

HOMOMORPHIC CRYPTOGRAPHY DEVELOPMENT USING SCHEMES

Priyanka¹, Dinesh²

^{1,2} Computer Science & Engineering Department, M.D.U, (India)

ABSTRACT

This paper presents the study of homomorphic cryptography. . In this paper a study of various papers and articles is done, and in this paper we explain the schemes of homomorphic cryptography. The main part of this paper covers the privacy or security of data communication by using schemes

Keywords: Homomorphic, RSA, Pailler, Gentry, Complexity Cipher

I. INTRODUCTION

From earlier study, we know that public key cryptography is discovered by Diffie and Hellman in [12] in 1976. Privacy of digital data has become necessary especially when internet has become an indispensable part of private and work lives. To achieve confidentiality application such as online banking, electronic voting, virtual network etc. are homomorphic and homomorphic schemes.

Homomorphic cryptosystems were introduced by Rivest, Adleman, and Dertouzos in 1978 [31].

Fully homomorphic cryptosystems or privacy homomorphisms were introduced by Rivest, Adleman, and Dertouzos in 1978 [37]. In their paper they asked for a way to allow a third, untrusted party to carry out extensive computation on encrypted data, without having to decrypt first. Unfortunately, shortly after its publication, major security flaws were found in the original proposed schemes of Rivest et al. The search for fully homomorphic cryptosystems began.

Over the years a lot of either additively (Paillier [35] 1999, Goldwasser-Micali [23] 1984, Naccache-Stern [34] 1998) or multiplicatively (El Gamal [14] 1984, RSA [37] 1978) homomorphic schemes have been introduced to the world. The demand for a fully homomorphic cryptosystem rose again in 1991 when Feigenbaum et al. [15] asked: "Is there an encryption function $Enc()$ such that both $Enc(x+y)$ and $Enc(x \cdot y)$ are easy to compute from $Enc(x)$ and $Enc(y)$?" and was answered in 2009. Craig Gentry published his fully homomorphic cryptosystem [19] in the summer of 2009.

Although not yet useful for practical applications, it ended the long search for the in 1978 emerged question about the existence of privacy homomorphism

II. OBJECTIVE

The main objective of homomorphic cryptography is to ensure privacy of data in communication and storage processes, such as the ability to delegate computations to untrusted parties. If a user could take a problem explained in one algebraic system and encode it into a problem in a various algebraic system in a way that decoding back to the original algebraic system is hard, then the user could encode expensive computations and

send them to the untrusted party. This untrusted party then performs the corresponding computation in the second algebraic system, returning the result to the user. Upon receiving the result, the user can decode it into a solution in the original algebraic system, while the untrusted party learns nothing of which computation was actually performed

III BRIEF OF HOMOMORPHIC ENCRYPTION

The security requirements for data and algorithms has become very necessary in the last few years. Due to the excessive growth of technology, a great variety of attacks on digital goods and technical devices has increased or increasing day by day. Some possibilities exist for storing and reading data securely i.e Secure data encryption. The problem becomes more complex when asking for the possibility to compute (publicly) with encrypted data or to modify functions in such a way that they are still executable while our privacy is ensured. That is where homomorphic cryptosystems can be used. Even in 1978 this was a highly important matter, it is even more important nowadays. However the partial homomorphic properties of schemes like RSA, Paillier, ElGamal, etc. have been acknowledged ever since, it was not before 2009 when a young IBM researcher published the first working fully homomorphic cryptosystem, based on lattices

IV. SCHEMES OF HOMOMORPHIC CRYPTOGRAPHY

The development of homomorphic cryptography is based on three schemes. They are as follows-

RSA-i.e Multiplicatively homomorphic schemes

Paillier-i. e Additively homomorphic schemes

Gentry-i.e Algebraically homomorphic schemes

4.1 RSA

It is based on multiplicative. It is also called multiplicatively Schemes In 1978, Rivest, Shamir, and Adleman published their public-key cryptosystem, which only uses elementary ideas from number theory, in their paper "A Method for Obtaining Digital signatures and Public-Key Cryptosystems" [37]. It was one of the first homomorphic cryptosystem. The RSA cryptosystem is the most widely used public-key cryptosystem. It may be used to provide both secrecy and digital signatures and its security is based on the intractability of the integer factorization

4.2 Paillier

It is based on the Additive. It is also called Additively Schemes. As we observe from earlier study that RSA scheme has a multiplicative homomorphic property. This means it is possible to perform multiplications with the encryptions of messages without losing or tampering with their underlying information. This is possible since the operation "multiplication" in the ciphertext space $(Z_n; \cdot)$ can be compared with the operation "multiplication" in the plaintext space $(Z_n; \cdot)$.

The Paillier scheme is known to be additively homomorphic. What might seem confusing at first is the fact that the two group operations are different, namely the product of two ciphertexts will decrypt to the sum of their plaintexts. In comparison to that, the product of two RSA ciphertexts decrypt to the product of their plaintexts. Hence the Paillier scheme is additively homomorphic and RSA multiplicatively.

4.3 Gentry

It is based on the Algebraic equations. It is also called Algebraically homomorphic encryption. Pascal Paillier introduced his cryptosystem in the 1999 published paper "Public-Key Cryptosystems Based on Composite Degree Residuosity Classes" [35]. The proposed technique is based on composite residuosity classes, whose computation is believed to be computationally difficult. It is a probabilistic asymmetric algorithm for public key cryptography and inherits additive homomorphic properties. In the decades before Gentry discovered his novel method to gain homomorphic encryption, many researchers worldwide tried to find more powerful and therefore more complex schemes to achieve the fully homomorphic property. Gentry uses a method which no other researcher tried before. Instead of directly creating a superior scheme, he would build one from a "somewhat" homomorphic scheme, if its decryption circuit is sufficiently simple. He realized that he could build a fully homomorphic scheme from any scheme that is bootstrappable, i.e., could homomorphically compute a slightly augmented version of its own decryption circuit

V. DECRYPTION COMPLEXITY

The aim of this initial construction of a somewhat homomorphic encryption scheme was to obtain a scheme that is bootstrappable. Up to now we do not know what bootstrappability even means and why it is a necessary prerequisite. Informally speaking a scheme is bootstrappable if it can homomorphically evaluate its own decryption circuit. Unfortunately this is not the case in this initial scheme [20]. In order to obtain a scheme that can be transformed into a fully homomorphic encryption scheme it is crucial to lower the complexity of the decryption circuit

VI. CONCLUSION

From the above study we conclude that homomorphic encryption works on the privacy of data communication. We make our communication secure by applying schemes. We can make our data encrypt by algebraically, additively and multiplicatively. It makes our online processes confidential

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