



BULK ARRIVAL AND DEPARTURE IN NON-RELIABLE SERVER

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ABSTRACT

In most queueing situations, it is considered that clients arrive at a service facility individually. In many real-world queueing scenarios, however, this assumption is broken. Customers at a bank or a reservation desk call coming in on a phone keyboard, machinery in need of maintenance, merchandise for shipping at a yard, mail arriving at a post office, People arriving in convoys at a port, a theatre, a restaurant, and so on are instances of queueing scenarios when customers do not arrive in bulk or groups. Mathematically and from a practical point of view, scenarios, when the size of an arriving group is a random variable, are more general and are likewise challenging to handle.

Keywords: Bulk Arrival and departure of customer, FCFS, Non-reliable Server, Queueing Model, Service breakdown, Queue Length.

INTRODUCTION

In addition to solving a number of practical issues in telephony, Erlang set the groundwork for queueing theory in terms of assumptions and techniques of analysis, which are still widely employed in modern communications and computer systems. Analysis of operational difficulties was something Erlang did well before anyone else. Operational research can be traced back to some of his work. A generic name for a customer is a server. Customers are people who arrive at a facility in need of a service and are able to use it. When a customer or unit is fed into the system, it is referred to as a "server" or "service channel." This can include customers at a bank or reservation counter or phone lines, machines with a serviceman needing maintenance, and so on. A server is a component of a computer system that provides a service to customers. The term "in service" refers to someone who is receiving assistance from a customer service representative. When a "customer" arrives at a server that is already full, they form or join a queue. In our daily lives, we encounter a variety of scenarios when we must wait in a queue. Some examples of this include patients waiting in front of the doctor's room, machines that need to be fixed, and automobiles that need a signal at a traffic light. The queues will continue to grow as long as there are people waiting to get in. There will be a wide range of queue formations around the world. It's crucial to remember that the client and the server are both involved in a waiting line. In order to better understand congestion, queueing systems are used as simple mathematical models. Any time "customers" desire



"service" from some facility, a queueing system is likely to develop. Typically, the arrival of customers and service timings are thought to be random. In the event that all of the "servers" are already full, new clients will typically wait in line for the next available server to open. The arrival pattern, the service mechanism, and queue discipline are the three components of a simple queueing system. Stochastic processes connected with queues are usually responsible for defining queues' characteristics from a probabilistic perspective. Stochastic processes were important to how Kendall (1951, 1953) approached queueing theory. More and more books have been written about queueing theory and the many different ways it can be applied over time. See Prabhu (1987), Takagi (1991), Dshalalow (2005, 2006), as well as the works of Takagi and Boguslavsky (2005, 2006) for a list of publications (1990). The last group comprises important monographs whose titles were first published in Russian. Hlynka's website has a list of the novels he's written. Waiting lines, or "queues," are the subject of queueing theory. A queue is formed when the demand for service is greater than the capacity to offer it at the time. As soon as a client or unit arrives at a service facility, they are forced to wait in a queue if the facility doesn't have direct access to the service they're looking for. There are also instances where customers abandon the system without joining the queue or without receiving assistance after waiting for some time. However, in all but the simplest queues, it is extremely difficult to determine the state probabilities. However, in many cases, the so-called equilibrium or steady-state distribution can be determined over a long period of time. The initial circumstances of the system have no effect on this distribution, which means it is stationary. Little's law is a very useful formula for systems in equilibrium where all that is needed are the expected values. For the most part, the parameters that drive queueing models are believed to be pre-determined in queueing theory's great majority of research.

BASIC CHARACTERISTICS

A queueing system's fundamental properties are as follows:

1. Customers' patterns of arrival and entry; patterns of service;
2. An organization's capacity to provide service
3. The system's capability; and
4. WAITING in line
5. Number of phases of service.

NOTATIONS: Here we have assumed different

case as; $i = 0$ represents the server turned

off then the probability is given by $P_0(r)$ where

$r = 0, 1, 2, \dots, N-1$; $i = 1$ The server is running; then the probability is $P_1(r)$, where $r =$

$1, 2, \dots$; $i = 2$ Represents when the server is turned on and found to be in a broken state denoted by $P_2(r)$.

STEADY-STATE EQUATIONS

We observe that only $P_n(0)$ exists when $n = 0, 1, 2, \dots, c-1, c, \dots, r-1, r$; both $P_n(0)$ and $P_n(1)$ exist when $n = r+1, r+2, \dots, R-2, R-1$; only $P_n(1)$ exists when



$$n = R, R+1, \dots, N. \text{ Further } P(0) = P(1) = 0 \text{ if } n > N.$$

The equations for the steady state are as follows:

$$0 = -(10 - \hat{1}) p \phi P_0(0) + (m - \hat{1}) P_1(0)$$

$$0 = - \left[\left(\frac{n}{(N-1)} \right) (\lambda_0 - \epsilon) + n(\mu - \epsilon) \right] P_n(0) + (n+1)(\mu - \epsilon) P_{n+1}(0) + \left(\frac{n-1}{(N-1)} \right) (\lambda_0 - \epsilon) P_{n-1}(0) \quad (n = 2, 3, \dots, c-1)$$

$$0 = - \left[\left(\frac{n}{(N-1)} \right) (\lambda_0 - \epsilon) + c(\mu - \epsilon) \right] P_n(0) + c(\mu - \epsilon) P_{n+1}(0) + \left(\frac{n-1}{(N-1)} \right) (\lambda_0 - \epsilon) P_{n-1}(0) \quad (n = c, c+1, \dots, r-1)$$

$$0 = - \left[\left(\frac{r}{(N-1)} \right) (\lambda_0 - \epsilon) + c(\mu - \epsilon) \right] P_r(0) + c(\mu - \epsilon) P_{r+1}(0) + c(\mu - \epsilon) P_{r+1}(1) + \left(\frac{r-1}{(N-1)} \right) (\lambda_0 - \epsilon) P_{r-1}(0)$$

$$0 = - \left[\left(\frac{n}{(N-1)} \right) (\lambda_0 - \epsilon) + c(\mu - \epsilon) \right] P_n(0) + c(\mu - \epsilon) P_{n+1}(0) + \left(\frac{n-1}{(N-1)} \right) (\lambda_0 - \epsilon) P_{n-1}(0) \quad (n = r+1, r+2, \dots, R-2)$$

$$0 = - \left[\left(\frac{R-1}{(N-1)} \right) (\lambda_0 - \epsilon) + c(\mu - \epsilon) \right] P_{R-1}(0) + \left(\frac{R-2}{(N-1)} \right) (\lambda_0 - \epsilon) P_{R-2}(0)$$

$$0 = - \left[\left(\frac{r+1}{(N-1)} \right) (\lambda_1 - \epsilon) + c(\mu - \epsilon) \right] P_{r+1}(1) + c(\mu - \epsilon) P_{r+2}(1)$$

$$0 = - \left[\left(\frac{n}{(N-1)} \right) (\lambda_1 - \epsilon) + c(\mu - \epsilon) \right] P_n(1) + c(\mu - \epsilon) P_{n+1}(0) + \left(\frac{n-1}{(N-1)} \right) (\lambda_1 - \epsilon) P_{n-1}(1) \quad (n = r+2, r+3, \dots, R-1)$$

$$0 = - \left[\left(\frac{R}{(N-1)} \right) (\lambda_1 - \epsilon) + c(\mu - \epsilon) \right] P_R(1) + c(\mu - \epsilon) P_{R+1}(1)$$

$$+ \left(\frac{R-1}{(N-1)} \right) (\lambda_0 - \epsilon) P_{R-1}(0) + \left(\frac{R-1}{(N-1)} \right) (\lambda_1 - \epsilon) P_{R-1}(1)$$

$$0 = - \left[\left(\frac{n}{(N-1)} \right) (\lambda_1 - \epsilon) + c(\mu - \epsilon) \right] P_n(1) + c(\mu - \epsilon) P_{n+1}(1) + \left(\frac{n-1}{(N-1)} \right) (\lambda_1 - \epsilon) P_{n-1}(1) \quad (n = R+1, R+2, \dots, N-1)$$

$$0 = -c(\mu - \epsilon) P_N(1) + \left(\frac{n-1}{(N-1)} \right) (\lambda_1 - \epsilon) P_{N-1}(1)$$

(2.3.1) provides us with the following information:

$$P_1(0) = \left[\frac{\lambda_0 - \epsilon}{(\mu - \epsilon)} \right] p' P_0(0)$$

With the result in (2.3.2), we arrive at the following conclusion

$$P_2(0) = \left[\frac{1}{(N-1)} \right] \left[\frac{(\lambda_0 - \epsilon)^2}{2(\mu - \epsilon)^2} \right] p' P_0(0)$$

Recursively deriving from the conclusion in (2.3.3) above, we arrive to,



$$P_n(0) = \left[\frac{1}{N-1} \right]^{n-1} \left[\frac{(\lambda_0 - \epsilon)^n}{n(\mu - \epsilon)^n} \right] p' P_0(0), \quad n = 1, 2, \dots, c-1$$

The loss probability is provided by $n = c$ if $n > c$.

$$P_c(0) = \left[\frac{1}{N-1} \right]^{c-1} \left[\frac{(\lambda_0 - \epsilon)^c}{c(\mu - \epsilon)^c} \right] p' P_0(0),$$

Recursively deriving from the above result in (2.3.4)

$$P_n(0) = \left[\frac{1}{N-1} \right]^{n-1} \left[\frac{(\lambda_0 - \epsilon)^n}{(\mu - \epsilon)^n} \right] \frac{(n-1)!}{c^{n-c}} p' P_0(0), \quad n = c+1, c+2, \dots, r$$

As a result of applying (2.3.15) to

$$P_{r+1}(0) = \left[\frac{r!}{(N-1)^r} \right] \left[\frac{(\lambda_0 - \epsilon)^{r+1}}{(\mu - \epsilon)^{r+1}} \right] \frac{1}{c^{r-c+1}} p' P_0(0) - P_{r+1}(1)$$

Recursively deriving from the above result in (2.3.6)

$$P_n(0) = \left(\frac{(n-1)!}{(N-1)^{n-1}} \right) \left(\frac{\lambda_0 - \epsilon}{\mu - \epsilon} \right)^n \frac{1}{c^{n-c}} p' P_0(0) - \left[\left(\frac{\lambda_0 - \epsilon}{c(N-1)(\mu - \epsilon)} \right)^{n-r-1} (n-1)P_{n-r-1} + \left(\frac{\lambda_0 - \epsilon}{c(N-1)(\mu - \epsilon)} \right)^{n-r-2} (n-1)P_{n-r-2} + \dots + \left(\frac{\lambda_0 - \epsilon}{c(N-1)(\mu - \epsilon)} \right)^{n-R+1} (n-1)P_{n-R+1} \right] P_{r+1}(1), \quad n = r+1, r+2, \dots, R-1$$

(2.3.7) gives us this result.

$$P_{r+1}(1) = \frac{p' \frac{(R-1)!}{(N-1)^{R-1}} \left(\frac{\lambda_0 - \epsilon}{\mu - \epsilon} \right)^R \frac{1}{c^{R-c}} P_0(0)}{\left\{ \frac{(R-1)!}{r!} \left(\frac{\lambda_0 - \epsilon}{c(N-1)(\mu - \epsilon)} \right)^{R-r-1} + \frac{(R-1)!}{(r+1)!} \left(\frac{\lambda_0 - \epsilon}{c(N-1)(\mu - \epsilon)} \right)^{R-r-2} + \dots + \frac{(R-1)!}{(R-2)!} \left(\frac{\lambda_0 - \epsilon}{c(N-1)(\mu - \epsilon)} \right) + 1 \right\}}$$

(2.3.8) provides us with the following information:

$$P_{r+2}(1) = \left[\frac{(r+1)}{(N-1)} \left(\frac{\lambda_1 - \epsilon}{c(\mu - \epsilon)} \right) + 1 \right] P_{r+1}(1)$$

Recursively, we use the result from (2.3.9) to build our own.

$$P_n(1) = \left[\left(\frac{\lambda_1 - \epsilon}{c(N-1)(\mu - \epsilon)} \right)^{n-r-1} (n-1)P_{n-r-1} + \left(\frac{\lambda_1 - \epsilon}{c(N-1)(\mu - \epsilon)} \right)^{n-r-2} (n-1)P_{n-r-2} + \dots + \left(\frac{\lambda_1 - \epsilon}{c(N-1)(\mu - \epsilon)} \right)^{n-R} (n-1)P_{n-R} \right] P_{r+1}(1), \quad n = r+1, r+2, \dots, R-1, R$$

Recursively deriving from the results in (2.3.10) and (2.3.11)

$$P_n(1) = \left[\left(\frac{\lambda_1 - \epsilon}{c(N-1)(\mu - \epsilon)} \right)^{n-r-1} (n-1)P_{n-r-1} + \left(\frac{\lambda_1 - \epsilon}{c(N-1)(\mu - \epsilon)} \right)^{n-r-2} (n-1)P_{n-r-2} + \dots + \left(\frac{\lambda_1 - \epsilon}{c(N-1)(\mu - \epsilon)} \right)^{n-R} (n-1)P_{n-R} \right] P_{r+1}(1), \quad n = R+1, R+2, \dots, N-1, N$$

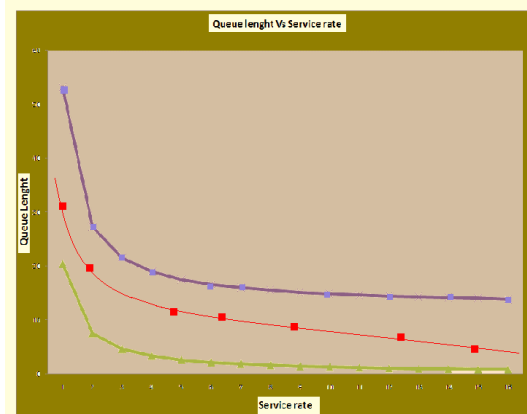
Hence

$$P_n(1) = \left[\left(\frac{\lambda_1 - \epsilon}{c(N-1)(\mu - \epsilon)} \right)^{n-r-1} (n-1)P_{n-r-1} + \left(\frac{\lambda_1 - \epsilon}{c(N-1)(\mu - \epsilon)} \right)^{n-r-2} (n-1)P_{n-r-2} + \dots + \left(\frac{\lambda_1 - \epsilon}{c(N-1)(\mu - \epsilon)} \right)^{n-R} (n-1)P_{n-R} \right] P_{r+1}(1), n = r+1, r+2, \dots, N-1, N$$

Consequently, from (2.3.13) to (2.3.21), we see that all steady-state probabilities are stated in terms of P0 (0).

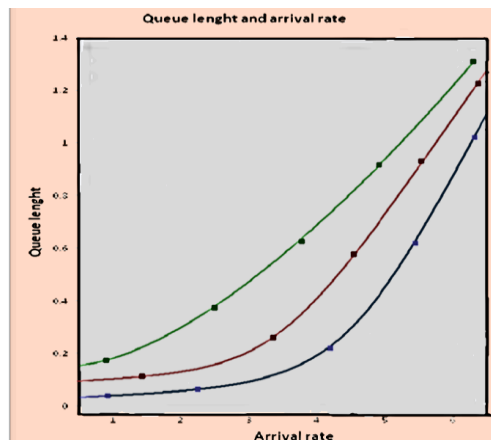
THE MOTIVATION

Hospitals, production systems, banks, and restaurants, for example, all use time-dependent solutions of single servers with feedback and retrial queues. Priority patients can proceed directly to treatment if they have a doctor's appointment scheduled at a specific time. As a result, patients are forced to wait in huge lines. This form of queuing model can be studied with the help of a time-dependent solution. Patients are given prompt attention if a doctor is available in the clinic. If the doctor is swamped, patients will have to wait in an endless line. Patients' requests are halted and must be resubmitted at a later time. The retry queue is a term for this type of repeated attempt. Retrial sufferers create a lineup, and only those at the front of that line can ask for his assistance. It immediately jumps to the head of the line to receive the services the patients demand when they are not pleased with his or her service or are in need of additional assistance.



This graph shows that the number of customers decreases when the service rate increases.

Graph-I



This graph shows that after the repair rate number of customers increases in the system.



Graph-II

AIMS AND OBJECTIVES

By employing time-dependent solutions for single and multi-server queues, we hope to shorten queue lengths in real-world situations. In a hospital, for example, doctors are available in the clinic, allowing patients to receive care right away. As a result, the doctor is swamped with patients, and the line becomes ever longer. Patients' requests are halted and must be resubmitted at a later time. The retry queue is a term for this type of repeated attempt. Retrial consumers and comments from non-retrial customers can be modelled as time-dependent solutions in this hospital management system. Patients can bypass the feedback backlog and receive service fast and within a time restriction if the server provides effective service. To avoid having to wait in a feedback queue, the consumer does some prior work before receiving his service. This study's goal is to determine

- An investigation of a single server feedback queue with and without a time-dependent solution is being conducted.
- A time-dependent solution of retry and feedback queue
- To give a time-dependent solution for a single server feedback queue when the server is on vacation
- To investigate a multi-server feedback and retry queue solution that is time dependent.
- Show numerical evidence on feedback and retrial queue with varied catastrophic effects.
- To determine the average queue length's asymptotic behaviour

CONCLUSION

In this paper, we have studied a Queueing Model of having a Non-reliable server in which the arrival of customers is in bulk. There are many situations in real life in which this type of problem may exist. The number of customers in the system is calculated after finding the value when the system is in an idle state, working state and when the system is in a broken state. Various graphs are plotted by using tabulated values. It can be concluded from the graph that if the service rate increases queue length is also decrease. Analyzed single server batch arrival queuing systems with double threshold policy, early setup operation and vacations in which the server is typically subject to unpredictable breakdowns while serving the customers

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