



A Study of Thermoelasticity and Wave Propagation

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Abstract

The theory of Thermoelasticity and wave propagation play a crucial role in many geophysical applications, including oil drilling, gas hydrate detection, seismic monitoring, and hydrogeology etc. Seismic wave propagation in porothermoelastic solid has been the subject of explorations by many analyzers in different fields like earth sciences, geophysics, earthquake engineering etc. One of the most prominent ways of information about interior of earth is called seismic activity. Seismic activity is also helpful in the study and prediction of earthquake and tsunamis. In the present work we offer an analysis of the most popular studies that describe basics of thermoelasticity and wave propagation.

Introduction

The dynamical equations formulated by Biot [1] have been serving as a basis to study wave propagation problems in poroelastic media. Green et al. [2] investigated thermoelastic material behavior in the absence of energy dissipation using both nonlinear and linear theories. They also investigated linearized thermoelasticity, which has the following properties: (i) unlike classical thermoelasticity, which is characterized by the Fourier law, the heat flow does not involve energy dissipation; (ii) a constitutive equation for an entropy flux vector is determined by the same potential function that also determines the stress; and (c) it allows heat to be transmitted as thermal waves at finite speed. In addition, a generic uniqueness thesis for linear thermoelasticity without energy dissipation is established. Fox, N [3] proposed a generalization of thermoelasticity based on a physically motivated modification to Fourier's law of heat conduction. Constitutive equations that are valid for finite deformations and temperature variations are postulated and then reduced to canonical form using conventional nonlinear continuum mechanics techniques. To illustrate novel aspects of the nonlinear theory, exact solutions are provided. In order to create a generalized dynamical theory of thermoelasticity, Lord et al. [5] used a form of the heat transport equation that takes into account the time required for the heat flow to accelerate. The theory accounts for the temperature-strain rate coupling; however the resulting coupled equations are both hyperbolic. As a result, the existing coupled theory of thermoelasticity no longer contains the dilemma of an infinite velocity of propagation. With the use of the generalized theory, a solution is discovered that compares favorably



to one already discovered using the traditional coupled theory. A theory of wave propagation in isotropic poroelastic media saturated by two immiscible Newtonian fluids has been investigated by Tuncay et al. [7]. By volume averaging the microscale balance and constitutive equations and assuming modest deformations, the macroscopic constitutive relations, as well as the mass and momentum balance equations, are produced. When Darcy's law is assumed to be true, the momentum transfer terms are described in terms of intrinsic and relative permeabilities and compare favorably to a known solution obtained using the conventional coupled theory. In a novel way, the coefficients of macroscopic constitutive interactions are stated in terms of measurable quantities. The hypothesis proves that there are three compressional waves and one rotational wave. The third compressional wave is dependent on the slope of the capillary pressure-saturation relation and is connected to the pressure differential between the fluid phase and solid phase. In the present study, we employ the volume-averaging technique to investigate the wave propagation properties of a linearly elastic porous medium saturated with two immiscible Newtonian fluids.

Wang et al.[9] provided a set of generalized thermo-poroelasticity equations. The LS and GL theories are represented by the equations, depending on certain relaxation times. By modifying the two Maxwell-Vernotte-Cattaneo relaxation times in the heat equation, they additionally tackle the generalization of the original LS theory. The related plane-wave analysis is carried out to demonstrate the velocity and attenuation differences between the LS and GL theories. Next, we construct an algorithm to simulate wave propagation. The algorithm computes waveforms and snapshots that show how waves propagate across various thermo-poroelastic media. They used a direct-grid method in this algorithm.

Carcione et al [10] provided a numerical technique for simulation of wave propagation in linear thermoelastic media. This algorithm is based on an extended Fourier law of heat transport and is analogous to the Maxwell model of viscoelasticity. A grid approach that is based on the Fourier differential operator is used to compute the wave field, and two time-integration procedures are used to cross-check the results of the analysis. Because the existence of a slow quasi-static mode, also known as the thermal mode, renders the differential equations rigid and unstable for explicit timestepping methods, initially, a second-order time-splitting approach is used to solve the unstable component analytically, and a Runge-Kutta method is used to solve the regular equations. In contrast, a first-order explicit Crank-Nicolson algorithm produces more dependable results for low thermal conductivity values. The accuracy of these two time-stepping techniques is second and first-order, respectively. In the calculation of the spatial derivatives, the Fourier differential offers spectral accuracy. The model predicts three propagation modes: a fast compressional or (elastic) P wave, a slow thermal P diffusion/wave (the T wave), which resemble the fast and slow P waves of poroelasticity, respectively, and a shear wave. When the thermal conductivity is low, the thermal



mode is diffuse, and when it is high, the mode is wave-like. The wavefront of the fast P wave is defined by three velocities: the isothermal velocity in the uncoupled case, the adiabatic velocity at low frequencies, and a higher velocity at high frequencies.

Wanting, et al.[11] discussed on Characteristics of wave propagation in thermoelastic medium. The propagation of seismic waves is significantly influenced by the thermoelastic properties in this medium. Theoretically, the fast longitudinal wave, the slow longitudinal wave (also known as the thermal wave for short), and the transverse wave would all propagate in the thermoelastic medium according to the Lord-Shulman hyperbolic coupled thermoelastic equation with a relaxation time correction term. The transverse wave is unaffected by the thermal properties of the medium, while the first two longitudinal waves are thermal dissipation attenuation waves. They analyzed the effects of changes in thermal conductivity, thermal expansion coefficient, and specific heat on wave velocity and attenuation, using a combination of plane wave dispersion analysis and Green's function numerical simulation to study the dispersion and attenuation characteristics of two thermal dissipation attenuating waves. Thermal conductivity has been found to be the primary factor in determining crucial changes in wave velocity and attenuation. Research has demonstrated that the crucial variations in wave velocity and attenuation are determined mostly by thermal conductivity. The amplitude of wave velocity and attenuation is significantly impacted by the thermal expansion coefficient. The first two thermoelastic coefficients are accounted for in the specific heat. Finally, the wave field snapshots are simulated using the second-order Green's function in the frequency domain of thermoelastic dynamics to demonstrate the behavior of longitudinal, transverse, and thermal waves during propagation in the thermoelastic medium.

Again Wanting, et al [12] represented another theory on thermoelastic waves. Unlike the traditional forward-modeling algorithms, they have used a unique finite-difference (FD) solver of the Lord-Shulman thermoelasticity equations to generate synthetic seismograms that account for the effects of the thermal characteristics (expansion coefficient, thermal conductivity, and specific heat). Since the differential equations become rigid and unstable when explicit time-stepping is used due to the presence of a sluggish quasistatic mode (the thermal mode), they resort to a time splitting approach. The spatial derivatives are calculated using a rotational staggered-grid FD approach, and the waves are absorbed at the boundaries using an unsplit convolutional perfectly matched layer for best performance at the grazing incidence. An analysis of the algorithm's stability under modeling conditions is performed. An attenuation of both the rapid P-wave (and E-wave) and the slow thermal P-wave (or T-wave) is shown by the numerical experiments, illustrating the impacts of the thermoelasticity features. Both the fast and slow P-waves in poroelasticity have features with these propagation modes. In terms of elastic waves, the thermal expansion coefficient significantly affects both velocity dispersion and attenuation. The T mode takes on a wave form at high thermal

conductivities and high frequencies due to the thermal conductivity's effect on the relaxation period of the thermal diffusion process.

Sharma, et al [13] used Biot's theory to study the propagation of plane-harmonic seismic waves in a transversely isotropic liquid-saturated porous solid. They also analyse the presence of three more quasi waves, in addition to SH waves, and provide analytical expressions for their propagation velocity. Existing waves have been observed to have velocities that change depending on the direction of propagation. It has been determined how Rayleigh-type surface waves travel along the free surface of transversely isotropic liquid-saturated porous solids, and the corresponding frequency equation has been derived. Numerical considerations have been given to analyze the Role of elastic constants in the existence of body waves and Rayleigh waves with real velocities. It has been shown that the propagation velocities vary depending on the direction of the wave

For simplifying the difficulties of seismic wave reflection and refraction at the interface, it is commonly assumed that the interface between two elastic half-spaces is welded. However, the welding contact at the interface may be compromised by the presence of liquid in the porous skeleton. The two media are probably very loosely bound together by a very small layer of viscous liquid at the interface. By recalling this Vashisth et al [14] studied the problem of reflection and transmission of a plane periodic wave incident on the loosely bonded interface between elastic solid and a liquid-filled porous solid. They assumed that the interface oversees like a dislocation, preserving stress continuity but permitting a limited amount of slip and normal displacement is continuous and shearing stress is related to slip velocity.

Energy ratios are determined numerically for a certain model and plotted for various degrees of bonding. The results for a welded contact and a smooth interface are obtained as particular cases. It is found that there is a dissipation of energy at a loosely bonded interface except for normal and grazing angles of incidence. This problem may also be useful to study the phenomena taking place in cracked materials (Griffith 1920) as well as to detect cracks in solids by non-destructive testing. Though ultrasonic techniques are extensively employed, the theoretical problem of reflection and refraction of plane harmonic waves at a loosely bonded interface between two such solid half-spaces does not seem to have been studied so far.

Sharma [15] Biot's theory of wave propagation in saturated porous solids has been amended to examine thermoelastic wave propagation in poroelastic medium. In this context, plane harmonic waves travelling through an isotropic poroelastic media are taken into account. Wave-induced temperature in the medium, fluid and solid particle displacements, and their relations are derived. Both the temperature and the thermoelastic coupling parameters are used to adapt the resulting Christoffel equations. These equations describe the presence and movement of four waves through a given medium. One is a non-attenuating transverse wave, and the other three are attenuating



longitudinal waves. The transverse wave is unaffected by the medium's temperature. For a numerical simulation of liquid-saturated sandstone, the velocities and attenuation of the longitudinal waves are calculated. Numerical examples are shown to illustrate how these longitudinal waves change depending on both thermal and poroelastic parameters.

M D Sharma [17] studied "Wave propagation across the boundary between two dissimilar poroelastic solids. In this context two dissimilar isotropic porous media are taken in welded contact at a plane interface between them. At the interface of two solids, a new parameter is introduced to indicate the potential strength of connections between the surface pores of the two solids. Need for continuity at the shared boundary is illustrated by describing many distinct sets of boundary conditions. At the porous-porous interface, we derive a set of boundary conditions to describe the partial coupling of surface pores. An imperfect bond can form between two saturated porous substances on the basis of such a partial connection. The tangential sliding that represents the flaw in the welded bonding at the planar interface causes some of the strain energy to be lost. In a porous, isotropic, fluid-saturated medium, there are three sorts of waves that can travel through it. When a wave hits an interface, it's reflected three times and refracted three times. For each of the three types of incident waves, the energy distribution within the reflected and refracted waves is analyzed. At the plane contact between kerosene-saturated sandstone and water-saturated lime-stone, a numerical example calculates the energy shares of reflected and refracted waves. Various boundary conditions are discussed, and their effects on these energy distributions are contrasted.

M D Sharma [17] derived a mathematical model for the wave propagation in anisotropic generalized thermoelastic medium. To model the wave propagation phenomenon in anisotropic thermoelastic media, two systems of equations have been developed. The first one depicts the medium's modified Christoffel equations, while the second one relates the temperature of the medium to the motion of its particles. There are three thermal parameters that control the overall effect of thermodynamics on wave propagation, and they are defined by a combination of the frequency and thermal coefficients. A method is described for precisely determining the speeds and attenuations of four quasi-waves travelling in such a medium. Both isotropic thermoelastic propagation and anisotropic elastic propagation are reduced to special instances. In the case of general anisotropy, an analogy has been established between thermoelastic wave propagation and poroelastic wave propagation. For a realistic numerical model, the variations in phase velocities and attenuation factors with the direction of phase propagation are estimated. Through the three thermal parameters described in the study, the influence of frequency, thermal conductivity, relaxation time, specific heat, and anisotropic symmetries on the velocities and attenuations of quasi-waves is numerically demonstrated.

M D Sharma [19] presented a model for studying surface waves in a general anisotropic poroelastic medium. This method is used for isotropic media; a complicated secular equation is derived to explain



the propagation of surface waves at the stress-free plane of a nondissipative porous medium. The most important aspect is that the resultant equation may be solved using iterative numerical methods since it can be analytically divided into real and imaginary parts. This secular equation's root determines the apparent phase velocity in a specified direction on the surface and denotes the existence of surface waves. The model of a crustal rock undergoes numerical work. For the top three anisotropies—triclinic, monoclinic, and orthorhombic—the propagation of surface waves is investigated numerically.

Conclusion

After reviewing the literature on thermoelasticity and wave propagation, it is clear that these are highly multidisciplinary subjects with a wide range of scientific and engineering applications.

A thorough understanding of how materials behave under mechanical and thermal stress is possible due to the combination of thermodynamic and elasticity concepts known as thermoelasticity. Engineering fields directly benefit from the understandings obtained from study on wave propagation and thermoelasticity. This covers, among other things, materials science, civil engineering, and aeronautical engineering.

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