International Journal of Advance Research in Science and Engineering
Volume No. 08, Issue No. 10, October 2019
www.ijarse.com
IJARSE
ISSN 2319 - 8354

PRE-TEST ESTIMATOR OF SCALE PARAMETER OF CLASSICAL PARETO DISTRIBUTION

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ABSTRACT

A pre-test estimator of scale parameter of classical Pareto distribution is considered when shape parameter is unknown. The test makes use of information about the shape parameter in form of a guess value to give an improved estimator of the scale parameter. The expression for bias and mean square error of the estimator are derived. Comparison of the proposed estimator with respect to usual unbiased estimator has been made on the basis of numerical findings.

Key Words : Classical Pareto distribution, Preliminary test, Bias, Mean square error, Relative efficiency.

Introduction :

Many socio-economic and other naturally occurring quantities are distributed according to certain statistical distribution with very long right tail. Pareto distribution fits well on most of such distributions. Davis and Feldstein [1] have viewed that Pareto distribution is a potential model for life testing problems.

The probability density function (p.d.f.) of a classical Pareto distribution is

$$f(X; a, \sigma) = a\sigma^{a} X^{-(a+1)} \quad ; X \geq \sigma, a > 0$$

where a and σ are respectively shape and scale parameter.

Let x_1, x_2.......,x_n be a random sample of size n from a classical Pareto distribution whose p.d.f. is given above.

For unknown a the unbiased estimator of σ is given by

$$\hat{\sigma}_u = [1 - \{(n-1)\hat{a}\}^{-1}]\hat{\sigma}$$

where

$$\hat{\sigma} = x_{(1)} = \min(x_1, x_2 \ldots\ldots x_n)$$

and

$$\hat{a} = \left[(1/n) \sum_{i=1}^{n} \log(x_i / x_{(i)}) \right]^{-1}$$

are MLE's of σand a respectively.

If a = a0 is known, an unbiased estimator of σ is

$$\sigma_u^* = [1 - (na_0)^{-1}]\hat{\sigma}$$

The problems of estimation of scale parameter of classical Pareto distribution have been considered by different authors like Likes [2]. Saxena and Johnson [3], Rohtagi and Saleh [4] and others when a prior information of parameter is not available.

Sometimes, the research workers may have some information about the parameter of the population prior to sample data for investigation. This information may be obtained either from the investigator's past experience or from similar type of surveys done by some reliable agencies. The use of this additional information results in increase in the precision of the estimator. Recently Singh and others [5] considered the problem of estimation for scale parameter of a classical Pareto distribution when its prior information is known. In present paper, we have studied the property of the estimator of the scale parameter using information of the shape parameter.

2. The proposed Estimator σ_{PT}^*

Suppose a guess value a_0 of the shape parameter is available. A preliminary test may be conducted for hypothesis $H_0 : a = a_0$ against $H_1 : a \neq a_0$ for estimating scale parameter σ. If the hypothesis is accepted, we use σ_u^* otherwise $\hat{\sigma}_u$ as an estimator of σ. If α is pressigned level of significance, the hypothesis H_0 is accepted if

$$r_1 \leq \frac{2na_0}{\hat{a}} \leq r_2,$$

where r_1 and r_2 are such that

$$P[\chi^2_{2(n-1)} > r_2] = \alpha / 2$$

$$P[\chi^2_{2(n-1)} < r_1] = \alpha/2$$

and $\chi^2_{2(n-1)}$ is a chi-square variate with 2(n-1) d.f. Therefore, the proposed estimator of σ is

$$\sigma^*_{PT} = \begin{cases} [1-(na_0)^{-1}] & ; \text{if } t_1 \le \hat{a} \le t_2 \\ [1-((n-1)\hat{a})^{-1}]\hat{\sigma} & ; \text{otherwise} \end{cases}$$

where, $t_1 = \dfrac{2a_0 n}{r_2}$ and $t_2 = \dfrac{2a_0 n}{r_1}$

3. Bias, Mean Square Error and Relative Efficiency of σ^*_{PT}

3.1 Bias

Since $\hat{a}$ and $\hat{\sigma}$ are independently distributed, the value of $E(\sigma^*_{PT})$ is written as,

$$E(\sigma^*_{PT}) = \sigma + [1-(na_0)^{-1}]\int_{t_1}^{t_2} f_1(\hat{a})d\hat{a}.\int_{\sigma}^{\infty}\hat{\sigma}f_2(\hat{\sigma})d\hat{\sigma}$$

$$-[\int_{t_1}^{t_2} f_1(\hat{a})d\hat{a} - (n-1)^{-1}\int_{t_1}^{t_2}(\hat{a})^{-1}f_1(\hat{a})d\hat{a}].\int_{\sigma}^{\infty}\hat{\sigma}.f_2(\hat{\sigma}).d\hat{\sigma}$$

where

$$f_1(\hat{a}) = [(na)^{n-1}/(\Gamma(n-1)(\hat{a})^n)]\exp[-(na/\hat{a})], \quad \hat{a} > 0$$

and

$$f_2(\hat{\sigma}) = na\sigma^{na}/(\hat{\sigma})^{(na+1)}; \quad \hat{\sigma} > 0$$

are p.d.f. of $\hat{a}$ and $\hat{\sigma}$ respectively. On evaluating and simplifying, we have

$$E[\sigma^*_{PT}] = \sigma + [1-(na_0)^{-1}][na\sigma/(na-1)].P$$

$$-na\sigma/(na-1)[P-(na)^{-1}Q] \quad \text{...(1)}$$

where

$$P = \frac{1}{\Gamma(n-1)}[l_{x_2}(n-1) - l_{x_1}(n-1)]$$

$$Q = \frac{1}{\Gamma(n)}[l_{x_2}(n) - l_{x_1}(n)],$$

and $x_1 = ar_1/2a_0$, $x_2 = ar_2/2a_0$,

$$l_x(m) = \int_0^x e^{-t}t^{m-1}dt$$

Therefore,

Bias = $E(\sigma^*_{PT}) - \sigma = \sigma(na-1)^{-1}[Q - naP/na_0]$

3.2. Mean Square Error

MSE of σ^*_{PT} is given by

$$MSE\ (\sigma^{*\,2}_{PT}) = E(\sigma^{*\,2}_{PT}) - 2\sigma E(\sigma^*_{PT}) + \sigma^2 \quad (2)$$

Now,

$$E(\sigma^{*\,2}_{PT}) = \int_{t_1}^{t_2}\int_{\sigma}^{\infty}[1-(na_0)^{-1}]^2(\hat{\sigma})^2 f_1(\hat{a})f_2(\hat{\sigma})d\hat{a}d\hat{\sigma}$$

$$+\int_{0}^{\infty}\int_{\sigma}^{\infty}[1-((n-1)\hat{a})^{-1}]^2(\hat{\sigma})^2 f_1(\hat{a})f_2(\hat{\sigma})d\hat{a}d\hat{\sigma}$$

$$-\int_{t_1}^{t_2}\int_{\sigma}^{\infty}[1-((n-1)\hat{a})^{-1}]^2(\hat{\sigma})^2 f_1(\hat{a})f_2(\hat{\sigma})d\hat{a}d\hat{\sigma}$$

On evaluating and simplifying, we obtain

$$E(\sigma^{*2}_{PT}) = [(na_0)^{-2}(na_0)^{-1}][na\sigma^2 P/(na-2)] + [1+(n-1).$$
$$(na-2)a)^{-1}]\sigma^2 + [2\sigma^2 Q/(na-2)]$$
$$-[\sigma^2 R/((n-1)(na-2)a)] \quad (3)$$

where

$$R = \frac{1}{\Gamma(n+1)}\left[l_{x_2}(n+1) - l_{x_1}(n+1)\right]$$

Using (1) and (3) in (2) and simplifying, we have

$$MSE(\sigma^*_{PT}) = [na(na-2na_0-1)\sigma^2 P/((na-1)(na-2)na_0^2]$$
$$+[2\sigma^2 Q/((na-1)(na-2))]$$
$$+[\sigma^2/((n-1)(na-2)a)][1-R]$$

Also,

$$MSE(\hat{\sigma}_u) = \sigma^2/(n-1)(na-2)a] \;\; ; \;\; na > 2$$

The relative efficiency of σ^*_{PT} with respect to $\hat{\sigma}_u$ is defined as

$$REF(\sigma^*_{PT}, \hat{\sigma}_u) = \frac{MSE(\hat{\sigma}_u)}{MSE(\sigma^*_{PT})}$$

Thus,

$$REF(\sigma^*_{PT}, \hat{\sigma}_u) = [1 + \{n(n-1)a^2(na - 2na_0 - 1)P/((na-1)(na_0)^2)\}$$
$$+ \{2(n-1)aQ/(na-1)\} - R]^{-1}$$

Values of $REF(\sigma^*_{PT}, \hat{\sigma}_u)$

The values of relative efficiencies for different values of a, a_0, n and α are presented in tables 1 to 4. These tables show that the preliminary test estimator σ^*_{PT} has higher efficiencies than $\hat{\sigma}_u$ when the guess value a_0 is near to a and n is small and the gain in efficiencies decrease with increase of α for these set of values (a, a_0) and n. Also for small n, if the value of α is selected large, the interval increases in which σ^*_{PT} is efficient than $\hat{\sigma}_u$.

Table 1

n = 5

α	a=1.25	a = 0.75	a = 0.90	a = 1.00	a = 1.00	a = 2.50
	a_0 = 0.50	a_0 = 1.00	a_0 = 1.05	a_0 = 1.00	a_0 = 0.80	a_0 = 1.25
0.01	0.4820	0.9794	1.1009	1.2223	1.3104	0.6870
0.02	0.4964	0.9780	1.0893	1.2054	1.2914	0.6917
0.05	0.5365	0.9815	1.0712	1.1705	1.2455	0.7082
0.10	0.5968	0.9903	1.0569	1.1331	1.1898	0.7375
0.20	0.7017	1.0043	1.0432	1.0882	1.1177	0.7938
0.40	0.8564	1.0168	1.0294	1.0427	1.0446	0.8855
0.60	0.9440	1.0166	1.0192	1.0226	1.0141	0.9458

Table 2

n = 7

α	a=1.25	a = 0.75	a = 0.90	a = 1.00	a = 1.00	a = 2.50
	a_0 = 0.50	a_0 = 1.00	a_0 = 1.05	a_0 = 1.00	a_0 = 0.80	a_0 = 1.25
0.01	0.4517	0.9695	1.0630	1.1497	1.1760	0.6349
0.02	0.4834	0.9690	1.0550	1.1388	1.1641	0.6482
0.05	0.5583	0.9730	1.0426	1.1157	1.1367	0.6838

0.10	0.6536	0.9809	1.0329	1.0901	1.1042	0.7337
0.20	0.7871	0.9932	1.0240	1.0586	1.0626	0.8125
0.40	0.9258	1.0053	1.0158	1.0264	1.0209	0.9148
0.60	0.9782	1.0077	1.0103	1.0115	1.0045	0.9674

Table 3

n = 9

α	a=1.25	a = 0.75	a = 0.90	a = 1.00	a = 1.00	a = 2.50
	a_0 = 0.50	a_0 = 1.00	a_0 = 1.05	a_0 = 1.00	a_0 = 0.80	a_0 = 1.25
0.01	0.4672	0.9638	1.0429	1.1128	1.1125	0.6284
0.02	0.5181	0.9642	1.0368	1.1048	1.1037	0.6650
0.05	0.6254	0.9690	1.0277	1.0876	1.0845	0.7388
0.10	0.7417	0.9771	1.0208	1.0682	1.0627	0.8154
0.20	0.8709	0.9890	1.0149	1.0439	1.0359	0.9006
0.40	0.9666	1.0009	1.0099	1.0189	1.0103	0.9697
0.60	0.9921	1.0042	1.0065	1.0077	1.0009	0.9917

Table 4

n = 12

α	a=1.25	a = 0.75	a = 0.90	a = 1.00	a = 1.00	a = 2.50
	a_0 = 0.50	a_0 = 1.00	a_0 = 1.05	a_0 = 1.00	a_0 = 0.80	a_0 = 1.25
0.01	0.5349	0.9591	1.0258	1.0824	1.0624	0.6284
0.02	0.6138	0.9606	1.0215	1.0767	1.0560	0.6650
0.05	0.7516	0.9666	1.0153	1.0642	1.0432	0.7388
0.10	0.8634	0.9651	1.0110	1.0499	1.0300	0.8154
0.20	0.9458	0.9867	1.0077	1.0319	1.0151	0.9006
0.40	0.9910	0.9981	1.0055	1.0132	1.0025	0.9697
0.60	0.9983	1.0018	1.0039	1.0050	0.9988	0.9917

Revised paper

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