,29,57,61,113] have analyzed two unit system models assuming failure and repair times of the units as independently distributed random variables. But it has been found in many practical situations that failure and repair times are correlated random variables. With this concept of correlated failure and repair times various authors including [28,57,61] considered system models assuming bivariate exponential distribution of failure and repair times. Further in most of the system models authors have assumed that the machine device used for repairing the failed units, remains good forever. But in real life situations this assumption is not practicable and the repair machine may also fail during its working process. In case of nuclear reactors and marine equipments, robots are used for repair purposes and a robot again being a machine may also fail while performing its intended task. The concept of repair machine was introduced by Gupta and Chaudhary [45] in a two unit cold standby system with independent failure and repair times.

In the present study we investigate and analyze a two non-identical unit parallel system model with a repair machine having correlated failure and repair times of units. System model under investigation consists of two units one is priority and the other is non priority unit. Initially both the units work and repair machine is in good condition. Repair machine is used to repair the failed unit but if during the repair of units, repair machine fails then the repair of failed unit is discontinued and repair machine is taken up for its repair, and after the repair of repair machine the repair of failed unit is done afresh. The repair machine is given preventive maintenance after a random period of operation except when both the units are in failure mode. The priority unit is given preference in repair over non-priority unit. The random period of operation after which the repair machine is given preventive maintenance and the time of completion of preventive maintenance are independent exponential variates whereas failure and repair times of both the units are correlated random variables having joint density of the form

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$f_i(x,y) = \lambda_i\mu\,(1-r_i)e^{-(\lambda_i x+\mu_i y)} I_0\,(2\sqrt{\lambda_i\mu\, r_i xy})\;\; ; x, y, \mu, \lambda_i > 0, 0 \le r_i < 1$

Where,

$I_0(2\sqrt{\lambda_i\mu_i r_i xy})$ is modified Bessel function of type one and order zero and is defined as

$I_0(z) = \sum_{k=0}^{\infty} \frac{(z/2)^k}{(k!)^2}$

By using regenerative point technique the following measures of systems effectiveness are obtained:

1. Reliability of the system and Mean time to system failure.
2. Expected up time of the system (0, t) and in steady state.
3. Expected busy period of the repair machine and repairman.
4. Expected number of repairs by repair machine and repairman.
5. Net expected profit earned by the system in (0, t) and in steady state.

2.2. SYSTEM DESCRIPTION AND ASSUMPTIONS

1. The system consists of two non –identical unit working in parallel form named as priority and non-priority unit with a repair machine (R.M.).
2. Initially both the units are working.
3. Both the units having two modes – Normal (N) and Total failure (F).
4. The Repair machine repairs a failed unit but during repair it can also fail. In such a situation, the repair of a failed unit is discontinued and the repairman starts the repair of R.M. as a single repairman is always available with system.
5. After a random period of time preventive maintenance is given to R.M. but when both the units are failed then preventive maintenance is not given to R.M.

6. The failure and repair times of both the priority and non-priority units are taken to be correlated random variables having bivariate exponential distribution.

7. The failure and repair times of R.M. are taken to be exponential distribution with different parameters.

8. The random period of operation after which preventive maintenance is given to R.M. and period of completion of preventive maintenance both are exponentially distributed random variables with different parameters.

2.3. NOTATIONS AND SYMBOLS

$X_i (i = 1,2)$: Random variables representing failure time of priority/ non-priority unit.

$Y_i (i = 1,2)$: Random variables representing repair time of priority/ non- priority unit.

$f_i(x, y)$: Joint p.d.f of (X_i, Y_i), given by

$$= \lambda_i \mu (1 - r_i) e^{-(\lambda_i x + \mu_i y)} I_0(2\sqrt{\lambda_i \mu r_i xy}); x, y, \mu_i, \lambda_i > 0, 0 \le r_i < 1, i = 1,2$$

$g_i(x)$: Marginal p.d.f of X_i, given by

$$= \lambda_i (1 - r_i) e^{-\lambda_i (1-r_i) x} ; x, \lambda_i > 0, 0 \le r_i < 1, i = 1,2$$

$h_i(y)$: Marginal p.d.f of Y_i, given by

$$= \mu (1 - r_i) e^{-\mu_i (1-r_i) y} ; y, \mu > 0, 0 \le r_i < 1, i = 1,2$$

$k_i(y|x)$: Conditional p.d.f of Y_i given $X_i = x$

$$= \mu_i e^{-(\mu_i y + \lambda_i r_i x)} I_0(2\sqrt{\lambda_i \mu_i r_i xy}) ;$$

$$x, y, \mu_i, \lambda_i > 0, 0 \le r_i < 1, i = 1,2$$

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α_1 : Rate of giving preventive maintenance to Repair machine

β_1 : Rate of completion of preventive maintenance

α_2 : Failure rate of Repair machine

β_2 : Repair rate of Repair machine

SYMBOLS FOR THE STATES OF THE SYSTEM

N_{10}/N_{20} : Priority / Non-Priority unit in normal mode and operative.

F_{1r}/F_{2r} : Priority / Non-Priority unit under repair.

F_{1w}/F_{2w} : Priority / Non-Priority unit waiting for repair.

RM_0/RM_r : Repair machine operative / under repair.

RM_g : Repair machine in good condition and non- functioning.

RM_P : Repair machine under preventive maintenance. With the help of the above

symbols the possible states of the system are:

$S_0 = [N_{10}, N_{20}, RM_g]$ $S_1 = [F_{1r}, N_{20}, RM_0]$

$S_2 = [N_{10}, F_{2r}, RM_0]$ $S_3 = [N_{10}, N_{20}, RM_P]$

$S_4 = [F_{1w}, N_{20}, RM_r]$ $S_5 = [F_{1r}, F_{2w}, RM_0]$

$S_6 = [F_{1w}, N_{20}, RM_P]$ $S_7 = [N_{10}, F_{2w}, RM_r]$

$S_8 = [F_{1w}, F_{2w}, RM_r]$ $S_9 = [N_{10}, F_{2w}, RM_P]$

$S_{10} = [F_{1w}, F_{2w}, RM_P]$

The transition diagram along with all the transitions is shown in Fig.2.1.

TRANSITION DIAGRAM

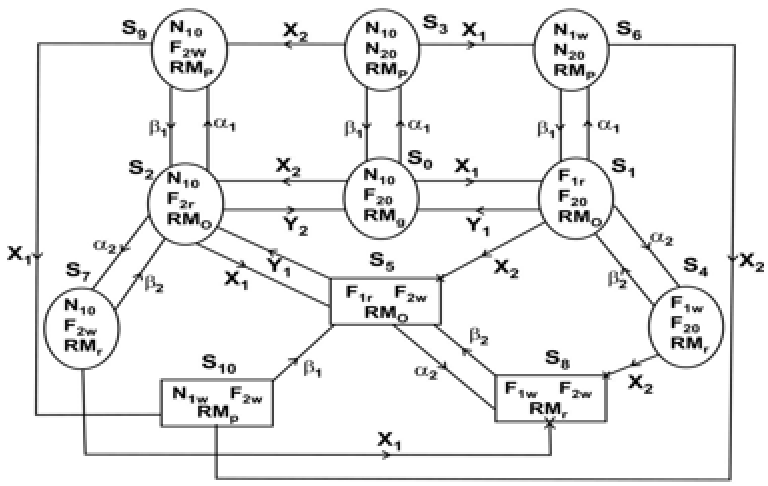

Fig. 2.1

2.4. TRANSITION PROBABILITIES AND SOJOURN TIMES

Let $T_0(\equiv 0)$, T_1, T_2 .... denotes the regenerative epochs and X_n denotes the state visited at epoch T_{n^+} i.e just after the transition at T_n . Then $\{X_n, T_n\}$ constitute a Markov-Renewal process with state space E, set of regenerative states and

$$Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \le t | X_n = i]$$

is the semi Markov kernel over E.

Then the transition probability matrix of the embedded Markov chain is

$$P = (p_{ij}) = (Q_{ij}(\infty)) = (Q(\infty)) \qquad \textbf{(1)}$$

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(a) The various conditional direct and indirect transition probabilities may be obtained as follows:

$$Q_{10|x}(t) = \int_0^t dK_1(u|x)\, e^{-[\alpha_1+\alpha_2+\lambda_2(1-r_2)]u}$$

$$Q_{14|x}(t) = \alpha_2 \int_0^t e^{-[\alpha_1+\alpha_2+\lambda_2(1-r_2)]u}\, \bar{K}_1(u|x)\, du$$

$$Q_{16|x}(t) = \alpha_1 \int_0^t e^{-[\alpha_1+\alpha_2+\lambda_2(1-r_2)]u}\, \bar{K}_1(u|x)\, du$$

$$Q^{(5)}_{12|x}(t) = \lambda_2(1-r_2) \int_0^t e^{-[\alpha_1+\alpha_2+\lambda_2(1-r_2)]u}\, \bar{K}_1(u|x)\, du \int_u^t \frac{dK_1(v|x)\, e^{-\alpha_2 v}}{\bar{K}_1(u|x)\, e^{-\alpha_2 u}}$$

$$- \frac{\lambda_2(1-r_2)}{[\alpha_1+\lambda_2(1-r_2)]} \int_0^t e^{-\alpha_2 v}\left[1 - e^{-[\alpha_1+\lambda_2(1-r_2)]v}\right] dK_1(v|x)$$

$$Q^{(5)}_{18|x}(t) = \lambda_2(1-r_2) \int_0^t e^{-[\alpha_1+\alpha_2+\lambda_2(1-r_2)]u}\, \bar{K}_1(u|x)\, du \int_u^t \frac{\alpha_2 e^{-\alpha_2 v}\, \bar{K}_1(v|x)}{\bar{K}_1(u|x)\, e^{-\alpha_2 u}}$$

$$- \frac{\alpha_2\lambda_2(1-r_2)}{[\alpha_1+\lambda_2(1-r_2)]} \int_0^t e^{-\alpha_2 v}\left[1 - e^{-[\alpha_1+\lambda_2(1-r_2)]v}\right] \bar{K}_1(v|x)$$

$$Q_{20|x}(t) = \int_0^t dK_2(u|x)\, e^{-[\alpha_1+\alpha_2+\lambda_1(1-r_1)]u}$$

$$Q_{25|x}(t) = \lambda_1(1-r_1) \int_0^t e^{-[\alpha_1+\alpha_2+\lambda_1(1-r_1)]u}\, \bar{K}_2(u|x)\, du$$

$$Q_{27|x}(t) = \alpha_2 \int_0^t e^{-[\alpha_1+\alpha_2+\lambda_1(1-r_1)]u} \bar{K}_2(u|x)\, du$$

$$Q_{29|x}(t) = \alpha_1 \int_0^t e^{-[\alpha_1+\alpha_2+\lambda_1(1-r_1)]u} \bar{K}_2(u|x)\, du$$

$$Q_{52|x}(t) = \int_0^t d\, K_1(u|x) e^{-\alpha_2 u}$$

$$Q_{58|x}(t) = \alpha_2 \int_0^t e^{-\alpha_2 u} \bar{K}_1(u|x)\, du$$

Unconditional transition probabilities are:

$$Q_{01}(t) = \lambda_1(1-r_1) \int_0^t e^{-[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]u}\, du$$

$$= \frac{\lambda_1(1-r_1)}{[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}[1 - e^{-[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]t}]$$

$$Q_{02}(t) = \lambda_2(1-r_2) \int_0^t e^{-[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]u}\, du$$

$$= \frac{\lambda_2(1-r_2)}{[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}[1 - e^{-[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]t}]$$

$$Q_{03}(t) = \alpha_1 \int_0^t e^{-[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]u}\, du$$

$$= \frac{\alpha_1}{[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}[1 - e^{-[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]t}]$$

$$Q_{30}(t) = \beta_1 \int_0^t e^{-[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]u}\, du$$

$$= \frac{\beta_1}{[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}[1 - e^{-[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]t}]$$

$$Q_{36}(t) = \lambda_1(1-r_1) \int_0^t e^{-[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]u}\, du$$

$$= \frac{\lambda_1(1-r_1)}{[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}[1 - e^{-[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]t}]$$

$$Q_{39}(t) = \lambda_2(1-r_2) \int_0^t e^{-[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]u}\, du$$

$$= \frac{\lambda_2(1-r_2)}{[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}[1 - e^{-[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]t}]$$

$$Q_{41}(t) = \beta_2 \int_0^t e^{-[\beta_2+\lambda_2(1-r_2)]u}\, du = \frac{\beta_2}{[\beta_2+\lambda_2(1-r_2)]}[1 - e^{-[\beta_2+\lambda_2(1-r_2)]t}]$$

$$Q_{45}^{(8)}(t) = \lambda_2(1-r_2)\int_0^t e^{-[\beta_2+\lambda_2(1-r_2)]u}du \int_u^t \beta_2 e^{-\beta_2(v-u)}dv$$

$$= \beta_2 \int_0^t e^{-\beta_2 v}[1 - e^{-\lambda_2(1-r_2)v}]dv$$

$$Q_{61}(t) = \beta_1 \int_0^t e^{-[\beta_1+\lambda_2(1-r_2)]u}\, du = \frac{\beta_1}{[\beta_1+\lambda_2(1-r_2)]}[1 - e^{-[\beta_1+\lambda_2(1-r_2)]t}]$$

$$Q_{6,10}(t) = \lambda_2(1-r_2) \int_0^t e^{-[\beta_1+\lambda_2(1-r_2)]u}\, du = \frac{\lambda_2(1-r_2)}{[\beta_1+\lambda_2(1-r_2)]}[1 - e^{-[\beta_1+\lambda_2(1-r_2)]t}]$$

$$Q_{72}(t) = \beta_2 \int_0^t e^{-[\beta_2+\lambda_1(1-r_1)]u}\, du = \frac{\beta_2}{[\beta_2+\lambda_1(1-r_1)]}[1 - e^{-[\beta_2+\lambda_1(1-r_1)]t}]$$

$$Q_{75}^{(8)}(t) = \lambda_1(1-r_1)\int_0^t e^{-[\beta_2+\lambda_1(1-r_1)]u}du \int_u^t \beta_2 e^{-\beta_2(v-u)}\, dv$$

$$= \beta_2 \int_0^t e^{-\beta_2 v}[1 - e^{-\lambda_1(1-r_1)v}]dv$$

$$Q_{85}(t) = \beta_2 \int_0^t e^{-\beta_2 u}\, du = 1 - e^{-\beta_2 t}$$

$$Q_{92}(t) = \beta_1 \int_0^t e^{-[\beta_1+\lambda_1(1-r_1)]u}\, du = \frac{\beta_1}{[\beta_1+\lambda_1(1-r_1)]}[1 - e^{-[\beta_1+\lambda_1(1-r_1)]t}]$$

$$Q_{9,10}(t) = \lambda_1(1-r_1) \int_0^t e^{-[\beta_1+\lambda_1(1-r_1)]u}\, du = \frac{\lambda_1(1-r_1)}{[\beta_1+\lambda_1(1-r_1)]}[1 - e^{-[\beta_1+\lambda_1(1-r_1)]t}]$$

$$Q_{10,5}(t) = \beta_1 \int_0^t e^{-\beta_1 u}\, du = 1 - e^{-\beta_1 t} \qquad \textbf{(2-28)}$$

(b) Steady state probabilities:

By taking the limit as $t \to \infty$, we obtain the following steady state transition probabilities:

First we find the following conditional direct and indirect steady-state probabilities of transition:

$$p_{10|x} = \int dK_1(u|x)e^{-[\alpha_1+\alpha_2+\lambda_2(1-r_2)]u} = k^*[\alpha_1 + \alpha_2 + \lambda_2(1-r_2)|x]$$

$$p_{12|x}^{(5)} = \frac{\lambda_2(1-r_2)}{[\alpha_1+\lambda_2(1-r_2)]}\int dK_1(v|x)e^{-\alpha_2 v}[1 - e^{-[\alpha_1+\alpha_2+\lambda_2(1-r_2)]v}]$$

$$-\frac{\lambda_2(1-r_2)}{[\alpha_1+\lambda_2(1-r_2)]}[k^*(\alpha_2|x) - k^*(\alpha_1+\alpha_2+\lambda_2(1-r_2)|x)]$$

Similarly,

$$p_{14|x} = \frac{\alpha_2}{[\alpha_1+\alpha_2+\lambda_2(1-r_2)]}[1 - k^*(\alpha_1+\alpha_2+\lambda_2(1-r_2)|x)]$$

$$p_{16|x} = \frac{\alpha_1}{[\alpha_1+\alpha_2+\lambda_2(1-r_2)]}[1 - k^*(\alpha_1+\alpha_2+\lambda_2(1-r_2)|x)]$$

$$p^{(5)}_{18|x} = \frac{\lambda_2(1-r_2)}{[\alpha_1+\lambda_2(1-r_2)]}[1 - k^*(\alpha_2|x)] - \frac{\lambda_2(1-r_2)}{[\alpha_1+\lambda_2(1-r_2)][\alpha_1+\alpha_2+\lambda_2(1-r_2)]}[1 - k^*(\alpha_1+\alpha_2+\lambda_2(1-r_2)|x)]$$

$$p_{20|x} = k^*[\alpha_1+\alpha_2+\lambda_1(1-r_1)|x]$$

$$p_{25|x} = \frac{\lambda_1(1-r_1)}{[\alpha_1+\alpha_2+\lambda_1(1-r_1)]}[1 - k^*(\alpha_1+\alpha_2+\lambda_1(1-r_1)|x)]$$

$$p_{27|x} = \frac{\alpha_2}{[\alpha_1+\alpha_2+\lambda_1(1-r_1)]}[1 - k^*(\alpha_1+\alpha_2+\lambda_1(1-r_1)|x)]$$

$$p_{29|x} = \frac{\alpha_1}{[\alpha_1+\alpha_2+\lambda_1(1-r_1)]}[1 - k^*(\alpha_1+\alpha_2+\lambda_1(1-r_1)|x)]$$

$$p_{52|x} = k^*(\alpha_2|x) \quad p_{58|x} = 1 - k^*(\alpha_2|x)$$

Unconditional steady state probabilities of transition are:

$$p_{01}\ p_{03}\ p_{36} = \frac{\lambda_1(1-r_1)}{[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]} \qquad p_{02}\ p_{30}\ p_{39} = \frac{\lambda_2(1-r_2)}{[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}$$

$$= \frac{\alpha_1}{[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]} \qquad = \frac{\beta_1}{[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}$$

$$= \frac{\lambda_1(1-r_1)}{[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]} \qquad = \frac{\lambda_2(1-r_2)}{[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}$$

$$p_{41} = \frac{r_2}{[\beta_2+\lambda_2(1-r_2)]} \qquad p^{(8)}_{45} = 1 - \frac{r_2}{[\beta_2+\lambda_2(1-r_2)]}$$

$$p_{61} = \frac{\beta_1}{[\beta_1+\lambda_2(1-r_2)]} \qquad p_{6,10} = \frac{\lambda_2(1-r_2)}{[\beta_1+\lambda_2(1-r_2)]}$$

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$p_{72} = \frac{\beta_2}{[\beta_2+\lambda_1(1-r_1)]}$ $p_{75}^{(8)} = 1 - \frac{\beta_2}{[\beta_2+\lambda_1(1-r_1)]}$

$p_{92} = \frac{\beta_1}{[\beta_1+\lambda_1(1-r_1)]}$ $p_{9,10} = \frac{\lambda_1(1-r_1)}{[\beta_1+\lambda_1(1-r_1)]}$

$p_{85} = p_{10,5} = 1$

Also,

$p_{10} = \int p_{10|x} g_1(x)dx$

$= \lambda_1(1-r_1) \int k^*(\alpha_1 + \alpha_2 + \lambda_2(1-r_2)|x)e^{-[\lambda_1(1-r_1)]x}dx$

$= \frac{\mu_1(1-r_1)}{[\alpha_1+\alpha_2+\lambda_2(1-r_2)+\mu_1(1-r_1)]}$

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Similarly,

$$p_{14} = \frac{\alpha_2}{[\alpha_1+\alpha_2+\lambda_2(1-r_2)+\mu_1(1-r_1)]} \qquad p_{12}^{(5)} = \frac{\lambda_2\mu_1(1-r_1)(1-r_2)}{[\alpha_2+\mu_1(1-r_1)][\alpha_1+\alpha_2+\lambda_2(1-r_2)+\mu_1(1-r_1)]}$$

$$p_{16} = \frac{\alpha_1}{[\alpha_1+\alpha_2+\lambda_2(1-r_2)+\mu_1(1-r_1)]} \qquad p_{18}^{(5)} = \frac{\alpha_2\lambda_2(1-r_2)}{[\alpha_2+\mu_1(1-r_1)][\alpha_1+\alpha_2+\lambda_2(1-r_2)+\mu_1(1-r_1)]}$$

$$p_{20} = \frac{\mu_2(1-r_2)}{[\alpha_1+\alpha_2+\lambda_1(1-r_1)+\mu_2(1-r_2)]} \qquad p_{25} = \frac{\lambda_1(1-r_1)}{[\alpha_1+\alpha_2+\lambda_1(1-r_1)+\mu_2(1-r_2)]}$$

$$p_{27} = \frac{\alpha_2}{[\alpha_1+\alpha_2+\lambda_1(1-r_1)+\mu_2(1-r_2)]} \qquad p_{29} = \frac{\alpha_1}{[\alpha_1+\alpha_2+\lambda_1(1-r_1)+\mu_2(1-r_2)]}$$

$$p_{52} = \frac{\mu_1(1-r_1)}{[\alpha_2+\mu_1(1-r_1)]} \qquad p_{52} = 1 - \frac{\mu_1(1-r_1)}{[\alpha_2+\mu_1(1-r_1)]} \qquad \textbf{(29-54)}$$

It can be easily seen that the following results hold good:

$$p_{01} + p_{02} + p_{03} = 1 \qquad p_{10} + p_{12}^{(5)} + p_{14} + p_{16} + p_{18}^{(5)} = 1$$

$$p_{20} + p_{25} + p_{27} + p_{29} = 1 \qquad p_{30} + p_{36} + p_{39} = 1$$

$$p_{41} + p_{45}^{(8)} = 1 \qquad p_{52} + p_{58} = 1$$

$$p_{61} + p_{6,10} = 1 \qquad p_{72} + p_{75}^{(8)} = 1$$

$p_{92} + p_{9,10} = 1$ $\qquad p_{85} = p_{10,5} = 1$ **(55-64)**

(c) Mean sojourn times:

The mean sojourn time in state S_i denoted by Ψ_i is defined as the expected time taken by the system in state S_i before transiting to any other state. To obtain mean sojourn time Ψ_i, in state S_i, we observe that as long as the system is in state S_i, there is no transition from S_i to any other state. If T_i denotes the sojourn time in state S_i then mean sojourn time Ψ_i in state S_i is:

$$\Psi_i = E[T_i] = \int P(T_i > t)dt$$

First we obtain the following conditional mean sojourn times:

$$\Psi_{1|x} = \int e^{-[\alpha_1+\alpha_2+\lambda_2(1-r_2)]t}\, \bar{K}_1(t|x)dt$$

$$= \frac{1}{[\alpha_1+\alpha_2+\lambda_2(1-r_2)]}[1 - k_1^*(\alpha_1 + \alpha_2 + \lambda_2(1 - r_2)|x)]$$

$$\Psi_{2|x} = \int e^{-[\alpha_1+\alpha_2+\lambda_1(1-r_1)]t}\, \bar{K}_2(t|x)dt$$

$$= \frac{1}{[\alpha_1+\alpha_2+\lambda_1(1-r_1)]}[1 - k_2^*(\alpha_1 + \alpha_2 + \lambda_1(1 - r_1)|x)]$$

$$\Psi_{5|x} = \int e^{-\alpha_2 t}\, \bar{K}_1(t|x)dt = \frac{1}{\alpha_2}[1 - k_1^*(\alpha_2|x)] \qquad \textbf{(65-67)}$$

Unconditional mean sojourn times are given by

$$\Psi_0 = \int e^{-[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]t}\, dt = \frac{1}{[\alpha_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}$$

$$\Psi_3\ \Psi_4\ \Psi_6\ \Psi_7 \quad = \int e^{-[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]t}\, dt = \frac{1}{[\beta_1+\lambda_1(1-r_1)+\lambda_2(1-r_2)]}$$

$$= \int e^{-[\beta_2+\lambda_2(1-r_2)]t}\, dt = \frac{1}{[\beta_2+\lambda_2(1-r_2)]}$$

$$= \int e^{-[\beta_1+\lambda_2(1-r_2)]t}\, dt = \frac{1}{[\beta_1+\lambda_2(1-r_2)]}$$

$$= \int e^{-[\beta_2+\lambda_1(1-r_1)]t}\, dt = \frac{1}{[\beta_2+\lambda_1(1-r_1)]}$$

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$\Psi_{\beta_2} = \int e^{-\beta_2 t}\, dt = \frac{1}{}$

$\Psi = \int e^{-[\beta_1+\lambda_1(1-r_1)]t}\, dt = \frac{1}{[\beta_1+\lambda_1(1-r_1)]}$

$\Psi_{10} = \int e^{-\beta_1 t}\, dt = \frac{1}{\beta_1}$

Also,

$\Psi_1 = \int \Psi_{1|x}\, g_1(x)\, dx = \lambda_1 (1 - r_1) \int \Psi_{1|x}\, e^{-\lambda_1(1-r_1)u}\, dx = \frac{1}{[\alpha_1+\alpha_2+\mu_1(1-r_1)+\lambda_2(1-r_2)]}$

Similarly,

$\Psi_2 = \int \Psi_{2|x}\, g_2(x)\, dx = \frac{1}{[\alpha_1+\alpha_2+\lambda_1(1-r_1)+\mu_2(1-r_2)]}$

$\Psi_5 = \int \Psi_{5|x}\, g_1(x)\, dx = \frac{1}{[\alpha_2+\mu_1(1-r_1)]}$ **(68-78)**

2.6. GRAPHICAL STUDY OF THE SYSTEM MODEL

For more concrete study of system behavior, we plot MTSF and Profit functions with respect to λ_1 (failure rate of priority unit) for different values of r_1.

Fig. 2.2 shows the variations in MTSF in respect of λ_1 for different values of r_1 as 0.25, 0.50 and 0.75 while the other parameters are fixed as $\lambda_1 = 0.05$, $\mu_1 = 0.8$, $\mu_2 = 0.7$, $\alpha_1 = 0.05$, $\alpha_2 = 0.06$, $\beta_1 = 0.4$, $\beta_2 = 0.5$, $r_2 = 0.50$. It is observed from the graph that MTSF decreases with the increase in the failure parameterλ_1 and increase with the increase in r_1.

Fig 2.3 represents the change in profit function P_1 and P_2 w.r.t. λ_1 for different values of r_1 as 0.25, 0.50 and 0.75 while the other parameters are fixed as $\lambda_1 = 0.05$,

$\mu_1 = 0.8, \mu_2 = 0.7, \alpha_1 = 0.05, \alpha_2 = 0.06, \beta_1 = 0.4, \beta_2 = 0.5, r_2 = 0.50, K_0 = 1000,$
$K_1 = 300$, $K_2 = 250$, $K_3 = 350$, $K_4 = 200$. From the graph it is seen that both profit functions decrease with the increase in failure rate λ_1 and increase with the increase in r_1. It is also observed that profit function P_2 is always higher as compared to profit function P_1 for fixed values of λ_1 and r_1.

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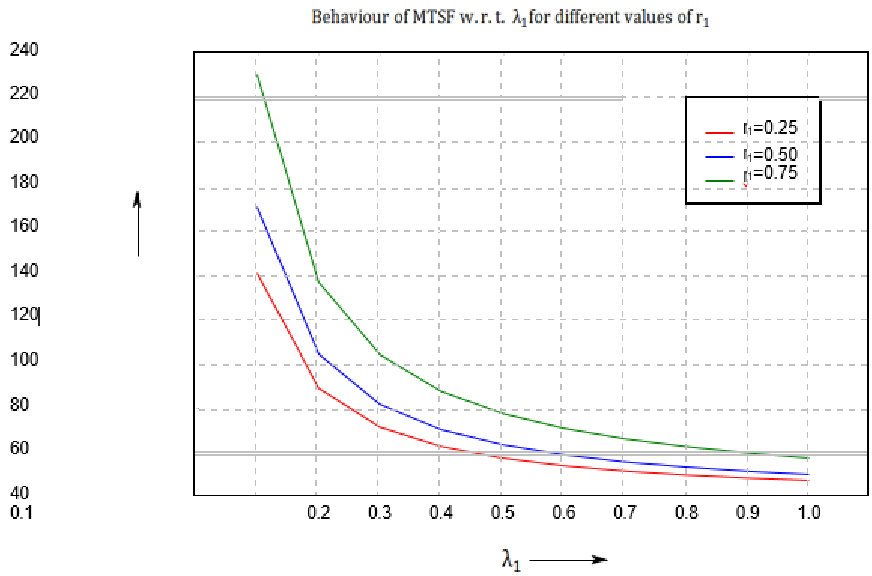

Fig.2.2

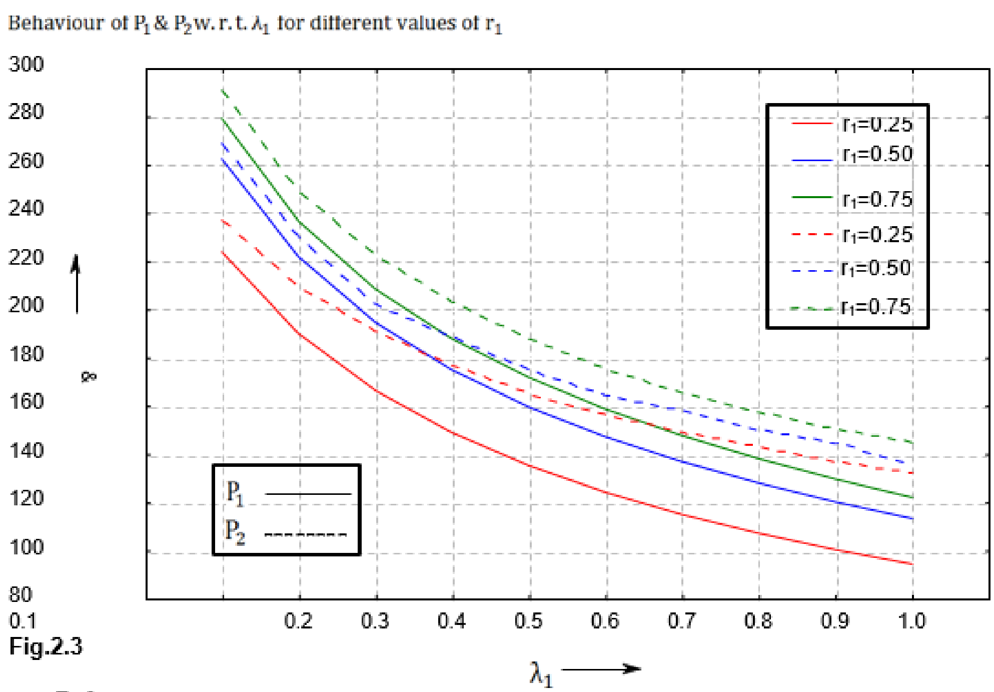

Fig.2.3

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