International Journal of Advance Research in Science and Engineering
Volume No. 11, Issue No. 02, February 2022
www.ijarse.com
IJARSE
ISSN 2319 - 8354

`V_4-Vertex Magic labeling for Bloom Graph

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Abstract

Let V_4 *be* an abelian group under multiplication. Let $g : E(G) \to V_4 - \{1\}$. The vertex magic labeling on V_4 is defined as the vertex labeling $g^*: V(G) \to V_4$ such that $g^*(v) = \prod_u g(uv)$ where the product is taken over all edges uv of G incident at v is a constant. A graph is said to be $V_4 -$ magic if its admits a vertex magic labeling on V_4. In this paper we investigate the results on Bloom graph and Cylinder graph.

Keyword: $B(m,n), C_{m,n}$,

AMS subject classification (2010): 05C78

1. Introduction

Laid foundation by Euler in the 18th Century, Graph Theory grew wider by Sedlack, Kong, Lee and Sun. Sedlack introduced Magic Labeling Bloom and Golomb connected Graph labeling to a wide range of applications such as Coding theory, Communication design, Radar, Circuit design, Astronomy, Network and X-ray crystallography.

Let V_4 be an abelian group under multiplication. Let $g: E(G) \to V_4 - \{1\}$. The vertex magic labeling on V_4 is defined as the vertex labeling $g^*: V(G) \to V_4$ such that $g^*(v) = \prod_u g(uv)$ where the product is taken over all edges uv of G incident at v is a constant. A graph is said to be V_4- magic if its admits a vertex magic labeling on V_4.

The result is verified for Bloom graph and Cylinder graph.

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2. Preliminaries:

Bloom Graph: The Bloom graph $B(m,n), m,n > 1$ is defined as follows:

$$V\big(B(m,n)\big) = \{(x,y) : 0 \le x \le m-1, 0 \le y \le n-1\}$$

Two distinct vertices (x_1, y_1) and (x_2, y_2) are adjacent if and only if

(i) $x_1 = x_2 - 1$ and $y_1 = y_2$

(ii) $x_1 = x_2 = 0$ and $y_1 + 1 \equiv y_2$ mod n

(iii) $x_1 = x_2 = m$ and $y_1 + 1 \equiv y_2$ mod n

(iv) $x_1 = x_2 - 1$ and $y_1 + 1 \equiv y_2$ mod n

(i) condition describes vertical edges

(ii) and (iii) condition describes horizontal edges in the topmost and lowermost rows respectively.

(iv) condition describes the slant edges.

3. Main Results:

Theorem 3.1: Bloom graph $B(m,n)$ is a V_4-magic graph.

Proof: :

Let G be a Bloom graph $B(m,n)$.

Let the vertex set $V(G)$ be $\{(x,y) : 0 \le x \le m-1; 0 \le y \le n-1\}$.

Define a function $g: E(G) \to \mathrm{V}_4 - \{1\}$

Let the edge labeling of G be

(i) Label the edges joining the vertices $(0,y)$ and $(0, y+1$ (mod $n)$ as "i" if they are adjacent.

(ii) Label the edges joining the vertices $(m-1, y)$ and $(m+1, y+1$ (mod $n))$ as "i" if they are adjacent

(iii) Label the edges joining the vertices (x,y) and $(x+1, y+1($mod $n))$ as "$-i$" if they are adjacent.

(iv) Label the edges joining the vertices (x,y) and $(x+1, y)$ as "$-i$" if they are adjacent.

Then $g^*: V(G) \to \mathrm{V}_4 - \{1\}$ is such that

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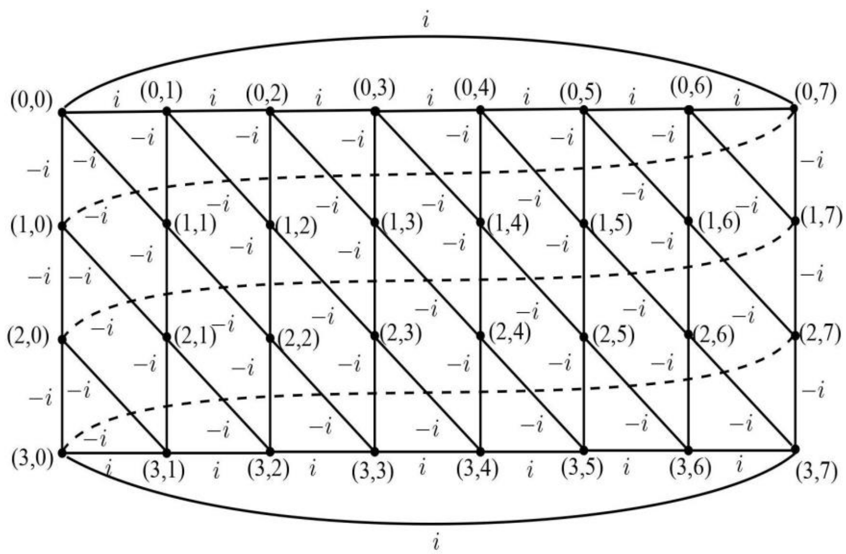
$g^*(x_i, y_j) = 1$ for all $i = 0$ to $m-1; j = 0$ to $n-1$

Thus each vertex of the Bloom graph gets the magic number $"1"$ satisfying the V_4-vertex magic labeling , Bloom graph becomes a V_4- magic graph in all cases, whether m and n are either odd or even or both even or both odd.

Illustration: $B(4,8)$ both m, n – even



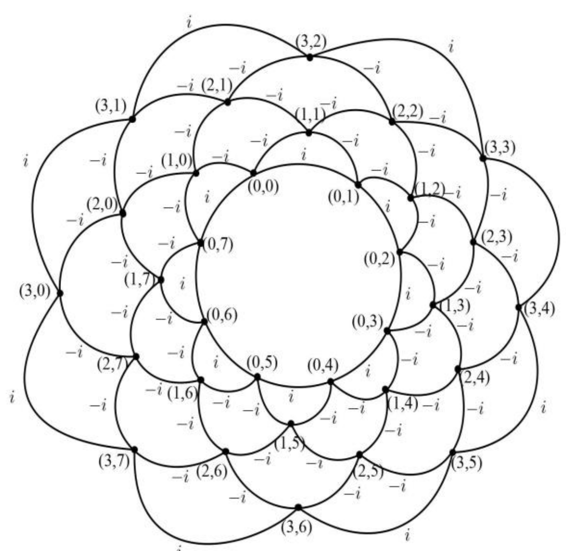


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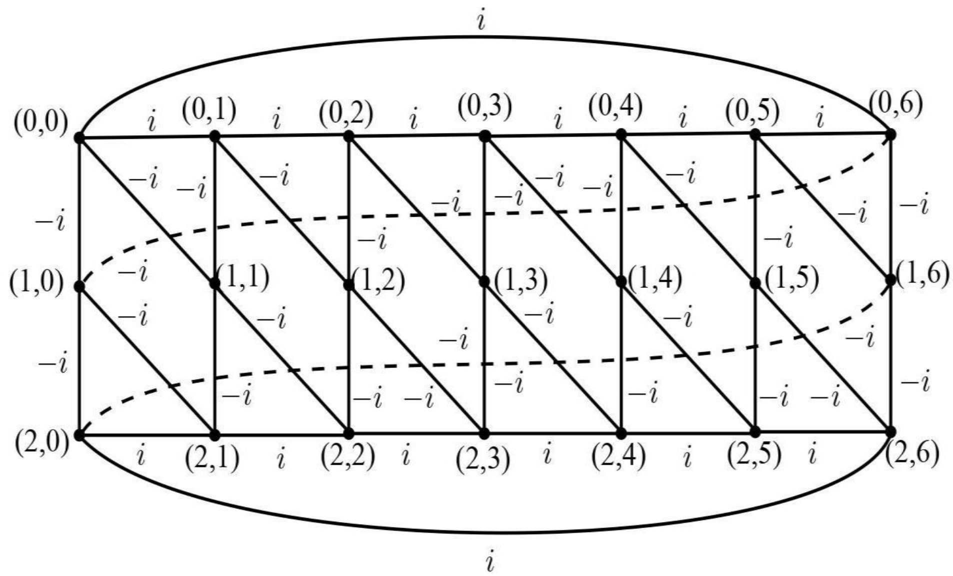
Illustration: $B(3,7)$ both m,n – odd



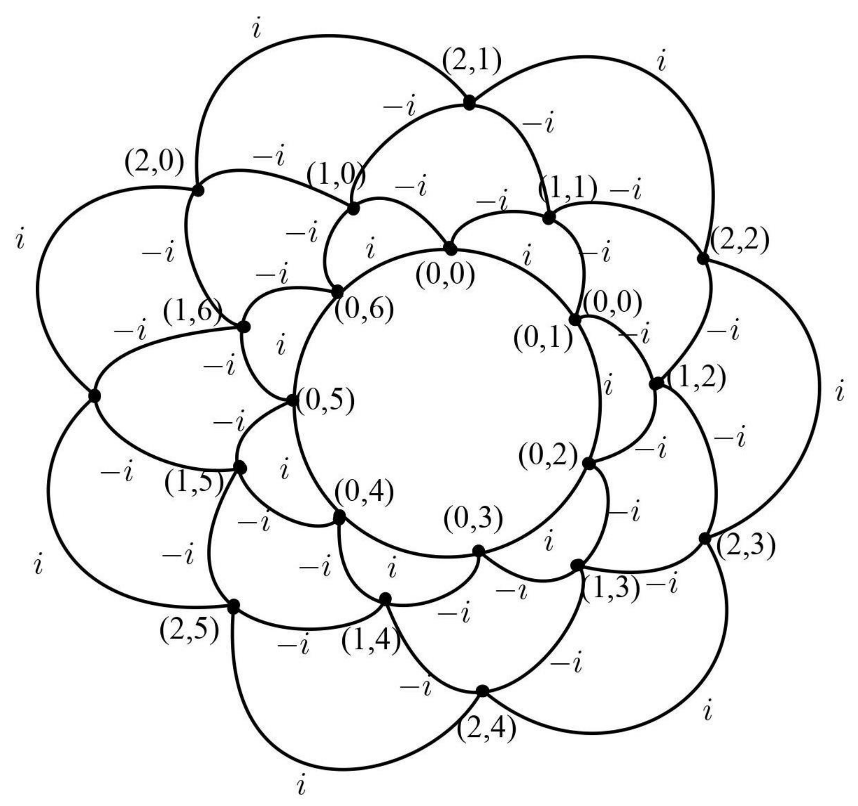


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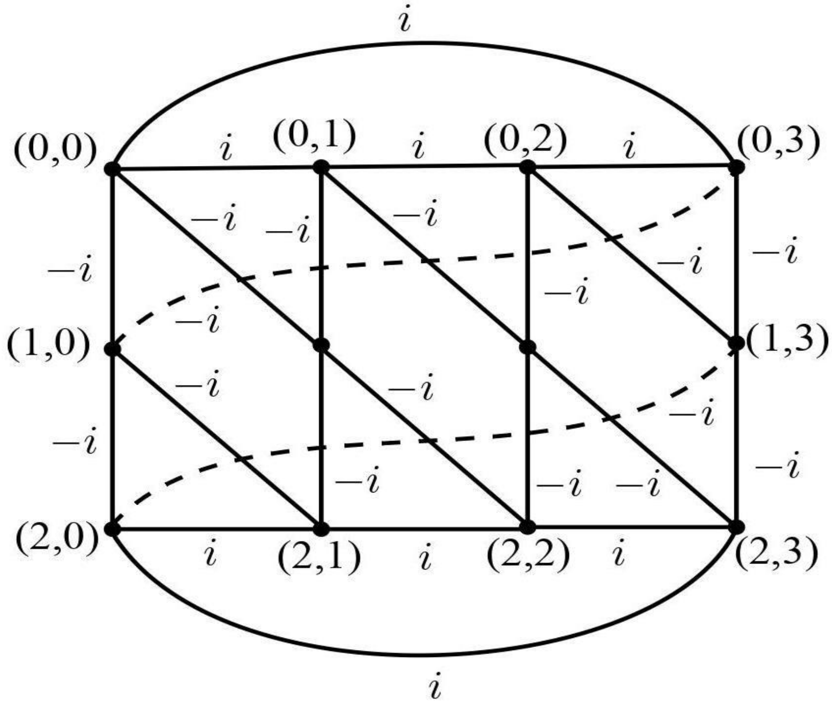
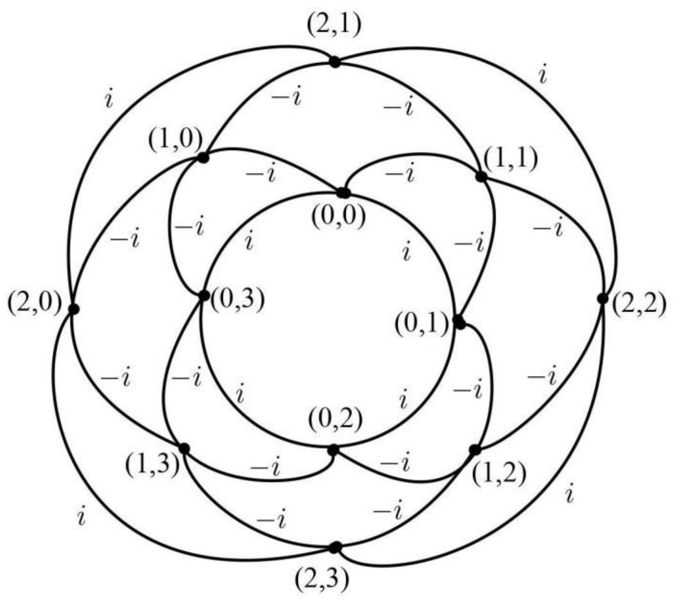
Illustration: $B(3,4)$





Theorem 3.2: For $m, n \geq 2,$ the cylinder graph $C_{m,n}$ is a V_4-magic graph.

Proof: Let G be a cylinder graph $C_{m,n}$.

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Let $V(G) = \{u_{pq} : 1 \leq p \leq m; 1 \leq q \leq n-1\}$ be the vertex set of G.

Let $E(G) = \{u_{pq}u_{p,q+1} : 1 \leq p \leq m, 1 \leq q \leq n-1\} \cup$

$\cup \{u_{pq}\, u_{p+1,q} : 1 \leq p \leq m-1, 1 \leq q \leq n\}$

$[u_{p,n+1} = u_{p_1}]$

Let us define a function $g: E(G) \to V_4 - \{1\}$ such that

$g(u_{pq}\, u_{pq+1}) = i$ for $p = 1, m, 1 \leq q \leq n$

$g(u_{pq}\, u_{pq+1}) = -1$ for $2 \leq p \leq m-1, 1 \leq q \leq n$

$g(u_{pq}\, u_{p+1,q}) = -1$ for $1 \leq p \leq m-1, 1 \leq q \leq n$

Then $g^*: V(G) \to V_4 - \{1\}$ is

$g^*(u_{pq}) = 1,$ for $1 \leq p \leq m; 1 \leq q \leq n$

This labeling holds for all cases whether m and n are either odd or even and $m, n \geq 2.$

Hence Cylinder graph becomes a V_4- vertex magic Labeling for $m, n \geq 2$.

Illustration: $C_{6,8}$

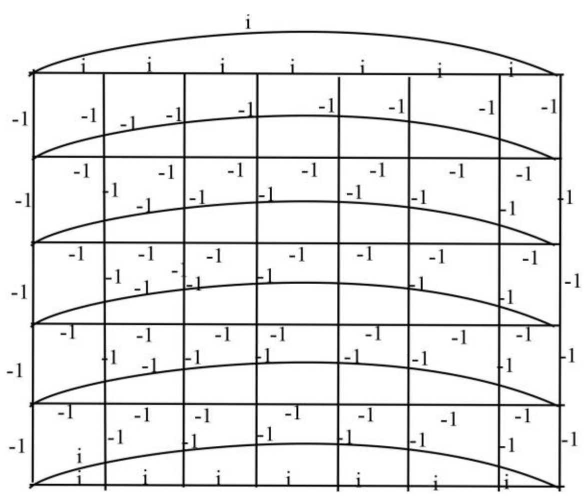

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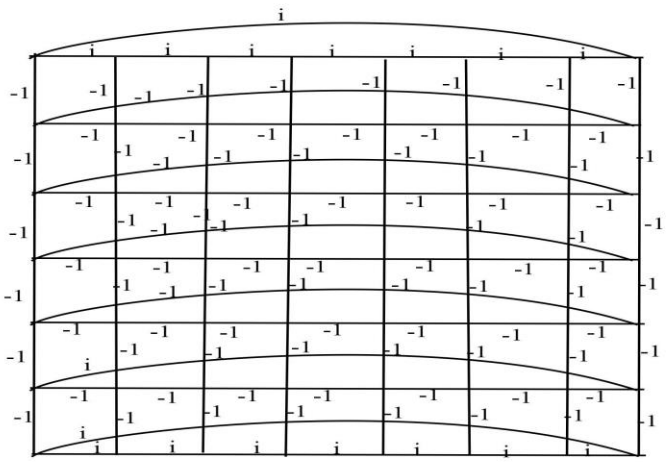
Illustration: $C_{7,9}$



Remark:

For $m, n \geq 2$, cylinder graph satisfies V_4- vertex magic graph labeling when i is replaced by $-i$ in all four cases throughout the graph. Thus Cylinder graph becomes V_4-magic graph.

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