



# A Peculiar Linearly Cover Variant of Amensalism in Mathematical Ecology with limited resources: Case(II)

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## Abstract

This model consists of two nonlinear differential equations of the first order. Both symbolise respective Amensal and Enemy Species. This paper identifies the effect of Amensal coefficient (c) and safety maintenance constant (d) on the constituted model. The Amensal Species has limited resources and a linearly variable cover for defence against the attacks of hostile species. To study closely the nature of the relationship between the two species, the graphs are meticulously illustrated where necessary.

## 1.Introduction:

On the quasi-linear basic balancing equations, K.V.L.N.Acharyulu and N.Ch.PattabhiRamacharyulu [2-9] investigated the local and global stabilities of an Ammensal-enemy eco-system. The author's previous research work also included a stability analysis for an Ammensal-enemy eco-system with various resources in various cases. Several authors[1,10-18] have opened new eras with effective methods for dealing with various situations in order to improve the interaction between species. The current study is primarily concerned with the establishment of interaction between the two species as a result of the effects of Amensal coefficient (c) and safety maintenance constant (d) .

## 2.Basic Equations of the Model:

In Mathematical Ecology, the equations of a strange linearly cover variant of Amensalism with Limited Resources are regarded as



$$dX/dt = aX - bX^2 - c(X - (d + eX))Y$$

$$dY/dt = fY - gY^2$$

Here  $x$  and  $y$  stand for Amensal and Enemy growth rates respectively. The natural growth rates of Amensal and enemy species are referred as  $a$  and  $f$ .  $c$  is the Amensal coefficient.

$b$  and  $g$  are the two species' decline rates due to natural resource restrictions.  $d + ex$  is the Amensal population, which protected from the attacks of enemy species by safety maintenance constants  $d$  and  $e$ .

**3. Case(i): The effect of  $c$  on Ammensal-Enemy Model**

For investigating the interaction between Ammensal and Enemy Species, the following values of concerned parameters are considered.

**Table(1)**

<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>e</b>	<b>f</b>	<b>g</b>	<b>X<sub>0</sub></b>	<b>Y<sub>0</sub></b>
<b>1.1100</b>	<b>0.889</b>	<b>changes</b>	<b>0.875</b>	<b>0.67</b>	<b>0.859</b>	<b>0.5</b>	<b>0.134</b>	<b>0.366</b>

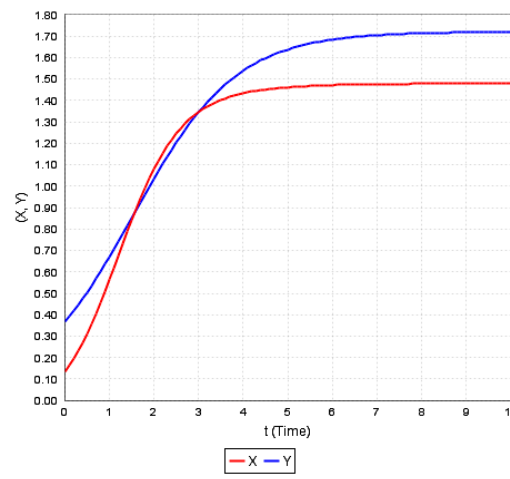
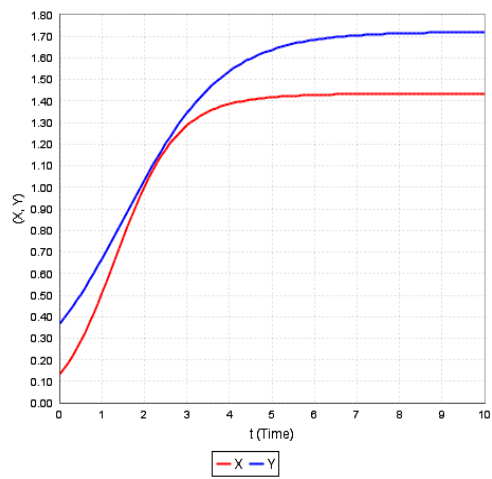
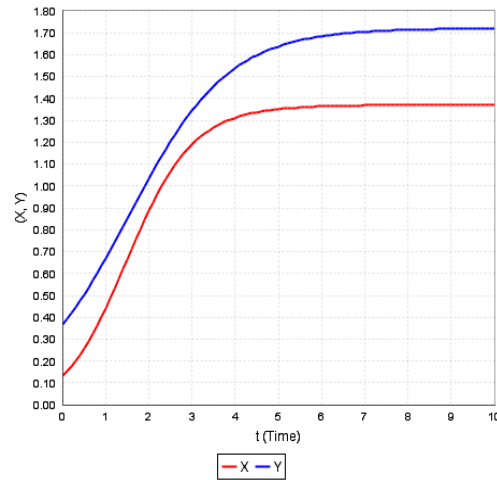
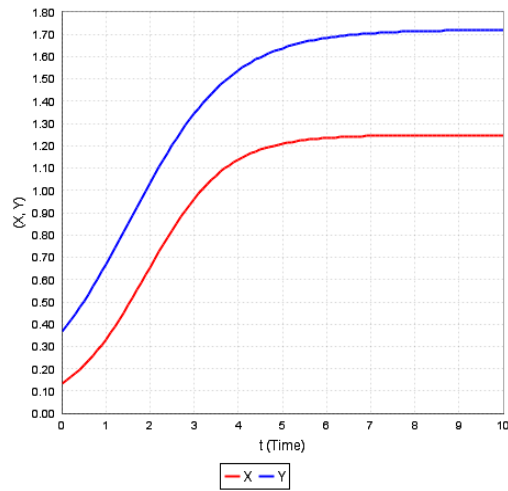
The effect of changing the parameter  $b$  on the model in view of possible Dominance time instincts .

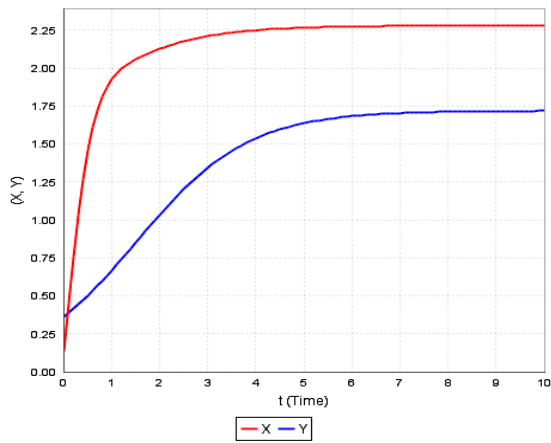
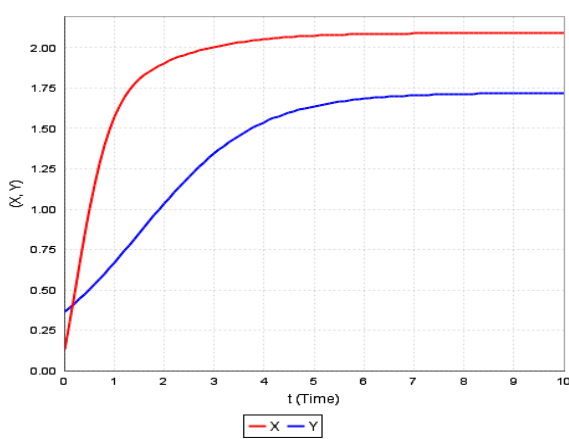
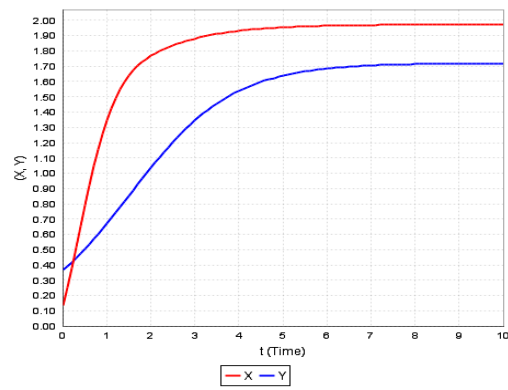
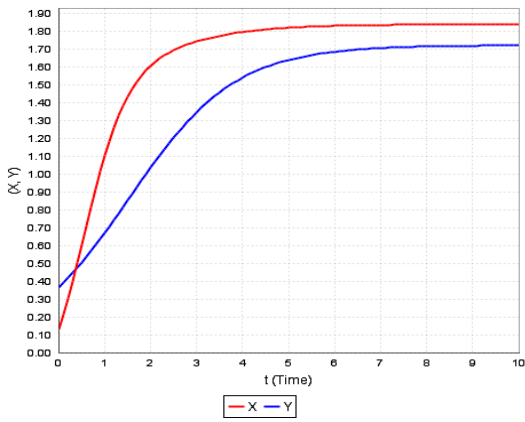
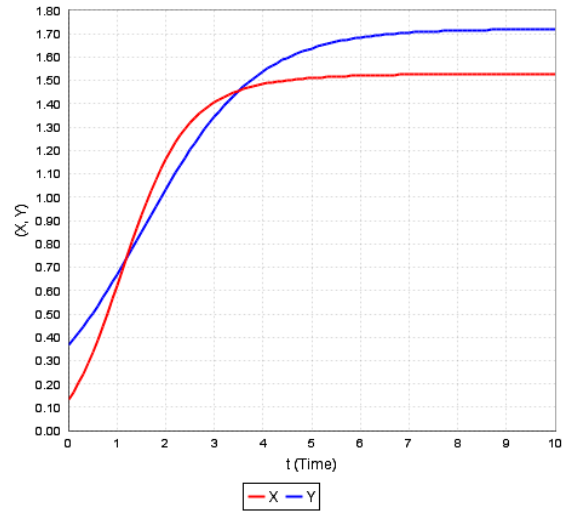
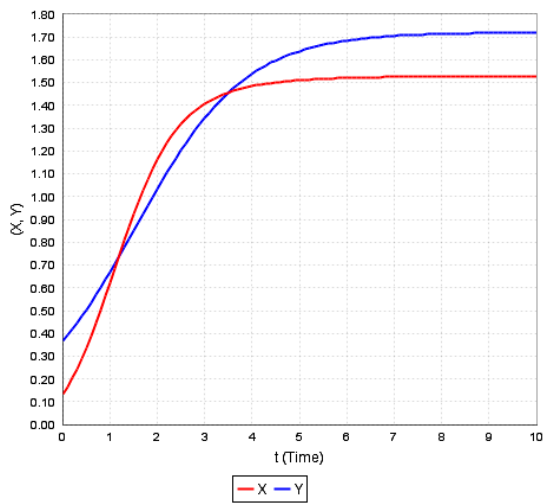
**Table(2)**

<b>S.NO</b>	<b>c</b>	<b>t<sub>1</sub><sup>*</sup></b>	<b>t<sub>2</sub><sup>*</sup></b>
<b>1</b>	<b>0</b>	<b>*</b>	<b>*</b>
<b>2</b>	<b>0.2050</b>	<b>*</b>	<b>*</b>
<b>3</b>	<b>0.3450</b>	<b>*</b>	<b>*</b>
<b>4</b>	<b>0.4550</b>	<b>1.6</b>	<b>3</b>
<b>5</b>	<b>0.5950</b>	<b>1.15</b>	<b>3.5</b>
<b>6</b>	<b>0.7650</b>	<b>0.95</b>	<b>4</b>
<b>7</b>	<b>1.0150</b>	<b>0.7</b>	<b>4.75</b>
<b>8</b>	<b>2.0800</b>	<b>0.38</b>	<b>*</b>
<b>9</b>	<b>3.2950</b>	<b>0.24</b>	<b>*</b>
<b>10</b>	<b>4.0300</b>	<b>0.2</b>	<b>*</b>
<b>11</b>	<b>4.9200</b>	<b>0.15</b>	<b>*</b>
<b>12</b>	<b>9.9400</b>	<b>0.07</b>	<b>*</b>

Fig(1) to Fig(10): when  $a=1.1100, b=0.889, d=0.875, e=0.67, f=0.859, g=0.5, X_0=0.134, Y_0=0.366$

Vs change in b



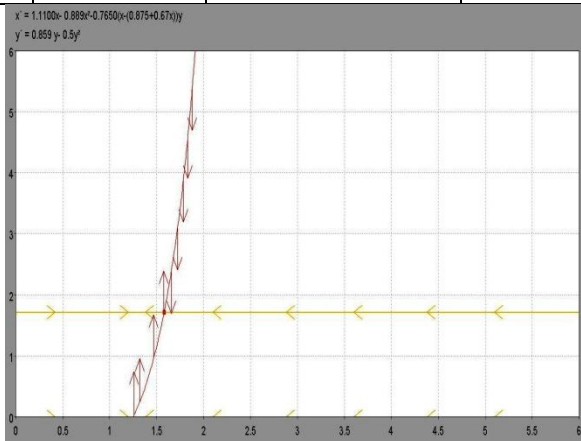


**4. Phase Plane Analysis:**

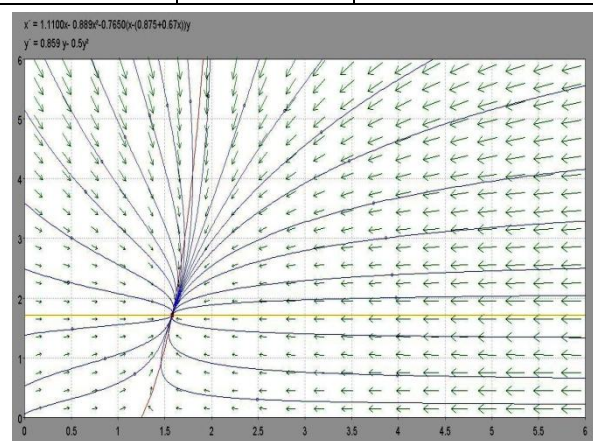
Phase plane analysis has been performed to analyse the model for establishing stability with equilibrium points whose Eigen values are known.

When  $a=1.1100, b=0.889, c=Changes, d=0.875, e=0.67, f=0.859, g=0.5$

S.No	Values of Parameter(c)	Equilibrium Point	Jacobian matrix	Eigenvalues	eigenvectors
1	0.2050	(1.3707, 1.718)	$\begin{bmatrix} -1.4434 & 0.086644 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1=-0.859$ $\lambda_2=-1.4434$	$E_1=(0.14666, 0.9891)^T$ $E_2=(0, 1)^T$
2	0.3450	(1.4351, 1.718)	$\begin{bmatrix} -1.6372 & 0.13849 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1=-0.859$ $\lambda_2=-1.6372$	$E_1=(0.17521, 0.9845)^T$ $E_2=(0, 1)^T$
3	0.4550	(1.4787, 1.718)	$\begin{bmatrix} -1.7771 & 0.17609 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1=-0.859$ $\lambda_2=-1.777$	$E_1=(0.18836, 0.982)^T$ $E_2=(0, 1)^T$
4	0.5950	(1.5277, 1.718)	$\begin{bmatrix} -1.9436 & 0.22066 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1=-0.859$ $\lambda_2=-1.9436$	$E_1=(0.19936, 0.9799)^T$ $E_2=(0, 1)^T$
5	0.7650	(1.5796, 1.718)	$\begin{bmatrix} -2.1323 & 0.2706 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1=-0.859$ $\lambda_2=-2.1323$	$E_1=(0.2078, 0.9781)^T$ $E_2=(0, 1)^T$



Phase Plane Figure (1)



Phase Plane Figure (2)



**5. Conclusions:** Change in  $c$  has a substantial impact on the Ammensal – Enemy Species. Initially, Ammensal Species is dominated by Enemy Species, but this is reversed with time. During this time, dominance reversal occurred twice at  $t_1^*$  and  $t_2^*$ . Thus,  $t_1^*$  decreases gradually and  $t_2^*$  rises gradually.

**6. Case(ii): The influence of  $d$  on Ammensal-Enemy Model:**

In order to investigate the interaction between Ammensal and Enemy Species, the following parameters are evaluated.

**Table(3):**

<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>e</b>	<b>f</b>	<b>g</b>	<b>X<sub>0</sub></b>	<b>Y<sub>0</sub></b>
<b>1.1100</b>	<b>0.889</b>	<b>2.515</b>	<b>changes</b>	<b>0.67</b>	<b>0.859</b>	<b>0.5</b>	<b>0.134</b>	<b>0.366</b>

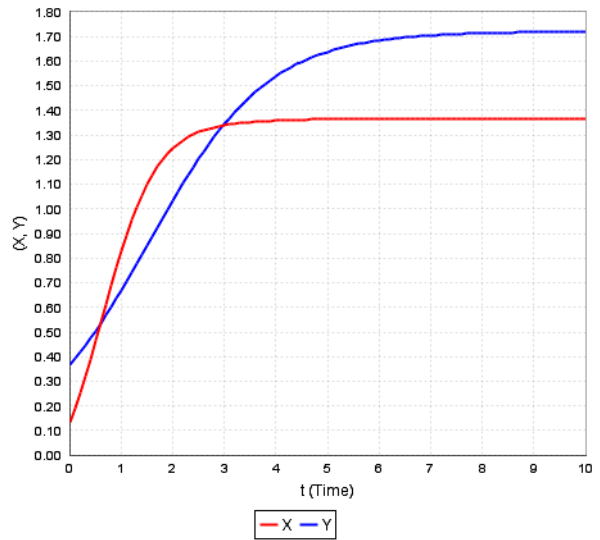
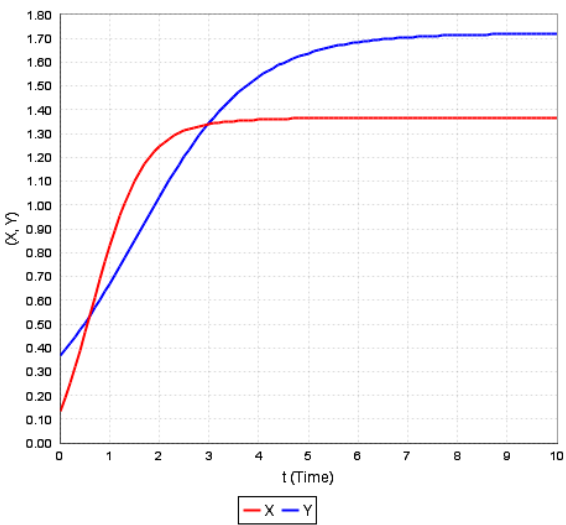
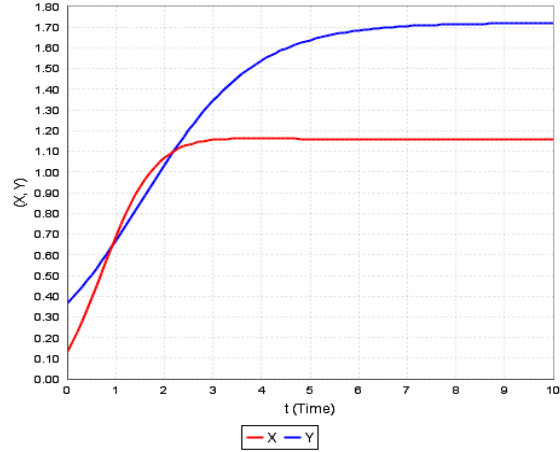
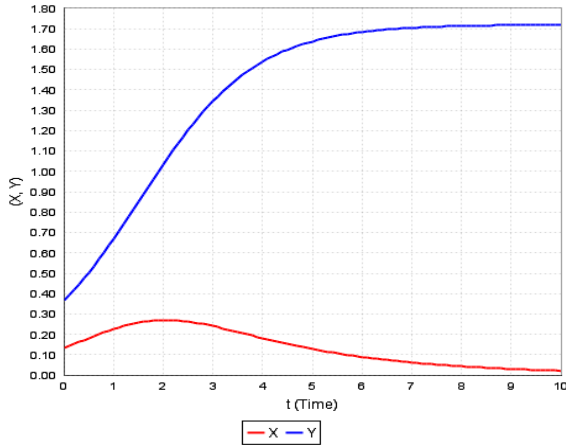
The effect of modifying the model's parameter  $c$  in account of prospective Dominant time instincts.

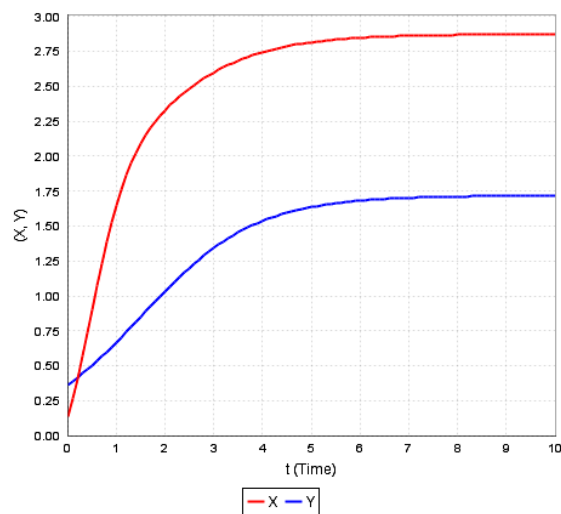
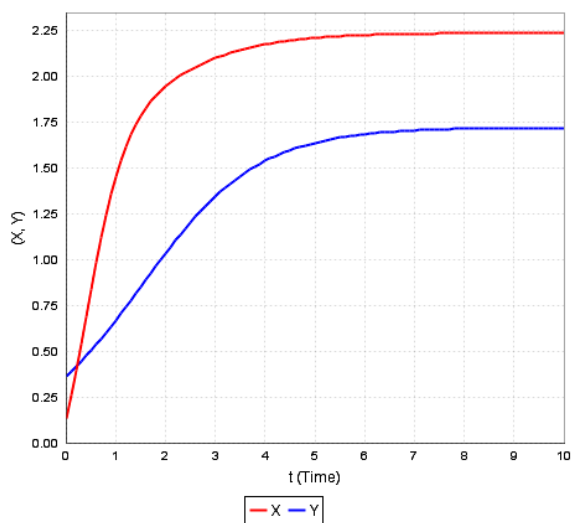
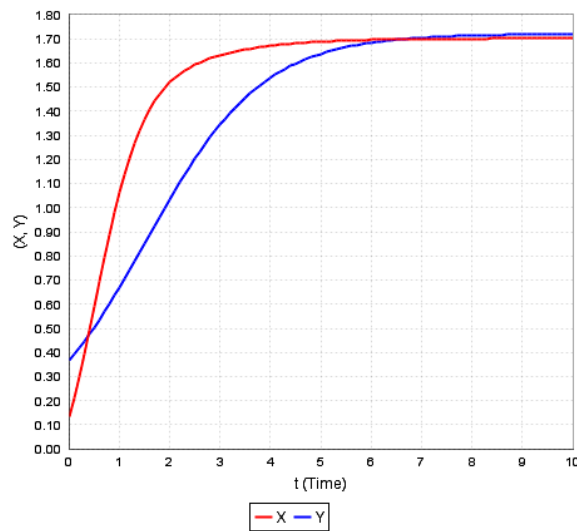
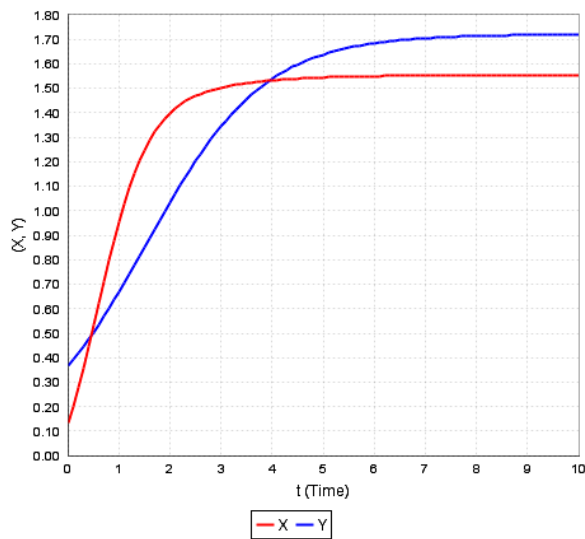
**Table(4):**

<b>S.No</b>	<b>d</b>	<b>t<sub>1</sub><sup>*</sup></b>	<b>t<sub>2</sub><sup>*</sup></b>
<b>1</b>	<b>0</b>	<b>*</b>	<b>*</b>
<b>2</b>	<b>0.2650</b>	<b>*</b>	<b>*</b>
<b>3</b>	<b>0.3600</b>	<b>0.95</b>	<b>2.19</b>
<b>4</b>	<b>0.4850</b>	<b>0.67</b>	<b>2.92</b>
<b>5</b>	<b>0.6100</b>	<b>0.45</b>	<b>3.9</b>
<b>6</b>	<b>0.7200</b>	<b>0.39</b>	<b>6.4</b>
<b>7</b>	<b>0.8900</b>	<b>0.3</b>	<b>*</b>
<b>8</b>	<b>1.1900</b>	<b>0.22</b>	<b>*</b>
<b>9</b>	<b>2.2050</b>	<b>0.11</b>	<b>*</b>
<b>10</b>	<b>3.0800</b>	<b>0.075</b>	<b>*</b>

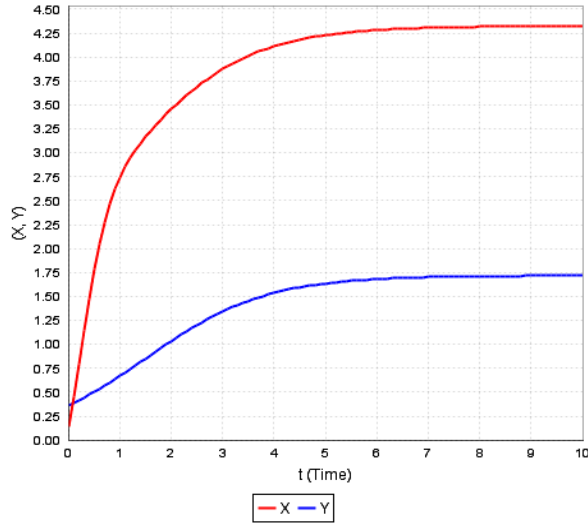
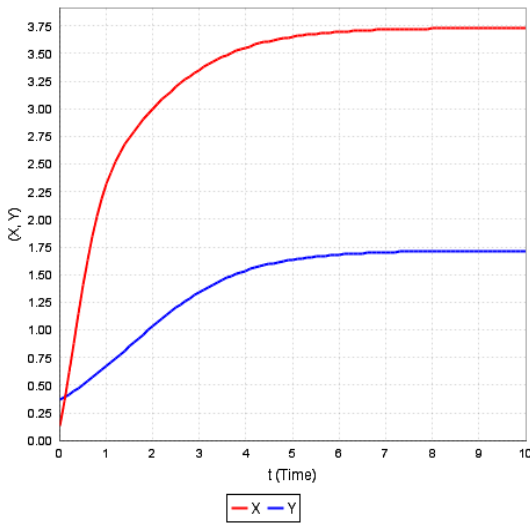
Fig (11) to Fig(20): when  $a=1.1100$ ,  $b=0.889$ ,  $c=2.515$ ,  $e=0.67$ ,  $f=0.859$ ,  $g=0.5$ ,  $X_0=0.134$ ,  $Y_0=0.366$

Vs change in b









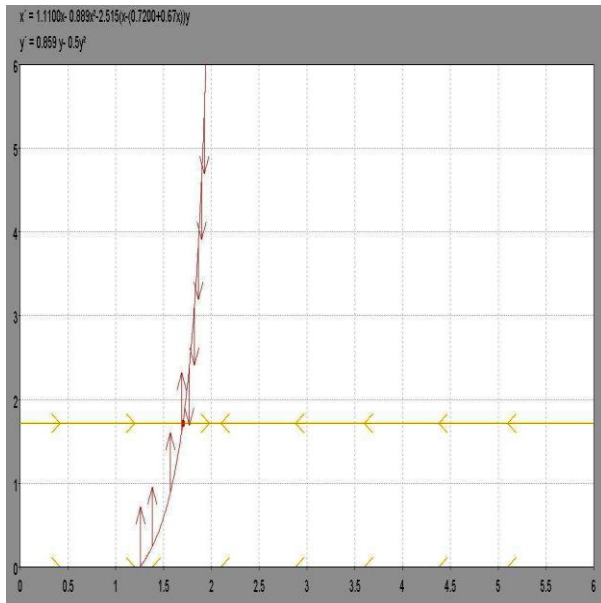
**7. Phase Plane Analysis:**

Phase plane analysis has been performed to analyse the model for establishing stability with equilibrium points whose Eigen values are known.

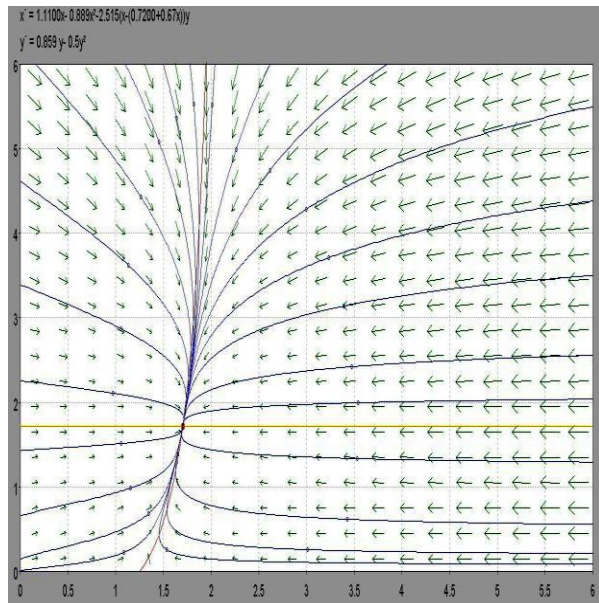
When  $a=1.1100, b=0.889, c=2.515, d=Changes, e=0.67, f=0.859, g=0.5$

S.No	Values of Parameter(d)	Equilibrium Point	Jacobian matrix	Eigenvalues	eigenvectors
1	0.2650	(0.9710, 1.718)	$\begin{bmatrix} -2.0424 & -0.13946 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -2.042$	$E_1 = (0.1170, -0.993)^T$ $E_2 = (0, 1)^T$
2	0.3600	(1.157, 1.718)	$\begin{bmatrix} -2.373 & -0.0548 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -2.373$	$E_1 = (0.0362, -0.999)^T$ $E_2 = (0, 1)^T$
3	0.4850	(1.3679, 1.718)	$\begin{bmatrix} -2.748 & 0.0844 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -2.748$	$E_1 = (0.0446, 0.999)^T$ $E_2 = (0, 1)^T$
4	0.6100	(1.5533, 1.718)	$\begin{bmatrix} -3.0777 & 0.24495 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -3.077$	$E_1 = (0.1097, 0.9939)^T$ $E_2 = (0, 1)^T$

5	0.7200	(1.7014, 1.718)	$\begin{bmatrix} -3.341 & 0.3986 \\ 0 & -0.859 \end{bmatrix}$	$\lambda_1 = -0.859$ $\lambda_2 = -3.341$	$E_1 = (0.1586, 0.9873)^T$ $E_2 = (0, 1)^T$



**Phase Plane Figure (3)**



**Phase Plane Figure (4)**

### 8. Conclusions:

Changes in the growth rate of Ammensal have a substantial effect on Ammensal-Enemy Species. The enemy species initially outnumber the ammensal species, but the ammensal species eventually outnumber the enemy species. In the meantime, dominance reversal happened at  $t_1^*$  and  $t_2$  twice. In this situation,  $t_1^*$  steadily declines and  $t_2^*$  gradually rises.

### 9. Overall Conclusions:

Change in the Parameter	Dominance Time	Nature
c increases	$t_1^*$ decreases $t_2^*$ increases	Nature of Dominance is reversed



<b>d increases</b>	<b>t<sub>1</sub>* decreases</b> <b>t<sub>2</sub>* increases</b>	<b>Nature of Dominance is reversed</b>

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