NEWTON-RAPHSON METHOD FOR SIMULATION OF MULTIPLE-EFFECT EVAPORATOR SYSTEM

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ABSTRACT

In the present Paper, A simplified model has been developed for the analysis of Triple-effect Multiple Effect Evaporator systems. For simplification, it is assumed that Boiling Point elevations are negligible. The model consists of linear equations which are solved using Newton-Raphson method.

Keywords: Simplified Model, Multiple Effect Evaporator, Newton-Raphson Method, Linear Equations

I. INTRODUCTION

Evaporation is an energy (heat)-driven separation process in which part of the solvent from a solution (with non-volatile solute) is removed by vaporization, resulting in a more concentrated solution. A solution containing a desired product is fed into the evaporator and it is heated by a heat source like steam. Because of the applied heat, the water in the solution is converted into vapour and is condensed while the concentrated solution is either removed or fed into a second evaporator for further concentration. The objective can be to concentrate the solution, to regenerate the solvent, or both. In many cases, however, the objective is to concentrate the solution. In these cases the solvent vapor is not regarded as product and it may or may not be recovered. Initially adopted in the production of salt with the aid of solar energy, evaporation has become the major Industrial concentration technique, water being the removed solvent in 99% of its applications. Evaporators are integral part of a number of process industries namely Pulp and Paper, Chlor-alkali, Sugar, Pharmaceuticals, Desalination, Dairy and Food processing, etc.

II. SINGLE EFFECT EVAORATION

In a single-effect operation, as the name implies, only one evaporator is employed. The feed mixture to be separated or concentrated is introduced to the effect. The energy required to evaporate the solvent is supplied by the latent heat of vaporization given up by the steam upon condensing. The feed upon entering this effect must be heated to the boiling point temperature of the effect at the operating pressure. Then the solvent, generally water, is evaporated and removed as a vapor.

III. MULTIPLE EFFECT EVAPORATORS

The vapor driven off the liquid during evaporation contains considerable energy. In an evaporator with just a single effect this vapor is usually passed through a condenser and removed from the system. The energy efficiency of the process can be dramatically increased by reusing this vapor to evaporate the product further in another effect. In order to do this the boiling temperature in the next effect must be sufficiently low to maintain an adequate temperature difference for the transfer of heat. This can be achieved by operating the second effect at a lower pressure. The reuse of the vapor in this way can be extended to a number of effects and is called multiple effect evaporation. For a multiple effect evaporator system the first effect is at the highest temperature and pressure and the last effect is at the lowest temperature and pressure.

In Multiple-Effect Evaporator, the vapor or steam produced in the first effect is introduced to the steam chest of the second effect and thus becomes the heating medium for the second effect. Similarly, the vapor from the second effect becomes the steam for the third effect.

Generally, the pure vapor above a solution is superheated because at a given pressure it condenses at a temperature below the boiling point temperature of the solution. The difference between the boiling point temperature of the solution and the condensation temperature of the vapor at the pressure of the vapor space is called the boiling point elevation of the effect.

To describe evaporator operation, the three terms, capacity, economy and consumption are commonly used. By capacity of an evaporator system is meant by the number of units of solvent evaporated per unit time. The economy of an evaporator system is the total number of units of solvent vaporized per unit of steam fed to the evaporator system. Steam consumption is the units of steam fed to the system per unit time, i.e. is the ratio of capacity to steam consumption.

IV. MODELING OF MULTIPLE EFFECT EVAPORATOR SYSTEMS

The design of evaporators is generally taken to mean the determination of the heat transfer area and the steam consumption required to effect a specified separation at a specified set of steady-state operating conditions.

4.1 Assumptions

1. Here we are describing the modeling equations for a triple effect evaporator system for the case where boiling point elevations are negligible.
2. The effect of composition on liquid enthalpy is neglected.
3. Vapor leaving an effect is at saturated condition.
4. Fouling resistance is negligible.
5. Each unit is perfectly mixed i.e., No pressure and velocity distribution and hence there is no need of momentum balance. The equations so obtained are generalized to include the case where boiling point elevations cannot be neglected.
6. Equal area effects are employed.

In the case of series operation with forward feed, depicted in figure 1, the thick liquor leaving the first effect becomes the feed for the second effect. For each effect added to the system, approximately one additional pound of solvent is evaporated per pound of steam fed to the first effect.

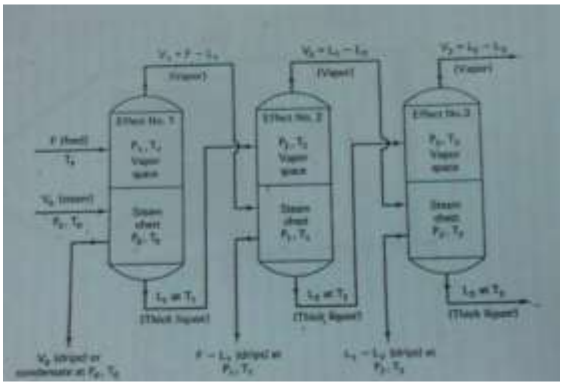

Figure: 1 A Triple- Effect Evaporator with Forward Feed. The Temperature Distribution Shown Is For A System with Negligible Boiling-Point Elevation

4.2 Nomenclature

F = feed rate, kg / hr;

L_j = Thick liquor rate, kg / hr, (where j= 1, 2, 3 for first, second and third effect respectively);

V_j= Vapor rate, kg / hr, (where j= 1, 2, 3 for first, second and third effect respectively);

X = Mass fraction of the solute in the feed;

x = Mass fraction of the solute in the thick liquor;

h_f = Enthalpy of the feed, kJ / kg;

h_j = Enthalpy of the thick liquor at the boiling point temperature of the evaporator , kJ / kg; for first, second and third effect respectively;

H_j = Enthalpy of the vapor at the boiling point of the evaporator, kJ / kg; for first, second and third effect respectively;

Q_j= Rate of heat transfer across the tubes (from the steam to the thick liquour), kJ / hr; for the first, second and third effect respectively

4.3 Model Equations

Effect no. 1

A Total Material Balance is given by:-

$F = V_1 + L_1$ ---- (1)

An enthalpy balance on the process stream yields

$Fh_f + Q – V_1 H_1 – L_1 h_1 = 0$

Since V1 = F – L1 , the previous equation can be rewritten as

$F (h_f – h_1) + Q – (F – L_1) (H_1 – h_1) = 0$ ---(2)

Where $Hj – hj = \lambda_j$

λj = The latent heat of vaporization of the solvent from the thick liquor at temperature T1 and pressure P1 (j = 1 , 2, 3, the effect number)

For the first effect, by Enthalpy balance on the steam,

$Q1 = Vo (Ho – ho) = \lambda o$ ---- (3)

And the rate of heat transfer is given by

$Q1 = U_1 A (To - T_1)$ ---- (4)

By eliminating the value of Q by equation 3, equation 2 can be rewritten as

$F (h_f - h_1) + Vo\ \lambda o - (F – L_1)\ \lambda_1 = 0$ ---- (5)

The equation 4 and 5 may be stated in functional notation as follows

Enthalpy Balance

$f_1 = F (h_f - h_1) + Vo\ \lambda o - (F – L_1)\ \lambda_1$----(6)

Heat Transfer Rate

$f_2 = U_1 A (To - T_1) – Vo\ \lambda 0$ ----(7)

Effect No. 2

By Enthalpy Balance on the steam, $Q_2 = (F – L_1)\ \lambda_1$

The Rate of heat transfer, $Q_2 = U_2 A (T_1 - T_2)$

Hence,Enthalpy Balance

$f_3 = L_1(h_1– h_2) + (F – \lambda_1) – (L_1 – L_2)\ \lambda_2$ ----(8)

Heat Transfer rate

$f_4 = U_2 A (T_1 - T_2) – (F – L_1)\ \lambda_1$ ----(9)

Effect no. 3

By Enthalpy Balance on steam, $Q_3 = (L_1 - L_2)\ \lambda_2$

The Rate of heat transfer, $Q_3 = U_3 A (T_2 - T_3)$

Hence,Enthalpy Balance

$f_5 = L_2(h_2 – h_3) + (L_1 - L_2)\ \lambda_2 - (L_2 - L_3)\ \lambda_3$----(10)

Heat transfer rate

$f_6 = U_3 A (T_2 - T_3) – (L_1 - L_2)\ \lambda_2$----(11)

Component Material Balance

The six independent equations can be solved for the six unknowns. In addition to these six independent equations, three additional equations that contain three additional independent variables x1 , x2, and x3 are given by

$FX - L_j x_j = 0,$ (j = 1, 2, 3) ----(12)

V. USE OF THE NEWTON – RAPHSON METHOD FOR SOLVING EVAPORATOR DESIGN PROBLEMS

The Newton – Raphson method consists of the repeated use of the linear terms of the Taylor series expansions of the functions f_1, f_2, f_3, f_4, f_5 and f_6 [eq (6) – eq (11)] ,i.e,

$$\frac{\partial f_1}{\partial V_o}(\Delta V_o) + \frac{\partial f_1}{\partial T_1}(\Delta T_1) + \frac{\partial f_1}{\partial L_1}(\Delta L_1) + \frac{\partial f_1}{\partial T_2}(\Delta T_2) + \frac{\partial f_1}{\partial L_2}(\Delta L_2) + \frac{\partial f_1}{\partial A}(\Delta A) = 0$$

----(13)

In equation 13, $\Delta V_o = Y_{o,k+1} - Y_{o,k}$, $\Delta T_1 = T_{1,k+1} - T_{1,k}$, $\Delta T_2 = T_{2,k+1} - T_{2,k}$, $\Delta L_1 = L_{1,k+1} - L_{1,k}$, $\Delta L_2 = L2_{,k+1} - L_{2,k}$ & $\Delta A = A_{k+1} - A_k$

Where subscripts k and k+1 denote the kth and k+1st trials. These six equations may be stated in compact form by means of the following matrix equation

$J_k\ \Delta X_k = -f_k$----(14)

Where Jk is called the Jacobian Matrix and, $\Delta X_k = X_{k+1} - X_k = [\Delta V_o, \Delta T_1, \Delta L_1, \Delta T_2, \Delta L_2, \Delta A]^T$

The subscripts k and k+1 denote that elemt of the matrices carrying these subscripts are those given by the kth and k+1st trials, respectively. The display of the elements of J_k and f_k is as follows:-

$$J_k = \begin{array}{|c|c|c|c|c|c|} \hline \frac{\partial f_1}{\partial V_o} & \frac{\partial f_1}{\partial T_1} & \frac{\partial f_1}{\partial L_1} & \frac{\partial f_1}{\partial T_2} & \frac{\partial f_1}{\partial L_2} & \frac{\partial f_1}{\partial A} \\ \hline \frac{\partial f_2}{\partial V_o} & \frac{\partial f_2}{\partial T_1} & \frac{\partial f_2}{\partial L_1} & \frac{\partial f_2}{\partial T_2} & \frac{\partial f_2}{\partial L_2} & \frac{\partial f_2}{\partial A} \\ \hline ---- & ---- & ---- & ---- & ---- & ---- \\ \hline ---- & ---- & ---- & ---- & ---- & ---- \\ \hline ---- & ---- & ---- & ---- & ---- & ---- \\ \hline \frac{\partial f_6}{\partial V_o} & \frac{\partial f_6}{\partial T_1} & \frac{\partial f_6}{\partial L_1} & \frac{\partial f_6}{\partial T_2} & \frac{\partial f_6}{\partial L_2} & \frac{\partial f_6}{\partial A} \\ \hline \end{array} \qquad f_k = \begin{array}{|c|} \hline f1 \\ \hline f2 \\ \hline f3 \\ \hline f4 \\ \hline f5 \\ \hline f6 \\ \hline \end{array} \qquad ----- (15)$$

If the function $f_1, f_2, \ldots, f_6$ and their partial derivatives which appear in J_k are continuous and the determinant of J_k is not equal to zero, then the Newton Raphson method will converge, provided a set of assumed values of the variables which are close enough to the solution set can be found .

If the changes in the specific heats with temperature in the neighborhood of the solution to equation (6) through equation (11) are negligible, then the sensible heat terms $(h_f - h_1)$, $(h_1 - h_2)$, $(h_2 - h_3)$ may be replaced by their respective equivalents; $C_p(T_f - T_1)$, $C_p(T_1 - T_2)$, and $C_p(T_2 - T_3)$. If the variation of the latent heats with temperature are also regarded as negligible in the neighborhood of the solution, then,

$$J_k = \begin{array}{|c|c|c|c|c|c|} \hline \lambda_0 & -FC_F & \partial f_1/\partial L_1 & \lambda_1 & 0 & 0 \\ \hline -\lambda_0 & -U_1A & 0 & 0 & 0 & U_1(T_0-T_1) \\ \hline 0 & L_1C_p & b_{33} & -L_1C_p & -\lambda_2 & 0 \\ \hline 0 & U_2A & \lambda_1 & -U_2\ A & 0 & U_2(T_1-T_2) \\ \hline 0 & 0 & \lambda_2 & L_2C_p & b_{33} & 0 \\ \hline 0 & 0 & -\lambda_2 & U_3A & \lambda_2 & U_3(T_2-T_3) \\ \hline \end{array}$$

Where, $b_{33} = C_p(T_1 - T_2) - (\lambda_1 + \lambda_2)$ & $b_{55} = C_p(T_2 - T_3) - (\lambda_2 + \lambda_3)$

VI. SOLUTION OF A TEE PROBLEM BY NEWTON- RAPHSON METHOD

We have to assume the value of following terms for the calculation procedure to design triple effect evaporator,

1- $(T_o - T_1)$, $(T_1 - T_2)$, and $(T_2 - T_3)$, 2- Solvent evaporated in first effect

3- Solvent evaporated in second effect, 4- Solvent evaporated in third effect

L_3 can be calculated by component material balance equation (12), as $L_3 = FX / x_3$

5- Area, A, 6- V_o, Vapor flow rate of steam

The following scaling procedure should be used…

1- Each functional equation is divided by the product $F\lambda_o$, and the new functional expression so obtained are denoted by g_j ($j = 1, 2, \ldots\ldots, 6$)

$$g_j = f_j / F\lambda_o$$

2- All flow rates are expressed as a function of the feed rate F, that is $L_j = l_j F$ and $V_j = v_j F$.

3- All the temperatures are expressed as a function of the steam temperature , $T_j = u_j T_o$, which defines the fractional temperature u_j.

4- The area of each effect is expressed as a fraction of a term proportional to the flow rate in the following manner;-

$A_j = a_j(F / 50)$, which defines the fractional area a_j.

After this scaling procedure has been applied to the functional expressions, the matrices J_k, ΔX_k, ndf_k take the following terms;-

$$J_k = \begin{bmatrix} 1 & b_{12} & \lambda_1/\lambda_0 & 0 & 0 & 0 \\ -1 & b_{22} & 0 & 0 & 0 & b_{26} \\ 0 & b_{32} & b_{33} & b_{44} & \lambda_2/\lambda_0 & 0 \\ 0 & b_{42} & \lambda_1/\lambda_0 & b_{44} & 0 & b_{46} \\ 0 & 0 & \lambda_1/\lambda_0 & b_{54} & b_{55} & 0 \\ 0 & 0 & \lambda_1/\lambda_0 & b_{64} & \lambda_2/\lambda_0 & b_{66} \end{bmatrix}$$

$\Delta X_k = [\ \Delta v_o\ \Delta u_1\ \Delta l_1\ \Delta u_2\ \Delta l_2\ \Delta a\ ]^T$

$f_k = [\ g_1\ g_2\ g_3\ g_4\ g_5\ g_6\ ]^T$

Where the elements of J_k consist of the partial derivatives of the g_j's with respect to the new set of variables (v_o, u_1, l_1, u_2, l_2, a)

$b_{12} = -C_pT_o\ /\lambda_o$, $b_{34} = -l_1 C_p T_o / \lambda_o$, $b_{22} = -U_{1\ a} T_o / 50\ \lambda_o$, $b_{44} = U_2\ a\ T_o / 50\ \lambda_o$,

$b_{32} = l_1 C_p T_o / \lambda_o$, $b_{54} = l_2 C_p T_o / \lambda_o$, $b_{42} = U_2\ a\ T_o\ / 50\ \lambda_o$, $b_{64} = U_3\ a\ T_0 / 50\ \lambda_o$,

$b_{26} = [\ U_1 (1 - u_1) To\ ] / 50\ \lambda o$, $b_{33} = [\ C_p (u_1 - u_2) T_o - (\lambda_1 + \lambda_2)]\ / \lambda_o$,

$b_{46} = [\ U_2 (u_1 - u_2) T_o\ ] / 50\ \lambda_o$, $b_{55} = [\ C_p (u_2 - u_3) T_o - (\lambda_2 + \lambda_3)\ ] / \lambda_o$, $b_{66} = [\ U_3 (u_2 - u_3) T_o\ ] / 50\ \lambda_o$

Then on the basis of the assumed values of the variables the matrices J_0, X_o, f_0 are used in the first trial. Convergence to within about six significant numbers is achieved in further trials. We get the value of v_0, u_1, l_1, u_2, l_2, Thus, the desired solution is.

1. Vapor rate, $V_0 = v_o F$, 2.Temperatures, $T_1 = u_1 T_o$, $T_2 = u_2 T_o$ 3.Flow rates $L_1 = l_1 F$, $L_2 = l_2 F$, $L_3 = F X / x_3$, 4.Area, $A = a F / 50$, 5.Compositions, $x1 = F X / L_1$ & $X_2 = F X / L_2$

VII. CONCLUSION

On the basis of above procedure we can design multiple effect evaporator using Newton-Raphson method. For Triple-effect evaporator, we can directly put the variables into the final solution and can find the solution for a specific practical problem. The formulation of the solution of problems in terms of the Newton-Raphson method is helpful because it forces one to display the independent equations and the independent variables.

VIII. RECOMMENDATIONS

1. This procedure can be further extended to include more effects. Here forward mixing is considered.
2. Backward mixing can also be taken into consideration.
3. The model should further be simulated with operating strategies i.e, condensate flash, feed and product flash, vapor bleeding etc. These strategies are beneficial for increasing the steam economy of a MEE system and they work as Energy reduction schemes.

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