

Optimizing a PID Controller for Simulated Single-Joint Arm Dynamics

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ABSTRACT

The purpose of this article is to obtain a mathematical model for a single-joint system to improve the step response of a prosthetic arm using a Proportional Integral Derivative control system in order to address past, present, and future error. A controller is tweaked by placing poles to get better performance and optimize the following parameters for best outcome: response time, steady state error and overshoot. A 2 degree of freedom robotic arm is implemented using hardware components to mimic the arm movement and analyzing it.

The action of the PID is simulated with the open loop unstable system which ensured the set-point tracking of the closed loop system and maintained the stability of the closed loop system as both the transient and the steady state of the system is greatly improved. The results gotten are analysed both in the time and frequency domain which showed that the controller discarded steady state offset, damped oscillations and reduced overshoot while system stability was guaranteed.

Keywords— PID controller, Prosthetic arm, single-joint system

I. INTRODUCTION

PID controllers are the most common and widely used controllers for industrial automation although modern control method is desired like backstepping method for nonlinear systems. The reason for their wide usage is a result of their simplicity which is not often rigorous but require just a few tasks of tuning the parameters of the controller. In this work we try to apply a PID controller because it has proven to have consistent performance where the Proportional part functions to ensure set-point tracking, the Integral part takes away steady state error and then the derivative action will damp most oscillations that occur at steady state. We introduce a unique method of determining the Proportional, Integral and Derivative gains by placing the poles of the closed-loop system and then used it to obtain closed loop gains for the system control. This method has proven to be consistent as tuning is not required but just predetermined values of gains are derived. Simulink is used to tune the PID controller according to the transfer function derived and hence it can be used to optimize the movement. The hardware of robotic arm with 1 dimension of motion is given in fig-1.

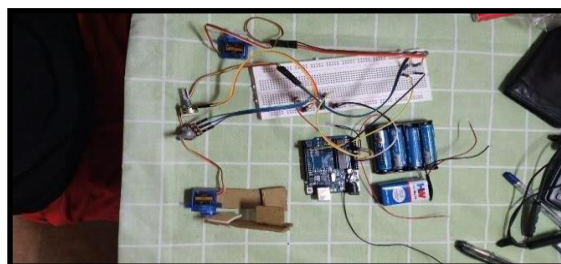


Figure-1

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III. INDENTATIONS AND EQUATIONS

1. Methodology

In order to provide information to the controller about whether the plant has performed its task or not, a closed loop system is used so the controller knows what the plant is actually doing. The output from the plant is monitored and feedback is provided to the controller, which is then compared with the system input to determine deviations from the expected output, allowing the controller to make any necessary adjustments. This allows the system to counteract errors and decrease response time given below in fig.2.

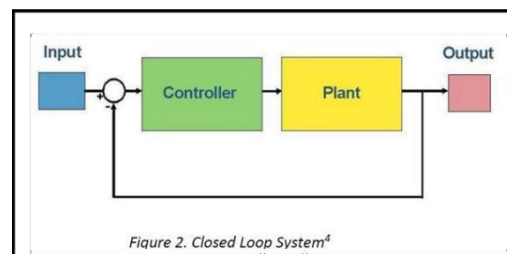


Figure-2

2. Response parameters

The main reason behind using a feedback loop is to minimize the errors. A few basic parameters defined below are focused on to improve the output response of the feedback loop. Each parameter represents the

behavior of a particular movement of the prosthetic arm controlled by an input voltage

- Rise time: The duration for which majority of the movement of arm is noticed.
- Overshoot: The displacement of arm past the desired output.
- Settling time: Time taken by arm to reach its final position.
- Steady-state error: The difference between the actual position of the arm and the desired position of the arm.
- Oscillation: The arm cycles back and forth between the desired location until it settles to its final position.

The optimal system would have the shortest possible rise time and settling time; as well as the smallest steady-state error and overshoot. The response parameters are explained in the fig. 3 given below.

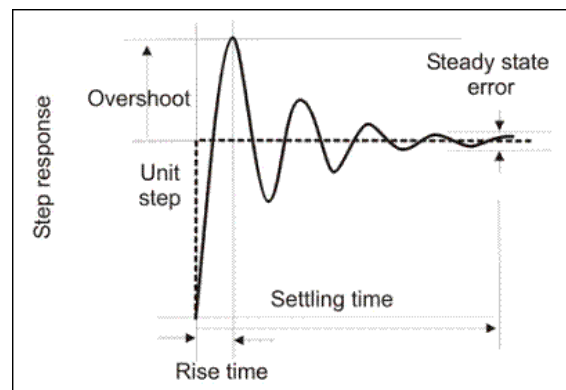
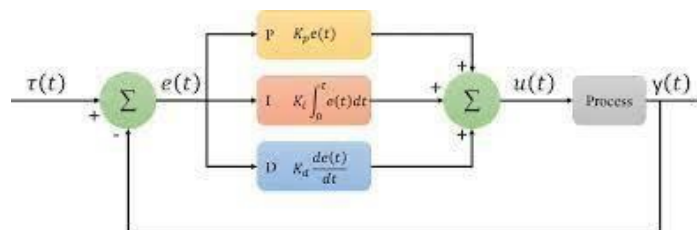


Figure-3

3. PID controller

The combination of proportional control action, integral control action and the derivative control action is called the PID-control action. The proportional controller stabilizes the gain but produces a steady state error. The integral controller reduces the steady state error. The derivative controller reduces the rate of change of error. The controller is manipulated with the help of constants K_p , K_i and K_d explained in the figure below.

Figure-4



Proportional Control: (K_p)

The proportional controller improves the steady-state tracking accuracy, disturbance signal rejection and relative stability of the system. It also increases the loop gain of the system which results in reducing the sensitivity of the system to parameter variations. The drawback in P-controller is that it develops a constant steady-state error.

Integral Control: (K_i)

The integral controller removes or reduces the steady error without the need for manual reset. Hence

the I-controller is sometimes called automatic reset. The drawback in integral controller is that it may lead to oscillatory response of increasing or decreasing amplitude which is undesirable, and the system may become unstable.

Derivative Control: (Kd)

The derivative controller focuses on the “future” error, which is done by taking the derivative of the error signal and multiplying by the KD constant. The derivative portion helps to improve overshoot, rise time, and settling time.

4. Arm Model

After considering the torque balance between inertia and friction, the torque for the elbow joint can be modelled by Ordinary Differential Equation (ODE):

$$J*\theta'' + f*\theta' = r + Mgl*\cos(\theta) \quad \text{---(1)}$$

The equation is non-linear and hence we assume that gravity will not affect the movement of the arm in horizontal motion. Now since the range of motion and gravitational field are perpendicular therefore the $\cos(\theta)$ component becomes zero:

$$J*\theta'' + f*\theta' = r \quad \text{---(2)}$$

J = inertia of arm.

θ = angle of rotation

f = friction coefficient of joint

r = actuator torque

The ordinary differential equation considers the torques, friction and inertia resisting an applied torque from the actuator. Since, we are modelling the output position to a step input, the system without control will never stop at a particular position. After adding a feedback loop, the actual and final position can be compared, and voltage can be increased or reduced to get to target position.

4.1 Arm model with resistance band

We need an extra torque dependent on the position itself, so a “spring” term was added to get a stable open loop response:

$$J*\theta'' + f*\theta' + Rb*\theta = r \quad \text{---(3)}$$

Rb = Resistance Band (Spring Constant)

To linearize the “spring” term, we assumed a straight trajectory for the resistive band instead of the actual circular path. All assumptions remain the same as the original arm model.

5. Derivation of transfer functions

Transfer function of arm

Start with the Arm Model. Take the Laplace and assume initial conditions are 0, as the arm is not moving initially:

$$\theta(s)*(Js^2 + fs) = T(s) \quad \text{---(4) Solve for the}$$

Transfer Function and multiply by (1/f)/(1/f).

$$G_{arm}(s) = \frac{\theta(s)}{T(s)} = \frac{1}{s(Tms+1)} \quad \text{where } Tm = \frac{I}{f} \quad \text{---(5)}$$

Assume Input Voltage and Torque are linearly proportional:

$$r=Y*EA \quad \text{---(6)}$$

$$T(s) = Y*Ea(s)$$

$$G_{arm}(s) = \frac{\theta(s)}{Ea(s)} = \frac{C}{Tms^2+s}$$

where $C = \frac{A}{f}$

Solve for constants X and Y.

$$Tm = \frac{I}{f} = 3.95*10^{-2}m^2 \text{ kg}$$

$$A = \frac{e}{Ea} = 2.31*10^{-1}mkg/V$$

$$C = \frac{A}{f} = 1.15mkg/V \quad \text{---(7)}$$

Transfer function of arm with resistance band

Start with the Arm Model with Resistance Band Equation. Take the Laplace and assume initial conditions are 0, as the arm is not moving initially.

$$\theta(s)(Js^2+fs+Rb) = T(s) \quad \text{---(8)}$$

Solve in a similar manner as the above Transfer function of the arm.

$$G_{arm \text{ modified}} = \frac{\theta(s)}{Ea(s)} = \frac{C}{Tms^2 + s + \frac{Rb}{f}} \quad \text{---(9)}$$

$$G_{PID}(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_D s$$

From the above diagram, writing equation for our system

$$G_{system}(s) = \frac{G_{arm}(s)G_{PID}(s)}{1 + G_{arm}(s)G_{PID}(s)}$$

$$G_{system}(s) = \frac{s^2(CK_D) + s(CK_P) + CK_i}{s^2(Tm) + s^2(1 + CK_D) + s(CK_P) + CK_i} \quad \text{---(10)}$$

6. Optimization methods

A method is chosen based on the response specific to the system its constraints.

Manual

The simplest optimization method is manual tweaking of PID parameters. A simulation can be set up in

MATLAB to evaluate the numeric value of the response for a certain time range. Using what is known about each of the different control terms in PID, parameters are changed and the updated response is evaluated for quality. This method is essentially guess-and-check.

Cohen-Coon

The Cohen-Coon optimization method is done by analysing the open-loop response of a system and getting timevalues for when the response is 50% of the steady state value and 63.2% of the steady state value. These time values are used with a predetermined table specific to this method to compute the values of the control constants. This method is most appropriate for systems that have a relatively long rise time.

Simulink

The Simulink module in MATLAB is a very user-friendly method, provided that the system model can be created. To start optimization, simply “tune” the PID controller block in the model. The program will show you the current and “tuned” response, the latter of which will change as the properties sliders are adjusted. Simply raising the sliders to max will usually not yield a perfect result, as there are tradeoffs between desirable system properties.

7. Result

Arm without resistive band

The uncontrolled system with no resistive band forms an infinite ramp. This corresponds to the arm spinning at a constant angular velocity. This is because a step torque is being input into the system, which eventually forms an equilibrium with the friction and inertia forces, and maintains a constant angular velocity. The arm is not actually going to an infinite position value, rather the arm keeps rotating, and the output can be rewritten as an angular position, along with a certain number of complete rotations.

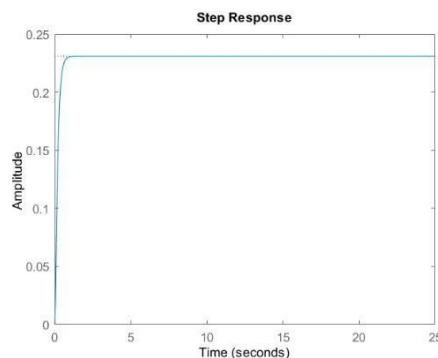


Figure-5 Steady state error of model with resistive

band:0.7691

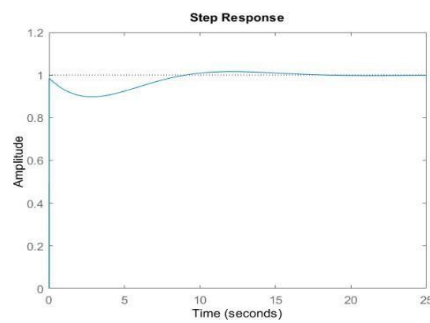
Rise time of model with resistive band:0.35463

Control constants

Kp is:19.1485

Ki is:9.077

Kd is:57.5846



Cohen-coon optimization method

Figure-6

Overshoot in Cohen-coon optimized control: 0.015883

Steady state error in Cohen-coon optimized control: 0.0010668

Rise time in Cohen-coon optimized control: 0.00136

Simulink

Arm without resistive band

The tuning settings for a plant with no resistance are simple. Both sliders are just put to the maximum value and the cleanest and fastest response is achieved. The physical behavior of this system is like the manually tuned system, with a rapid convergence on the target position, and no overshoot. There is a key difference in the controller itself, in that the derivative gain is much lower than that of the manually tuned system, which makes the system more resistant to noisy signals.

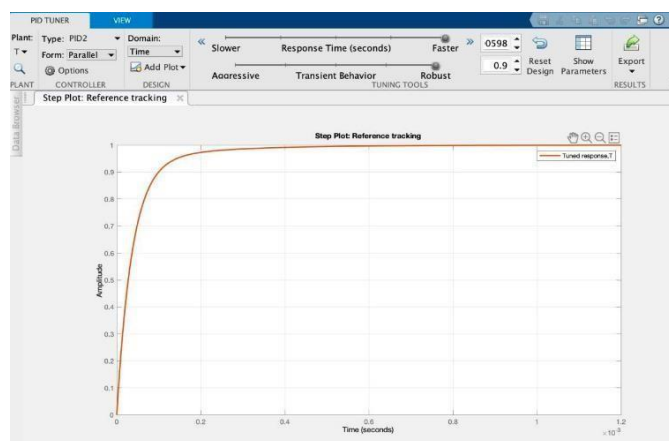


Figure-7

Arm with resistive band

The tuning settings for an arm with a resistance illustrate some of the tradeoffs involved in control system optimization. A faster response time can be specified, though this will come at the expense of either a significant steady state error (approximately 10%), or some overshoot in the response. Since this response is still very fast given the physical requirements, with a rise time of only 40 ms, stability of the response has been prioritized over response speed.

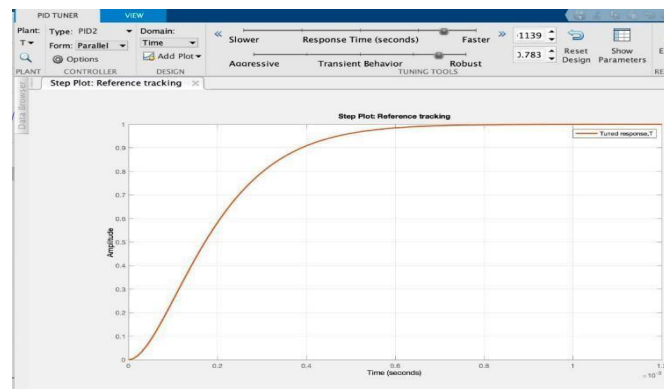


Figure-8

8. Model limitations

There are several limitations in our mathematical model. The main limitation in our model is our inability to include gravitational force, and thus an inability to model movement in the vertical plane. Another limitation in our model is the lack of multiple degrees of freedom. Theoretically, each joint would be independent of the surrounding joints when considering its own angular displacement, but the presence of inertia and momentum from other members would make each joint dependent on the other joints. This is made even more significant if we decide to model a multi-joint arm that is moving in the vertical plane with the influence of gravity.

9. Conclusion

We attempted to model a single joint arm that has a range of motion perpendicular to gravity, and with no external load. A torque was applied to the arm, and the forces that resist this torque are the inertia of the arm and the friction of the joint. It was found that this system alone would not produce a stable step response without a feedback loop, so a resistive band was added to stabilize the position in an open loop system. Different optimization methods were applied for the PID controllers of each of these systems. The methods utilized were manual tuning, Cohen-Coon optimization, and tuning in the Simulink module in MATLAB. It was determined that tuning with Simulink would give the best overall PID solution in both scenarios. The Simulink UI was very useful in quickly observing a range of solutions, as opposed to using the guess and check method of manual tuning. To improve this model, we can attempt to introduce gravity into this model and add multiple joints.

Introduction of multiple joints would create dependencies on the momentums of other joints. This effect would be magnified if gravity was also introduced into the model. An issue with the introduction of gravity in the model is the nonlinear equation that results from an accurate representation.

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