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AN ANALYSIS OF FUZZY BINARY SOFT MAPPINGS ON FUZZY BINARY SOFT CLASSES

Dr.Ashwani Sethi [1]

[1]University College of Basic Science & Humanities
[1]Guru Kashi University, Talwandi Sabo

ABSTRACT

In the present paper, we introduce fuzzy binary soft mapping on fuzzy binary soft topological spaces. It is defined on fuzzy binary soft classes of fuzzy binary soft sets over two initial sets $\mathbb{U}_{\xi}^{1}$, $\mathbb{U}_{\xi}^{2}$ with fixed set of parameters. In this extension we define fuzzy binary soft continuity using such classes of fuzzy binary soft mapping. We give needed examples. We prove the properties of fuzzy binary soft images and fuzzy binary soft inverse images in detail. The findings are given in the form of Theorems.

Keywords: Fuzzy binary soft set, Fuzzy binary soft mapping, fuzzy binary soft class and fuzzy binary soft continuity.

AMS Subject Classification (2020): 54A40, 54C05, 54C10.

1. INTRODUCTION

In 1965, Prof L. Zadeh [8] introduced new concept for uncertainty called fuzzy set theory. Molodtsov [6] initiated a soft set in 1999. Maji et al [4] investigated a new approach including both fuzzy sets and soft sets. Athar Kharal and B. Ahmad studied [2] fuzzy soft mapping on fuzzy soft sets.In 2016, Ahu Acikgoz and Nihal Das [1] first defined the basic structures of binary soft set theory in two universal sets and its basic properties. Further binary soft mapping was introduced by Sabir Hussain [7]. In 2020, We [3, 5] have introduced fuzzy binary soft set and fuzzy binary soft topological spaces over $\mathbb{U}_{\xi}^{1}$, $\mathbb{U}_{\xi}^{2}$. In this paper in section 2, we gave needed definition for our main result. In section 3, we introduced and studied fuzzy binary soft mapping on fuzzy binary soft classes and proved its properties.

2. PRELIMINARIES

Definition 2. 1 [3]

Let $\mathbb{U}_{\xi}^{1}$, $\mathbb{U}_{\xi}^{2}$ be the two initial universal sets, E_P be a set of parameters and $F_{fbs}(\mathbb{U}_{\xi}^{1})$, $F_{fbs}(\mathbb{U}_{\xi}^{2})$ denotes the set of all fuzzy sets over $\mathbb{U}_{\xi}^{1}$, $\mathbb{U}_{\xi}^{2}$ respectively. Also let $A_P \subseteq E_P$. Then (F_{fbs}, A_P) is said to be "***fuzzy binary soft set***

" over $\mathbb{U}_{\xi}^{1}, \mathbb{U}_{\xi}^{2}$, where F_{fbs} is a mapping given by $F_{fbs} : A_P \to F(\mathbb{U}_{\xi}^{1}) \times F(\mathbb{U}_{\xi}^{2})$, $F_{fbs}(p) = (S, T)$ for each $p \in A_P$

such that $S \subseteq \mathbb{U}_{\xi}^{1}$, $T \subseteq \mathbb{U}_{\xi}^{2}$.

Definition 2.2 [3]

Let (F_{fbs}, A_P) and (G_{fbs}, B_P) be two fuzzy binary soft sets over $\mathbb{U}_{\xi}^{1}, \mathbb{U}_{\xi}^{2}$, (F_{fbs}, A_P) is called a ***"fuzzy binary soft subset"*** of (G_{fbs}, B_P) if

(i) $A_P \subseteq B_P$,

(ii) $F_{fbs}(p)$ is the fuzzy subset of $G_{fbs}(p)$ for each $p \in A_P$ and is denoted by $(F_{fbs}, A_P) \tilde{\subseteq} (G_{fbs}, B_P)$, briefly.$(F_{fbs}, A_P)$ is said to be a ***"fuzzy binary soft superset"*** of (G_{fbs}, B_P) if (G_{fbs}, B_P) is a fuzzy binary soft subset of (F_{fbs}, A_P).

That is $(F_{fbs}, A_P) \tilde{\supseteq} (G_{fbs}, B_P)$.

Definition 2.3 [3]

The "***complement of a fuzzy binary soft set***" (F_{fbs}, A_P) is denoted by $(F_{fbs}, A_P)^c$ and is defined by $(F_{fbs}, A_P)^c \cong (F_{fbs}{}^c, \rceil A_P)$, where $F_{fbs}{}^c : \rceil A_P \to F_{fbs}(U^1) \times F_{fbs}(U^2)$ is a mapping given by $F_{fbs}{}^c(p) = (F_{fbs}(\rceil p))^c$ for all $\rceil p \in A_P$.

Definition 2.4 [3]

A fuzzy binary soft set (F_{fbs}, A_P) over $\mathbb{U}_{\xi}^{1}, \mathbb{U}_{\xi}^{2}$ is said to be a "***fuzzy binary null soft set"*** if for all $p \in A_P$, $F_{fbs}(p)$ is the null fuzzy set over $\mathbb{U}_{\xi}^{1}, \mathbb{U}_{\xi}^{2}$ and is denoted by $\tilde{0}$.

Definition 2.5 [3]

A fuzzy binary soft set (F_{fbs}, A_P) over $\mathbb{U}_{\xi}^{1}, \mathbb{U}_{\xi}^{2}$ is said to be a "***fuzzy binary absolute soft set***" if for all p $\in A_P$, $F_{fbs}(p)$ is the absolute fuzzy set over $\mathbb{U}_{\xi}^{1}, \mathbb{U}_{\xi}^{2}$ and is denoted by $\tilde{A}$.

Definition 2.6 [3]

"Union of two fuzzy binary soft sets" (F_{fbs}, A_P) and (G_{fbs}, B_P) over $\mathbb{U}_{\xi}^{1}, \mathbb{U}_{\xi}^{2}$ is the fuzzy binary soft set ,

where $C_P = A_P \cup B_P$ and for each $p \in C_P$,

$$H_{fbs}(p) = \begin{cases} F_{fbs}(p), & p \in A_p \backslash B_p \\ G_{fbs}(p), & p \in B_p \backslash A_p \\ F_{fbs}(p) \cup G_{fbs}(p), & p \in A_p \cap B_p \end{cases}$$

We write $(H_{fbs}, C_P) \cong (F_{fbs}, A_P) \tilde{\cup} (G_{fbs}, B_P)$.

Definition 2.7 [3]

"Intersection of two fuzzy binary soft sets" (F_{fbs}, A_P) and over $\mathbb{U}_\xi^1, \mathbb{U}_\xi^2$ is the fuzzy binary soft set (H_{fbs}, C_P), where $C_P = A_P \cap B_P$ and $H_{fbs}(p) = F_{fbs}(p) \cap G_{fbs}(p)$ for each $p \in C_P$, such that $F_{fbs}(p) = (S_1, T_1)$ for each $p \in A_P$ and $G_{fbs} = (S_2, T_2)$ for each $p \in B_P$. we denote it by $(H_{fbs}, P_C) \cong (F_{fbs}, A_P) \tilde{\cap} (G_{fbs}, B_P)$.

Definition 2.8. [5]

Consider the collection $\tilde{\tau}$ of fuzzy binary soft sets. Now $\tilde{\tau}$ is said to be a "***fuzzy binary soft topology***" on $\mathbb{U}_\xi^1, \mathbb{U}_\xi^2$ if

(i) $\tilde{\emptyset}, \tilde{A} \tilde{\in} \tilde{\tau}$
(ii) $\{(F_{fbs}, E_P)_i / i \in I\} \tilde{\in} \tilde{\tau}$ implies $\tilde{\bigcup}_{i\in I} (F_{fbs}, E_P)_i \tilde{\in} \tilde{\tau}$
(iii) $(F_{fbs}, E_P), (G_{fbs}, E_P) \tilde{\in} \tilde{\tau}$ implies $(F_{fbs}, E_P) \tilde{\cap} (G_{fbs}, E_P) \tilde{\in} \tilde{\tau}$.

Then $(\mathbb{U}_\xi^1, \mathbb{U}_\xi^2, \tilde{\tau}, E_P)$ is a "fuzzy binary soft topological space".

Definition: 2.9 [2]

Let $\widetilde{(X, E)}$ and $\widetilde{(Y, E')}$ be classes of fuzzy soft sets over X and Y with attributes from E and E', respectively. Let u : X → Y and p : E → E' be mappings. Then a mapping f = (u, p) : $\widetilde{(X, E)} \to \widetilde{(Y, E')}$ is defined as follows: for a fuzzy soft set (Λ, Σ) in $\widetilde{(X, E)}$, f(Λ, Σ) is a fuzzy soft set in $\widetilde{(Y, E')}$ obtained as follows: for $\beta \in p(E) \subseteq E'$ and y ∈ Y,

$$f(\Lambda, \Sigma)(\beta)(y) = \vee_{x\in u^{-1}(y)} (\vee_{\alpha \in p^{-1}(\beta)\cap\Sigma}\ \Lambda(\alpha))(x), \quad \text{if } u^{-1}(y) \neq \phi,\ p^{-1}(\beta) \cap \Sigma \neq \phi,$$
$$= 0 \quad \text{otherwise.}$$

f(Λ, Σ) is called a "***fuzzy soft image of a fuzzy soft set***" (Λ, Σ).

Definition: 2.10 [7]

Suppose (U_1, U_2, E) and (V_1, V_2, E') are two binary soft classes. Let u = m x n : $P(U_1) \times P(U_2) \to P(V_1) \times P(V_2)$; where m : $P(U_1) \to P(V_1)$, n : $P(U_2) \to P(V_2)$ and $P : E \to E'$ are mappings. Then binary soft mapping from binary soft class (U_1, U_2, E) to binary soft class (V_1, V_2, E') is denoted as $f : (U_1, U_2, E) \to (V_1, V_2, E')$ and is defined as : for a binary soft set (F, A) in (U_1, U_2, E), $(f(F, A), B)$; where $B = p(A) \subseteq E'$, is a binary soft set in (V_1, V_2, E') given by

$$f(F, A)(\beta) = u\left(\bigcup_{\alpha\in p^{-1}(\beta)\cap A} F(\alpha) \right), \text{ for } \beta \in B \subseteq E'.$$

$(f(F, A), B)$ is called a "***binary soft image of a binary soft set***" (F, A). If $B = E'$, then we shall write $(f(F, A), E')$ as $f(F, A)$.

3. FUZZY BINARY SOFT MAPPINGS ON FUZZY BINARY SOFT CLASSES

Definition: 3.1

Suppose $(\mathbb{U}_\xi^1, \mathbb{U}_\xi^2, E_p)$ and $(\mathbb{V}_\xi^1, \mathbb{V}_\xi^2, E_p^*)$ are two fuzzy binary soft classes. Let w = u x v : $F_{fbs}(\mathbb{U}_\xi^1) \times F_{fbs}(\mathbb{U}_\xi^2) \to F_{fbs}(\mathbb{V}_\xi^1) \times F_{fbs}(\mathbb{V}_\xi^2)$; where u : $F_{fbs}(\mathbb{U}_\xi^1) \to F_{fbs}(\mathbb{V}_\xi^1)$, v : $F_{fbs}(\mathbb{U}_\xi^2) \to F_{fbs}(\mathbb{V}_\xi^2)$ and e: $E_p \to E_p^*$ are mappings. Then fuzzy binary soft mapping is denoted as $f : (\mathbb{U}_\xi^1, \mathbb{U}_\xi^2, E_p) \to (\mathbb{V}_\xi^1, \mathbb{V}_\xi^2, E_p^*)$ and is defined as : for a fuzzy binary soft set (F_{fbs}, A_P) in $(\mathbb{U}_\xi^1, \mathbb{U}_\xi^2, E_p)$, $(f(F_{fbs}, A_P), B_p)$; where $B_p = e(A_p) \subseteq E_p^*$, is a fuzzy binary soft set in $(\mathbb{V}_\xi^1, \mathbb{V}_\xi^2, E_p^*)$ given by

$$f(F_{fbs}, A_P)(p^*)(a, b) = \bigcup_{(s,t)\in w^{-1}(a,b)} \left(\bigcup_{p\in e^{-1}(p^*)\cap A_p} F_{fbs}(p) \right)(s, t), \text{ for } p^* \in B_p \subseteq E_p^*$$
$$= 0 \quad \text{otherwise.}$$

(f (F_{fbs} , A_P), B_p) is called a "***fuzzy binary soft image of a fuzzy binary soft set***" (F_{fbs} , A_P). If $B_p = E_p$*, then we shall write (f (F_{fbs} , A_P), E_p*) as f (F_{fbs} , A_P).

Definition : 3.2

Let $f : (\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_p) \to (\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_p^*)$ be a fuzzy binary soft mapping from a fuzzy binary soft class $(\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_p)$ to another fuzzy binary soft class $(\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_p^*)$ and (G_{fbs} , C_P) be a fuzzy binary soft set in $(\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_p^*)$, where $C_P \subseteq E_p^*$. Then (f^{-1}(G_{fbs} , C_P), is a fuzzy binary soft set in $(\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_p)$ defined as: for $p \in e^{-1}(C_p) \subseteq E_p$,

$$f^{-1}(G_{fbs}, C_P)(p)(s,t) = G_{fbs}(e(p))(w(s,t)), \quad \text{for } e(p) \in C_P.$$
$$= 0 \quad \text{otherwise.}$$

(f^{-1}(G_{fbs} , C_P), D_P) is called a "***fuzzy binary soft inverse image***" of (G_{fbs} , C_P).
We can rewritten (f^{-1}(G_{fbs} , C_P), D_P) shortly as f^{-1}(G_{fbs} , C_P).

Example : 3.3

Consider the following sets:

$\mathbb{U}^1_\xi = \{s^1, s^2, s^3\}$, $\mathbb{U}^2_\xi = \{t^1, t^2, t^3\}$, $\mathbb{V}^1_\xi = \{a^1, a^2, a^3\}$, $\mathbb{V}^2_\xi = \{b^1, b^2, b^3\}$,

$E_p = \{p^1, p^2, p^3, p^4\}$, $E_p^* = \{p^{*1}, p^{*2}, p^{*3}\}$

and $(\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_p)$, $(\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_p^*)$ are fuzzy binary soft classes. Define $e : E_p \to E_p^*$, $u : F_{fbs}(\mathbb{U}^1_\xi) \to F_{fbs}(\mathbb{V}^1_\xi)$ and $v : F_{fbs}(\mathbb{U}^2_\xi) \to F_{fbs}(\mathbb{V}^2_\xi)$ as :

$u(s^1) = a^2$, $u(s^2) = a^3$, $u(s^3) = a^1$,
$v(t^1) = b^3$, $v(t^2) = b^1$, $v(t^3) = b^2$,
$e(p^1) = p^{*1}$, $e(p^2) = p^{*3}$, $e(p^3) = p^{*2}$, $e(p^4) = p^{*3}$.

Choose two fuzzy binary soft set in $(\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_p)$ and $(\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_p^*)$ respectively, as

$$(F_{fbs}, A_P) = \{(p^2, (\{\tfrac{s^1}{0.2}, \tfrac{s^2}{0.4}, \tfrac{s^3}{0.2}\}, \{\tfrac{t^1}{1}, \tfrac{t^2}{0}, \tfrac{t^3}{0.6}\})), (p^3, (\{\tfrac{s^1}{0}, \tfrac{s^2}{0.8}, \tfrac{s^3}{0.3}\}, \{\tfrac{t^1}{0.1}, \tfrac{t^2}{0.6}, \tfrac{t^3}{0.2}\})),$$
$$(p^4, (\{\tfrac{s^1}{0.4}, \tfrac{s^2}{0.7}, \tfrac{s^3}{0}\}, \{\tfrac{t^1}{0.9}, \tfrac{t^2}{0.2}, \tfrac{t^3}{0}\}))\}.$$

$$(G_{fbs}, B_P) = \{(p^{*1}, (\{\tfrac{a^1}{0.2}, \tfrac{a^2}{0.4}, \tfrac{a^3}{0.2}\}, \{\tfrac{b^1}{1}, \tfrac{b^2}{0}, \tfrac{b^3}{0.6}\})), (p^{*2}, (\{\tfrac{a^1}{0}, \tfrac{a^2}{0.8}, \tfrac{a^3}{0.3}\}, \{\tfrac{b^1}{0.1}, \tfrac{b^2}{0.6}, \tfrac{b^3}{0.2}\}))\}.$$

Hence the fuzzy binary soft mapping $f : (\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_p) \to (\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_p^*)$ is given as :

For a fuzzy binary soft set (F_{fbs} , E_P) in $(\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_p)$; (f (F_{fbs} , A_P), B_p), $B_p = e(A_p) = \{p^{*2}, p^{*3}\}$, is a fuzzy binary soft set in $(\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_p^*)$ obtained as:

$$f(F_{fbs}, A_P)\, p^{*2} \quad (a^1, b^1) = \cup_{(s,t)\in w^{-1}(a,b)}(\cup_{p\in e^{-1}(p^*)\cap A_p} F_{fbs}(p))(s,t) \quad (\text{since } e^{-1}(p^{*2})\cap A_p = \{p^3\})$$
$$= \cup_{s3,\,t2}(\cup_{p3} F_{fbs}(p))(s,t)$$
$$= \cup_{s3,\,t2}(\cup F_{fbs}(\{p^3\}))$$
$$= (\{\tfrac{s^3}{0.3}\}, \{\tfrac{t^2}{0.6}\})$$

Therefore, $f(F_{fbs}, A_P)\, p^{*2}(a^1, b^1) = (0.3, 0.6)$

$$f(F_{fbs}, A_P)\, p^{*2} \quad (a^2, b^2) = \cup_{(s,t)\in w^{-1}(a,b)}(\cup_{p\in e^{-1}(p^*)\cap A_p} F_{fbs}(p))(s,t) \quad (\text{since } e^{-1}(p^{*2})\cap A_p = \{p^3\})$$
$$= \cup_{s1,\,t3}(\cup_{p3} F_{fbs}(p))(s,t)$$
$$= \cup_{s1,\,t3}(\cup F_{fbs}(\{p^3\}))$$
$$= (\{\tfrac{s^1}{0}\}, \{\tfrac{t^3}{0.2}\})$$

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Therefore, $f(F_{fbs}, A_P)\, p^{*2}\, (a^2, b^2) = (0, 0.2)$

$f(F_{fbs}, A_P)\, p^{*2}\quad (a^3, b^3) = \cup_{(s,t)\in w^{-1}(a,b)}(\cup_{p\in e^{-1}(p^*)\cap A_P} F_{fbs}(p))(s, t)$ (since $e^{-1}(p^{*2})\cap A_p = \{p^3\}$)

$= \cup_{s2,\, t1}(\cup_{p3} F_{fbs}(p))(s, t)$

$= \cup_{s2,\, t1}(\cup F_{fbs}(\{p^3\}))$

$= (\{\frac{s^2}{0.8}\}, \{\frac{t^1}{0.1}\})$

Therefore, $f(F_{fbs}, A_P)\, p^{*2}\, (a^3, b^3) = (0.8, 0.1)$

By similar calculations we get,

$(f(F_{fbs}, A_P), B_P) = \{(p^{*2}, (\{\frac{a^1}{0.3}, \frac{a^2}{0}, \frac{a^3}{0.8}\}, \{\frac{b^1}{0.6}, \frac{b^2}{0.2}, \frac{b^3}{0.1}\})), (p^{*3}, (\{\frac{a^1}{0.2}, \frac{a^2}{0.4}, \frac{a^3}{0.7}\}, \{\frac{b^1}{0.2}, \frac{b^2}{0.6}, \frac{b^3}{1}\}))\}.$

Definition: 3.4

Let $f : (\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_P) \to (\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_P^*)$ be a fuzzy binary soft mapping and (F_{fbs}, A_P) and (G_{fbs}, B_P) are fuzzy binary soft sets in $(\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_P)$, then for $p^* \in E^*$, $(a, b) \in (\mathbb{V}^1_\xi, \mathbb{V}^2_\xi)$, the "***fuzzy binary soft union and intersection of fuzzy binary soft images***" $f(F_{fbs}, A_P)$ and $f(G_{fbs}, B_P)$ in $(\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_P^*)$ are defined as

$(f(F_{fbs}, A_P)\, \tilde{\cup}\, f(G_{fbs}, B_P))(p^*)(a, b) = f(F_{fbs}, A_P)(p^*)(a, b)\, \tilde{\cup}\, f(G_{fbs}, B_P)(p^*)(a, b),$

$(f(F_{fbs}, A_P)\, \tilde{\cap}\, f(G_{fbs}, B_P))(p^*)(a, b) = f(F_{fbs}, A_P)(p^*)(a, b)\, \tilde{\cap}\, f(G_{fbs}, B_P)(p^*)(a, b),$

Where $\tilde{\cup}$, $\tilde{\cap}$ denote fuzzy binary soft union and intersection of fuzzy binary soft images in $(\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_P^*)$.

Definition: 3.5

Let $f : (\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_P) \to (\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_P^*)$ be a fuzzy binary soft mapping and (F_{fbs}, A_P) and (G_{fbs}, B_P) are fuzzy binary soft sets in $(\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_P^*)$, then for $p \in E$, $(s, t) \in (\mathbb{U}^1_\xi, \mathbb{U}^2_\xi)$ the "***fuzzy binary soft union and intersection of fuzzy binary soft inverse images***" $f^{-1}(F_{fbs}, A_P)$ and $f^{-1}(G_{fbs}, B_P)$ in $(\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_P)$ are defined as

$(f^{-1}(F_{fbs}, A_P)\, \tilde{\cup}\, f^{-1}(G_{fbs}, B_P))(p)(s, t) = f^{-1}(F_{fbs}, A_P)(p)(s, t)\, \tilde{\cup}\, f^{-1}(G_{fbs}, B_P)(p)(s, t),$

$(f^{-1}(F_{fbs}, A_P)\, \tilde{\cap}\, f^{-1}(G_{fbs}, B_P))(p)(s, t) = f^{-1}(F_{fbs}, A_P)(p)(s, t)\, \tilde{\cap}\, f^{-1}(G_{fbs}, B_P)(p)(s, t),$

Where $\tilde{\cup}$, $\tilde{\cap}$ denote fuzzy binary soft union and intersection of fuzzy binary soft inverse images in $(\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_P)$.

Theorem : 3.6

Let $f : (\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_P) \to (\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_P^*)$ and $w : F_{fbs}(\mathbb{U}^1_\xi) \times F_{fbs}(\mathbb{U}^2_\xi) \to F_{fbs}(\mathbb{V}^1_\xi) \times F_{fbs}(\mathbb{V}^2_\xi)$; where $u : F_{fbs}(\mathbb{U}^1_\xi) \to F_{fbs}(\mathbb{V}^1_\xi)$, $v : F_{fbs}(\mathbb{U}^2_\xi) \to F_{fbs}(\mathbb{V}^2_\xi)$ and $e: E_P \to E_P^*$ be mappings. For fuzzy binary soft sets $f(F_{fbs}, A_P)$, $f(G_{fbs}, B_P)$ and a family of fuzzy binary soft sets $(F_{fbs}, A_P)_i$ in $(\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_P)$ we have

(i) $f(\tilde{\emptyset}) \tilde{\cong} \tilde{\emptyset}$

(ii) $f(\mathbb{U}^1_\xi, \mathbb{U}^2_\xi) \tilde{\subseteq} (\mathbb{V}^1_\xi, \mathbb{V}^2_\xi)$

(iii) If $(F_{fbs}, A_P) \tilde{\subseteq} (G_{fbs}, B_P)$ then $f(F_{fbs}, A_P) \tilde{\subseteq} f(G_{fbs}, B_P)$

(iv) $f((F_{fbs}, A_P)\, \tilde{\cup}\, (G_{fbs}, B_P)) \tilde{\cong} f(F_{fbs}, A_P)\, \tilde{\cup}\, f(G_{fbs}, B_P)$

In general, $f(\tilde{\cup}_i (F_{fbs}, A_P)_i \tilde{\cong} \tilde{\cup}_i (f(F_{fbs}, A_P)_i)$

(v) $f((F_{fbs}, A_P)\, \tilde{\cap}\, (G_{fbs}, B_P)) \tilde{\subseteq} f(F_{fbs}, A_P)\, \tilde{\cap}\, f(G_{fbs}, B_P)$

In general, $f(\tilde{\cap}_i (F_{fbs}, A_P)_i \tilde{\subseteq} \tilde{\cap}_i (f(F_{fbs}, A_P)_i)$.

Proof:

(i) , (ii) is obvious.

(iii) Consider, $(F_{fbs}, A_P) \tilde{\subseteq} (G_{fbs}, B_P)$

We have, $f(F_{fbs}, A_P)(p^*)(a, b) \quad = \cup_{(s,t)\in w^{-1}(a,b)}(\cup_{p\in e^{-1}(p^*)\cap A_p} F_{fbs}(p))(s, t)$

$$= \bigcup_{(s,t)\in w^{-1}(a,b)} \bigcup_{p\in e^{-1}(p*)\cap A_p} F_{fbs}(p)\ (s,t)$$
$$\subseteq \bigcup_{(s,t)\in w^{-1}(a,b)} \bigcup_{p\in e^{-1}(p*)\cap B_p} G_{fbs}(p)\ (s,t)$$
$$= f(G_{fbs}, B_P)\ (p^*)\ (a, b)$$

Hence, $f(F_{fbs}, A_P) \,\tilde{\subseteq}\, f(G_{fbs}, B_P)$

(iv) For $p^* \in E_p{}^*$ and $(a, b) \in (\mathbb{V}^1_\xi, \mathbb{V}^2_\xi)$, we show that

$f((F_{fbs}, A_P) \,\tilde{\cup}\, (G_{fbs}, B_P))(p^*)(a, b) = f(F_{fbs}, A_P)(p^*)(a, b) \,\tilde{\cup}\, f(G_{fbs}, B_P)(p^*)(a, b)$

Consider,

$f((F_{fbs}, A_P) \,\tilde{\cup}\, (G_{fbs}, B_P))(p^*)(a, b) = f((H_{fbs}, A_P \cup B_P)(p^*)(a, b)$ (say)

$$= \bigcup_{(s,t)\in w^{-1}(a,b)} \left(\bigcup_{p\in e^{-1}(p*)\cap(A_p\cup B_p)} H_{fbs}(p)\right)(s,t)$$

Where ,
$$H(p) = \begin{cases} F_{fbs}(p), & p \in A_p - B_p \cap e^{-1}(p*) \\ G_{fbs}(p), & p \in B_p - A_p \cap e^{-1}(p*) \\ F_{fbs}(p) \cup F_{fbs}(p), & p \in A_p \cap B_p \cap e^{-1}(p*) \end{cases}$$

for $p \in (A_P \cup B_P) \cup e^{-1}(p*)$.

Therefore, $f((F_{fbs}, A_P) \,\tilde{\cup}\, (G_{fbs}, B_P))(p^*)(a, b)$

$$= \bigcup_{(s,t)\in w^{-1}(a,b)} \left(\bigcup \begin{cases} F_{fbs}(p), & p \in A_p - B_p \cap e^{-1}(p*) \\ G_{fbs}(p), & p \in B_p - A_p \cap e^{-1}(p*) \\ F_{fbs}(p) \cup G_{fbs}(p), & p \in A_p \cap B_p \cap e^{-1}(p*) \end{cases} \right) \text{------(1)}$$

By definition 3.4 we have,

$(f(F_{fbs}, A_P) \,\tilde{\cup}\, f(G_{fbs}, B_P))(p^*)(a, b) = f(F_{fbs}, A_P)(p^*)(a, b) \,\tilde{\cup}\, f(G_{fbs}, B_P)(p^*)(a, b)$

$$= \left(\bigcup_{(s,t)\in w^{-1}(a,b)} \left(\bigcup_{p\in e^{-1}(p*)\cap A_p} F_{fbs}(p)\right)(s,t)\right)$$
$$\cup \left(\bigcup_{(s,t)\in w^{-1}(a,b)} \left(\bigcup_{p\in e^{-1}(p*)\cap B_p} G_{fbs}(p)\right)(s,t)\right)$$
$$= \bigcup_{(s,t)\in w^{-1}(a,b)} \left(\bigcup_{p\in e^{-1}(p*)\cap(A_p\cup B_p)} (F_{fbs}(p) \cup G_{fbs}(p))\right)(s,t)$$
$$= \bigcup_{(s,t)\in w^{-1}(a,b)} \left(\bigcup \begin{cases} F_{fbs}(p), & p \in A_p - B_p \cap e^{-1}(p*) \\ G_{fbs}(p), & p \in B_p - A_p \cap e^{-1}(p*) \\ F_{fbs}(p) \cup G_{fbs}(p), & p \in A_p \cap B_p \cap e^{-1}(p*) \end{cases} \right) \text{-----------(2)}$$

Therefore, from (1) and (2) we get,

$f((F_{fbs}, A_P) \,\tilde{\cup}\, (G_{fbs}, B_P)) \cong f(F_{fbs}, A_P) \,\tilde{\cup}\, f(G_{fbs}, B_P)$

(v) For $p^* \in E_p{}^*$ and $(a, b) \in (\mathbb{V}^1_\xi, \mathbb{V}^2_\xi)$, and using definition 3.4, we have

$f((F_{fbs}, A_P) \,\tilde{\cap}\, (G_{fbs}, B_P))(p^*)(a, b) = f((H_{fbs}, A_P \cap B_P)(p^*)(a, b)$ (say)

$$= \bigcup_{(s,t)\in w^{-1}(a,b)} \left(\bigcup_{p\in e^{-1}(p*)\cap(A_p\cap B_p)} H_{fbs}(p)\right)(s,t)$$
$$= \bigcup_{(s,t)\in w^{-1}(a,b)} \left(\bigcup_{p\in e^{-1}(p*)\cap(A_p\cap B_p)} [F_{fbs}(p) \cap G_{fbs}(p)]\right)(s,t)$$
$$= \bigcup_{(s,t)\in w^{-1}(a,b)} \left(\bigcup_{p\in e^{-1}(p*)\cap(A_p\cap B_p)} [F_{fbs}(p)(s,t) \cap G_{fbs}(p)\ (s,t)]\right)$$
$$\subseteq \left(\bigcup_{(s,t)\in w^{-1}(a,b)} \left(\bigcup_{p\in e^{-1}(p*)\cap A_p} F_{fbs}(p)\right)(s,t)\right)$$
$$\cap \left(\bigcup_{(s,t)\in w^{-1}(a,b)} \left(\bigcup_{p\in e^{-1}(p*)\cap B_p} G_{fbs}(p)\right)(s,t)\right)$$

$= f(F_{fbs}, A_P)(p^*)(a, b) \,\tilde{\cap}\, f(G_{fbs}, B_P)(p^*)(a, b)$, for $p^* = e(p)$

$= (f(F_{fbs}, A_P) \,\tilde{\cap}\, f(G_{fbs}, B_P))(p^*)(a, b)$

Hence $f((F_{fbs}, A_P) \tilde{\cap} (G_{fbs}, B_P)) \tilde{\subseteq} f(F_{fbs}, A_P) \tilde{\cap} f(G_{fbs}, B_P)$.

Hence the theorem.

Theorem : 3.7

Let $f : (\mathbb{U}^1_\xi, \mathbb{U}^2_\xi, E_p) \rightarrow (\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_p^*)$ and $w : F_{fbs}(\mathbb{U}^1_\xi) \times F_{fbs}(\mathbb{U}^2_\xi) \rightarrow F_{fbs}(\mathbb{V}^1_\xi) \times F_{fbs}(\mathbb{V}^2_\xi)$; where $u : F_{fbs}(\mathbb{U}^1_\xi) \rightarrow F_{fbs}(\mathbb{V}^1_\xi)$, $v : F_{fbs}(\mathbb{U}^2_\xi) \rightarrow F_{fbs}(\mathbb{V}^2_\xi)$ and $e: E_p \rightarrow E_p^*$ be mappings. For fuzzy binary soft sets $f(F_{fbs}, A_P)$, $f(G_{fbs}, B_P)$ and a family of fuzzy binary soft sets $(F_{fbs}, A_P)_i$ in $(\mathbb{V}^1_\xi, \mathbb{V}^2_\xi, E_p^*)$ we have

(i) $f^{-1}(\tilde{\emptyset}) \cong \tilde{\emptyset}$

(ii) $f^{-1}(\mathbb{V}^1_\xi, \mathbb{V}^2_\xi) \tilde{\subseteq} (\mathbb{U}^1_\xi, \mathbb{U}^2_\xi)$

(iii) If $(F_{fbs}, A_P) \tilde{\subseteq} (G_{fbs}, B_P)$ then $f^{-1}(F_{fbs}, A_P) \tilde{\subseteq} f^{-1}(G_{fbs}, B_P)$

(iv) $f^{-1}((F_{fbs}, A_P) \tilde{\cup} (G_{fbs}, B_P)) \cong f^{-1}(F_{fbs}, A_P) \tilde{\cup} f^{-1}(G_{fbs}, B_P)$

In general, $f^{-1}(\tilde{\bigcup}_i (F_{fbs}, A_P)_i \cong \tilde{\bigcup}_i (f^{-1}(F_{fbs}, A_P)_i)$

(v) $f^{-1}((F_{fbs}, A_P) \tilde{\cap} (G_{fbs}, B_P)) \cong f^{-1}(F_{fbs}, A_P) \tilde{\cap} f^{-1}(G_{fbs}, B_P)$

In general, $f^{-1}(\tilde{\bigcap}_i (F_{fbs}, A_P)_i \cong \tilde{\bigcap}_i (f^{-1}(F_{fbs}, A_P)_i)$.

Proof:

(i) , (ii) is obvious.

(iii) Consider, $(F_{fbs}, A_P) \tilde{\subseteq} (G_{fbs}, B_P)$

We have,
$$\begin{aligned} f^{-1}(F_{fbs}, A_P)(p)(s,t) &= F_{fbs}(e(p))(w(s,t)) \\ &= F_{fbs}(p^*)(w(s,t)), \qquad e(p) = p^* \\ &\subseteq G_{fbs}(p^*)(w(s,t)) \\ &= G_{fbs}(e(p))(w(s,t)) \\ &= f^{-1}(G_{fbs}, B_P) \end{aligned}$$

Hence, $f^{-1}(F_{fbs}, A_P) \tilde{\subseteq} f^{-1}(G_{fbs}, B_P)$

(iv) For $p \in E_p$ and $(s,t) \in (\mathbb{U}^1_\xi, \mathbb{U}^2_\xi)$, we have

$$\begin{aligned} f^{-1}((F_{fbs}, A_P) \tilde{\cup} (G_{fbs}, B_P))(p)(s,t) &= f^{-1}((H_{fbs}, A_P \cup B_P)(p)(s,t) \\ &= H_{fbs}(e(p))(w(s,t)), \qquad e(p) \in A_P \cup B_P,\ w(s,t) \in (V^1, V^2) \\ &= H_{fbs}(p^*)(w(s,t)), \text{ where } p^* = e(p) \end{aligned}$$

$$= \begin{cases} F_{fbs}(p*)(w(s,t)) & p* \in A_p - B_p \\ G_{fbs}(p*)(w(s,t)), & p* \in B_p - A_p \\ F_{fbs}(p*) \cup F_{fbs}(p*)(w(s,t)), & p* \in A_p \cap B_p \end{cases} \quad \text{--------(1)}$$

By definition 3.5 we have,

$$\begin{aligned} (f^{-1}(F_{fbs}, A_P) \tilde{\cup} f^{-1}(G_{fbs}, B_P))(p)(s,t) &= f^{-1}(F_{fbs}, A_P)(p)(s,t) \tilde{\cup} f^{-1}(G_{fbs}, B_P)(p)(s,t) \\ &= F_{fbs}(e(p))(w(s,t)) \tilde{\cup} G_{fbs}(e(p))(w(s,t)), \ p^* = e(p) \in A_P \cup B_P \end{aligned}$$

$$= \begin{cases} F_{fbs}(p*)(w(s,t)) & p* \in A_p - B_p \\ G_{fbs}(p*)(w(s,t)), & p* \in B_p - A_p \\ F_{fbs}(p*) \cup F_{fbs}(p*)(w(s,t)), & p* \in A_p \cap B_p \end{cases} \quad \text{-----------(2)}$$

Therefore, from (1) and (2) we get,

$f^{-1}((F_{fbs}, A_P) \tilde{\cup} (G_{fbs}, B_P)) \cong f^{-1}(F_{fbs}, A_P) \tilde{\cup} f^{-1}(G_{fbs}, B_P)$

(v) For $p \in E_p$ and $(s,t) \in (\mathbb{U}^1_\xi, \mathbb{U}^2_\xi)$, and using definition 3.5, we have

$$\begin{aligned} f^{-1}((F_{fbs}, A_P) \tilde{\cap} (G_{fbs}, B_P))(p)(s,t) &= f^{-1}((H_{fbs}, A_P \cap B_P)(p)(s,t) \ \text{(say)} \\ &= H_{fbs}(e(p))(w(s,t)), \ e(p) \in A_P \cap B_P \\ &= H_{fbs}(p^*)(w(s,t)), \text{ where } p^* = e(p) \end{aligned}$$

$= (F_{fbs}(p^*) \tilde{\cap} G_{fbs}(p^*))(w(s, t))$
$= F_{fbs}(e(p))(w(s, t)) \tilde{\cap} G_{fbs}(e(p))(w(s, t)),$
$= f^{-1}(F_{fbs}, A_P)(p)(s, t) \tilde{\cap} f^{-1}(G_{fbs}, A_P)(p)(s, t)$
$= [f^{-1}(F_{fbs}, A_P)(p) \tilde{\cap} f^{-1}(G_{fbs}, A_P)](p)(s, t)$

Hence $f^{-1}((F_{fbs}, A_P) \tilde{\cap} (G_{fbs}, B_P)) \tilde{\cong} f^{-1}(F_{fbs}, A_P) \tilde{\cap} f^{-1}(G_{fbs}, B_P)$

Hence the theorem.

4. CONCLUSION

Here we introduced "fuzzy binary soft mapping on fuzzy binary soft topological space. It is defined on fuzzy binary soft classes of fuzzy binary soft sets over two initial sets $\mathbb{U}^1_{\xi}$, $\mathbb{U}^2_{\xi}$ with fixed set of parameters. In this extension we defined fuzzy binary soft continuity using such classes of fuzzy binary soft mapping. Further we proved the properties of fuzzy binary soft images and fuzzy binary soft inverse images. It will supportive for additional exploration augmentation in fuzzy binary soft mapping.

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