

EFFECT OF THERMAL DIFFUSION ON STEADY LAMINAR FREE CONVECTIVE FLOW ALONG A POROUS HOT VERTICAL PLATE IN THE PRESENCE OF HEAT SOURCE WITH MASS TRANSFER IN ROTATING SYSTEM

¹Ram Krishin Diwedi, ²Vineet Bhardwaj, ³Rahul Johari

¹*Shri Varshney College, Aligarh, U.P.-202001, India*

²*Department of Applied Sciences and Humanities, Jamia Millia Islamia, New Delhi-110025, India*

³*St.John's College, Agra, U.P.-282002, India*

Email Id- rahuljohari.phy@gmail.com

ABSTRACT

The purpose of the present problem is to study the effect of thermal diffusion on steady laminar free convective flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source with mass transfer. The governing equations of motions are solved by a regular perturbation technique. The expressions of temperature and concentration are derived. The effects of Soret number (S_o) and rotation velocity parameter (E) on the primary and secondary velocities, concentration and the components of skin friction in x and y directions are discussed with the help of tables and graphs.

INTRODUCTION

The problem of free convection flow of an electrically conducting fluid past a vertical plate under the influence of a magnetic field attracted many scientists, in view of its application in Aerodynamics, Astrophysics, Geophysics and Engineering.

Laminar natural convection and heat transfer in fluids flow with and without heat source in channels with constant wall temperature was discussed by OSTRACH [1]. An analysis of laminar free convective flow and heat transfer on a flat plate parallel to the direction of governing body force was studied by OSTRACH [2]. Combined natural and forced convection laminar flow and heat transfer in fluid, with and without source channels, with linearly varying wall temperature was discussed by OSTARCH [3]. SASTRI [4] dealt heat transfer in the flow over a flat plate with suction and constant heat source. Also, SASTRI [5] studied a problem of heat transfer in the presence of temperature dependent heat source in the flow over a flat plate suction. Forced and natural flows were discussed by SCHLITCHINTING [6]. ECKERT and DRAKE [7] and BANSAL [8]. Free convection effects on the Stokes problem for an infinite vertical plate has been studied by SOUNDALGEKAR [9]. POP and SOUNDALGEKAR [10] investigated free convection flow past and accelerated vertical infinite plate. RAPTIS et al [11] studied effects of free convection currents on the flow of an electrically conducting fluid of an accelerated



vertical infinite plate with variable suction. SHARMA [12] investigated free convection effect on the flow past and infinite vertical, porous plate with constant suction and heat flux. KUMAR and VARSHNEY [13] studied steady laminar free convection flow of an electrically conducting fluid along a porous hot vertical plate in the presence of heat source with mass transfer. Recently VARSHNEY and RAM PRAKASH [14] have discussed steady laminar free convective flow along a porous hot vertical plate in the presence of heat source with mass transfer in rotating system.

Present study is an extension of the work of VARSHNEY and RAM PRAKASH [14] with thermal diffusion. The aim of present study is to investigate the effects of thermal diffusion and rotation on the velocity of the fluid.

FORMULATION OF THE PROBLEM

Consider the steady free convective flow with mass transfer and thermal diffusion of an electrically conducting viscous fluid past an infinite vertical porous plate at $Z^* = 0$. Let the fluid and the plate be a state of rigid rotation with constant angular velocity Ω and Z^* axis, taken normal to the plate. A constant transverse magnetic field B_0 is acting parallel to the axis of rotation. Taking the magnetic Reynolds number to be small, the induced magnetic field is neglected in comparison to applied magnetic field B_0 . Since the length of the plate is large therefore, all the physical variables depend on Z^* only.

In the present problem magnetic field B_0 is assumed to be constant throughout the motion. We further assume that the electric field is equal to zero. The governing equations of continuity, momentum, energy and diffusion for a free convective flow of an electrically conducting fluid along a hot, non-conducting porous vertical plate in the presence of heat source and rotation are given as:

$$\frac{dw^*}{dz^*} = 0$$

i.e.

$$w^* = -w_0 \text{ (constant)} \text{-----(1)}$$

$$w^* \frac{du^*}{dz^*} - 2\Omega v^* = \nu \frac{d^2u^*}{dz^{*2}} - \frac{\sigma B_0^2 \nu}{\rho} u^* + g\beta(T^* - T_\infty) + g\beta'(C^* - C_\infty) \text{-----(2)}$$

$$w^* \frac{dv^*}{dz^*} + 2\Omega u^* = \nu \frac{d^2v^*}{dz^{*2}} - \frac{\sigma \beta_0 \nu}{\rho} v^* \text{-----(3)}$$

$$\frac{dp^*}{dz^*} = 0$$

i.e. $p^* = \text{constant} \text{-----(4)}$

$$\rho C_p w^* \frac{dT^*}{dz^*} = k \frac{d^2T^*}{dz^{*2}} + S^*(T^* - T_\infty) \text{-----(5)}$$

$$w^* \frac{dC^*}{dz^*} = D^2 \frac{d^2C^*}{dz^{*2}} + D_1 \frac{d^2T^*}{dz^{*2}} \text{-----(6)}$$

where ρ is the density, g is the acceleration due to gravity, β is the coefficient of volume expansion, β' is the coefficient of concentration expansion, ν is the Kinematic viscosity, T_∞ is the temperature of the fluid in the free stream, σ is the electric conductivity, B_0 is the magnetic induction, D is the chemical molecular diffusivity, k is the thermal conductivity S^* is the coefficient of heat source, C_∞ is the concentration at Infinity, C_p is the specific heat at constant pressure Ω is the angular velocity D_1 is thermal diffusivity.

The boundary conditions of the problem are

$$u^* = 0, v^* = 0, T^* = T_w, C^* = C_w \text{ at } z^* = 0 \text{-----(7)}$$

$$u^* \rightarrow 0, v^* \rightarrow 0, T^* \rightarrow T_\infty, C^* \rightarrow C_\infty \text{ at } z^* \rightarrow \infty$$

on introducing the following non dimensional quantities

$$z = \frac{w_0 z^*}{v}, \quad u = \frac{u^*}{w_0}, \quad v = \frac{v^*}{w_0}, \quad \Theta = \frac{T^* - T_\infty}{T_w - T_\infty}, \quad \Phi = \frac{C^* - C_\infty}{C_w - C_\infty}$$

$$E = \frac{v \Omega}{w_0^2}, \quad S_c = \frac{v}{D} \text{ (Schmidt No.)}, \quad S = \frac{v^2 S^*}{k w_0^2} \text{ (Source Para.)}$$

$$M = \frac{\sigma v B_0^2}{\rho w^2} \text{ (Hartman No.)}, \quad P_r = \frac{\rho v C_p}{k} \text{ (Prandtl No.)},$$

$$G_r = g \beta v \frac{T_w - T_\infty}{w_0^3} \text{ (Grashof number),}$$

$$G_c = g \beta^* v \frac{C_w - C_\infty}{w_0^3} \text{ (Modified Grashof number),}$$

$$S_c = \frac{D(T_w - T_\infty)}{v(C_w - C_\infty)} \text{ (Soret number)}$$

In equations (2), (3), (5) and (6), we get

$$\frac{d^2 Q}{dz^2} + \frac{dQ}{dz} - M_1 Q = -G_r \Theta - G_c \Phi \text{-----(8)}$$

$$\frac{d^2 \Theta}{dz^2} + P_r \frac{d\Theta}{dz} + S \Theta = 0 \text{-----(9)}$$

$$\frac{d^2 \Phi}{dz^2} + S_c \frac{d\Phi}{dz} + S_0 S_c \frac{d^2 \Theta}{dz^2} = 0 \text{-----(10)}$$

Where $Q = u + iv, M_1 = M + 2iE$

The corresponding boundary conditions are

$$Q = 0, \Theta = 1, \Phi = 1 \text{ at } z = 0$$

$$Q \rightarrow 0, \Theta \rightarrow 0, \Phi \rightarrow 0 \text{ at } z \rightarrow \infty \text{-----(11)}$$

On solving equations (8) to (10) which are ordinary linear differential equation in Q, Θ, Φ with boundary conditions (11), we get

$$Q = A_6 e^{-A_1 z} - A_3 (G_r - G_c A_5) e^{-A_2 z} - (1 + A_5) G_c A_4 e^{-scz} \text{-----(12)}$$

$$\Theta = e^{-A_2 z} \text{-----(13)}$$

$$\Phi = -(1 + A_5) e^{-scz} - A_5 e^{-A_2 z} \text{-----(14)}$$

where

$$A_1 = \frac{[1 + \{1 + 4M_1\}^{1/2}]}{2} = \frac{1}{2}(1 + a) + \frac{1}{2}bi$$

$$A_2 = \frac{Pr + \{Pr^2 - 4S\}^{1/2}}{2}$$

$$A_3 = \frac{1}{A_2^2 - A_2 - M_1} = \frac{(A_2^2 - A_2 - M_1) + 2Ei}{(A_2^2 - A_2 - M)^2 + 4E^2}$$

$$A_4 = \frac{1}{S_c^2 - S_c - M_1} = \frac{(S_c^2 - S_c - M) + 2Ei}{(S_c^2 - S_c - M)^2 + 4E^2}$$

$$A_5 = \frac{S_0 S_c A_2}{A_2 - S_c}$$

$$A_6 = G_r A_3 + (1 + A_5) G_c A_4 - G_c A_3 A_5$$

The primary velocity u (real part of Q) and secondary velocity v (imaginary part of Q) from equation (12) are given

$$u = \left[\frac{\{(G_r - G_c A_5) K_1 + (1 + A_5) G_c K_3\} \cos(bz/2) + \{(G_r - G_c A_5) K_2 + (1 + A_5) G_c K_4\} \sin(bz/2)}{e^{-(1+a)z/2} - (G_r - G_c A_5) K_1 e^{-A_2 Z} - (1 + A_5) G_c K_3 e^{-S_c Z}} \right] \quad (15)$$

$$v = \left[\frac{\{(G_r - G_c A_5) K_2 + (1 + A_5) G_c K_4\} \cos(bz/2) - \{(G_r - G_c A_5) K_1 + (1 + A_5) G_c K_3\} \sin(bz/2)}{e^{-(1+a)z/2} - (G_r - G_c A_5) K_1 e^{-A_2 Z} - (1 + A_5) G_c K_3 e^{-S_c Z}} \right] \quad (16)$$

Where

$$a = \left[\frac{\{(1+4M)^2 + 64E^2\}^{\frac{1}{2}} - (1+4M)}{2} \right]^{1/2}$$

$$b = \left[\frac{\{(1+4M)^2 + 64E^2\}^{\frac{1}{2}} - (1+4M)}{2} \right]^{1/2}$$

$$K_1 = \frac{(A_2^2 - A_2 - M)}{(A_2^2 - A_2 - M)^2 + 4E^2}$$

$$K_2 = \frac{2E}{(A_2^2 - A_2 - M)^2 + 4E^2}$$

$$K_3 = \frac{(S_c^2 - S_c - M)}{(S_c^2 - S_c - M)^2 + 4E^2}$$

$$K_4 = \frac{2E}{(S_c^2 - S_c - M)^2 + 4E^2}$$

The components of Skin Friction at the plate in the x and y directions are given as

$$\tau_x = \left(\frac{\partial u}{\partial z} \right)_{z=0} = -\{(G_r - G_c A_5) K_1 + (1 + A_5) G_c K_3\} \left(\frac{b}{2} \right) + \{(G_r - G_c A_5) K_2 + (1 + A_5) G_c K_4\} (b/2) + (G_r - G_c A_5) K_1 A_2 + (1 + A_5) G_c K_3 S_c \quad (17)$$

$$\tau_y = \left(\frac{\partial v}{\partial z} \right)_{z=0} = -\{(G_r - G_c A_5) K_1 + (1 + A_5) G_c K_3\} \left(\frac{b}{2} \right) + \{(G_r - G_c A_5) K_2 + (1 + A_5) G_c K_4\} \left((1 + a)/2 \right) + (G_r - G_c A_5) K_2 A_2 + (1 + A_5) G_c K_4 S_c \quad (18)$$

RESULT AND DISCUSSION

The primary and secondary velocity profiles are tabulated in Table-1 and 2 and plotted in Fig.-1 and 2 having graphs from 1 to 3 for $M=1, G_r=4, G_c=4, P_r=0.71, S_c=0.6, S=0.05$ and different values of S_0 (Soret number) and (Rotation velocity parameter)

	So	E
For Graph-1	1	0.5



For Graph-2	2	0.5
For Graph-3	1	1.0

From Graphs-1 to 3 of Fig.-1, it is found that the primary velocity u increases sharply till $z=1.2$ (near the wall) after its primary velocity decreases sharply till $z=3.5$ then after its primary velocity decreases continuously with the increase in z . On comparing graph 2 and 3 with graph -1, It is observed that primary velocity increases with the increase in S_o and decreases with the increase in E .

The components of skin friction at the plate in x and y directions are tabulated in Table-3 at the same values as taken for velocity profile.

From Table-3, it is found that skin friction in x - direction increases with the increase in S_o , but it decreases with the increase in E . It is also observed that skin friction in y -direction decreases with the increase in S_o but increases with the increase in E .

The concentration profile is tabulated in Table- 4 and plotted in Fig.- 3 at the same values as taken for velocity profile from Fig.- 3 it is noticed that concentration increases with the increase in S_o .

Table-1: Values of primary velocity u at $M=1, G_r=5, G_c=4, P_r=0.71, S_c=0.6, S=0.05$ and different values of $S=0.05$ and different values of S_o and E .

Z	For Graph-1	For Graph-2	For Graph-3
0	0	0	0
1	2.273654	2.6463201	1.1895096
2	1.688846	2.1063884	0.8331227
3	1.048303	1.3920086	0.5004617
4	0.627878	0.8773665	0.2952502
5	0.372324	0.5415557	0.1739622

Table-2: Values of primary velocity u at $M=1, G_r=5, G_c=4, P_r=0.71, S_c=0.6, S=0.05$ and different values of $S=0.05$ and different values of S_o and E .

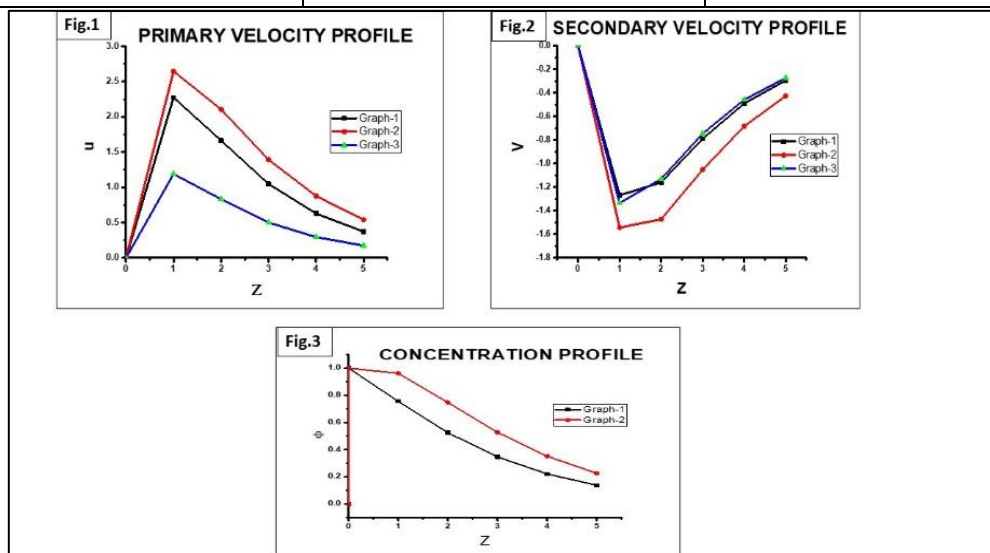
Z	For Graph-1	For Graph-2	For Graph-3
0	0	0	0
1	-1.269804	-1.544023	-1.335292
2	-1.160068	-1.473346	-1.131317
3	-0.790019	-1.052496	-0.746714
4	-0.491082	-0.68406	-0.458348
5	-0.295165	-0.427226	-0.274103

Table-3: Values of primary velocity u at $M=1, G_r=5, G_c=4, P_r=0.71, S_c=0.6, S=0.05$ and different values of $S=0.05$ and different values of S_o and E .

S_o	E	τ_x	τ_x
1	0.5	6.85605	-7.29050
2	0.5	7.48525	-7.83856
1	1	3.90123	-3.40537

Table-4: Values of concentration $P_r=0.71, S_c=0.6, S=0.05$ and different values of $S=0.05$ and different values of S_o

Z	Graph-1	Graph-2
0	1	1
1	0.755198	0.961417
2	0.524272	0.747168
3	0.346152	0.526858
4	0.221058	0.351292
5	0.137859	0.225859



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