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Bulk Queueing Analysis with Removable Server and Working Vacation

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Abstract

In this paper we will study about Queueing model where service are given to the customers in bulk as arrival pattern of the customer is one by one and there is single service point. Service time distribution is exponential with two rates. and queue discipline is first come first serve basis. We will find out expected queue length when server will be busy, empty and will be in working state on removable server.

Keywords: First come first serve, removable server, single server, Vacation, queue length.

INTODUCTION In this paper, the arrival of the customer is occurring one by one and service provide by the single server. If the queue is empty then server goes on vacation, when queue size is less than 'a' then server serve the customer with rate μ_1 and if the queue size is greater than 'a' then server serve the customer with a batch size 'd' of rate μ_2.The assumptiondescribe the system are , arrivals of the customers arrive for the service according to poisson law with parameter λ, the service time distribution follows exponentially with two different ratesμ_1 and μ_2, server serve the customer of batch sized if server is busy when batch size is less than 'a' , the Queue discipline follows first come first serve and the server is removed from its service as soon as the Queue is empty.

Notations

$P_{00}(t)$	:	Probability that there are no arrival and server is on vacation.
$P_{i1}(t)$	:	Probability that there are exactly 'i' arrival and server is an working vocation

$P_{i2}(t)$: Probability that there are exactly 'i' arrivals and server is busy

$X(t) = 0$ $y(t) = 0$ removal state

$x(t) < a y(t) = 1$ working vacation, $xt \geq a$ $y(t) = 2$ busy.

The difference-differential equations governing the mode are

$$P'_{00}(t) = -\lambda P_{00}(t) + \mu_1 P_{0,1}(t) + \mu_2 P_{02}(t) \quad (1)$$

$$P'_{01}(t) = -(\lambda + \mu_1) P_{0,1}(t) + \lambda P_{00}(t) + \mu_1 P_{11}(t) + \mu_2 d P_{12} \quad (2)$$

$$P'_{n1}(t) = -(\lambda + \mu_1) P_{n,1}(t) + \lambda P_{n-1}(t) + \mu_1 P_{n+1,1}(t) \ n \leq a \quad (3)$$

$$P'_{0,2}(t) = (\lambda + d\mu_2) P_{0,2}(t) + \lambda P_{a+1,1}(t) + d\mu_2 P_{n,2}(t) \quad (4)$$

$$P'_{n,2}(t) = (\lambda + d\mu_2) P_{n,2}(t) + \lambda P_{n,2}(t) + d\mu_2 P_{n+a,2}(t) \ n > a \quad (5)$$

Taking Laplace Transformation of Equation (1) – (5) we get

$$S\overline{P}_{00}(s) - P_{00}(s) = -\lambda \overline{P}_{00}(s) + \mu_1 \overline{P}_{01}(s) + \mu_2 \overline{P}_{02}(s) \quad (6)$$

$$\overline{P}_{00}(s) = \frac{1}{S+\lambda}(1 - \mu_1 \overline{P}_{01}(s) + \mu_2 \overline{P}_{02}(s)) \quad (7)$$

$$S\overline{P}_{01}(s) - P_{01}(s) = -(\lambda + \mu_1)\overline{P}_{01}(s) + \lambda \overline{P}_{00}(s) + \mu_1 \overline{P}_{11}(s) + \mu_2 d \overline{P}_{12}(s) \quad (8)$$

$$\overline{P}_{01}(s) = \frac{1}{(S+\lambda+\mu_1)}[\lambda \overline{P}_{00}(s) + \mu_1 \overline{P}_{11}(s) + \mu_2 d \overline{P}_{12}(s)] \quad (9)$$

$$\overline{P}_{n1}(s) = \frac{1}{S+\lambda+\mu_1}[\lambda \overline{P}_{n-1,1}(s) + \mu_1 \overline{P}_{n+1,1}(s) + \mu_2 d \overline{P}_{n+1,2}(s)] \quad (10)$$

$$S\overline{P}_{02}(s) - P_{0,2}(0) = -(\lambda + d\mu_2)\overline{P}_{02}(s) + \lambda \overline{P}_{a+1,1}(s) + d\mu_2 \overline{P}_{n,2}(s) \quad (11)$$

Consider $|z| = 1 - \delta$

$|g(z)| < |f(z)|$

then $f(z)$ and $\{f(z) + g(z)\}$ will have same number of zero as inside $z = 1 - \delta$ roots of $h(z) = 0$ is real and unique iff

$$P = \frac{\lambda}{\mu_1} < 1$$

$$Z = \frac{(S+\lambda+d\mu_2) \pm \sqrt{(S+\lambda+d\mu_2)^2 - 4\mu_2\lambda d}}{2d\mu_2} \quad (12)$$

$$\alpha = (S + \lambda + d\mu_2) + \sqrt{(S+\lambda+d\mu_2)^2 - 4\mu_2 d\lambda} \quad (13)$$

$$\beta = (S + \lambda + d\mu_2) - \sqrt{(S+\lambda+d\mu_2)^2 - 4\mu_2 d\lambda} \quad (14)$$

Let α is the unique positive real part

$$\overline{P}_{n1}(s) = \overline{P}_{01}(s)\alpha - \overline{P}_{02}(s)\ d\mu_2\ R^n$$

$$\overline{P}_{02}(s) = \frac{\overline{P}_{01}(s)\alpha\left(\frac{\lambda}{S+\lambda+\mu_1}\right)\left(\frac{\lambda}{(S+\lambda+d\mu_2)}\right)}{\left[1+d\mu_2\left(\frac{\lambda}{S+\lambda+\mu_1}\right)\left(\frac{\lambda}{S+\lambda+\mu_2}\right) < R^n\right]} \quad (15)$$

$$\overline{P}_{00}(s) = \frac{\mu_1}{S+\lambda}\overline{P}_{01}(s) + \frac{d\mu_2}{S+\lambda}\overline{P}_{01}(s)\ D_1 + \frac{1}{S+\lambda} + d\ \mu_2\lambda^2\alpha\ R^n (16)$$

$$\overline{P}_{00}(s) = \frac{1}{(S+\lambda)}[(\mu_1 + d\mu_2 D_1)\overline{P}_{01}(s) + 1] \quad (17)$$

$$\overline{P}_{n1}(s) = [\ \alpha - d\ \mu_2\ D_1\ R^{n+1}]\ \overline{P}_{01}(s) \quad (18)$$

$$\overline{P}_{n,2}(s) = \frac{\alpha\lambda^2 R^n}{(S+\lambda+\mu_1)(S+\lambda+d\mu_2) + d\mu_2\lambda^2\alpha R^n}\overline{P}_{01}(s) \quad (19)$$

According to normalizing condition

$$\sum_{i=0}^{n}\overline{P}_{i0}(s) + \overline{P}_{i1}(s) + \overline{P}_{i2}(s) = \frac{1}{S} \quad (20)$$

Steady state probabilities

$$P_{in} = \lim_{s\to 0} S\ \overline{P}_{1n}(s) \quad (21)$$

$$P_{00} = \left[\frac{\mu_1}{\lambda} + \frac{d\mu_2}{\lambda}k\right] P_{01} (22)$$

$$P_{n1} = \left[\left(\frac{\lambda}{\mu_1}\right)^n - k\mu^2 R^n\right] P_{01} (23)$$

$$P_{n2} = P_{01}\left[\frac{\lambda^2\alpha R^n}{(\lambda+\mu_1)(\lambda+\mu_2 d) + d\mu_2 a\lambda^2\alpha R^n}\right] \quad (24)$$

Expected Queue Length if server is removed

$$L_{q_R} = \sum_{i=0}^{n} n\,P_{00} = 0\ (25)$$

Hence it verify that server is on removed state of these is no customer is the queue.

Expected Queue length if server is on working vacation

$$L_{q_{wv}} = \sum_{n=1}^{a} n\ P_{n1}$$

$$L_{q_{wv}} = P_{01} a \left\{ \sum_{n=1}^{a-1} n \left(\frac{\lambda}{\mu_1} \right)^n - d\mu_2 R^n \frac{\lambda^2 \alpha}{(\lambda+\mu_1)(\lambda+d\mu_2)+d\mu_2\lambda^2\alpha R^n} \right\} \qquad (26)$$

Expected Queue length at busy period

$$L_{q_B} = \sum_{n=a}^{\infty} n \; P_{n2}$$

$$L_{qB} = P_{01} \sum_{n=a}^{\infty} n \frac{\lambda^2 \alpha R^n}{(\lambda+\mu_1)(\lambda+d\mu_2)+d\mu_2\lambda^2\alpha R^n} \qquad (27)$$

FIGURES AND TABLE InTable - I, value of working vacation queue length or busy period queue length at different arrival rates and service rates.. Fig. I shows the behavior of working vacation queue length. It is clear from the graph that if arrival rate λ is increases then queue length is also increases.

Table I

P_{01}	D	λ	μ_1	μ_2	a	L_{qwv}
0.06	0.2	0.1	0.2	0.2	0.125	0.005584592
0.06	0.2	0.1	0.2	0.3	0.125	0.005099943
0.06	0.2	0.1	0.2	0.4	0.125	0.004430666
0.06	0.2	0.1	0.2	0.5	0.125	0.003576759

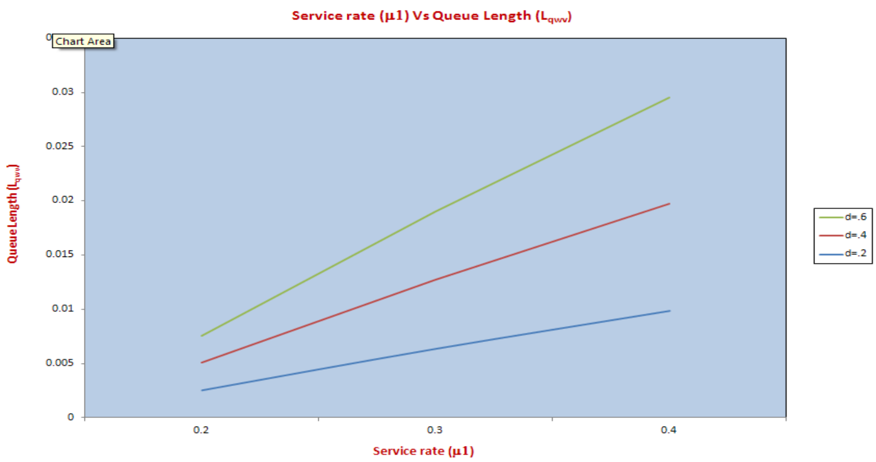

Fig. - I

Conclusion: It is clear from the Fig. - I that as batch size increases queue length in working vacation is also increases.

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