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STUDY OF HETEROGENEOUS SERVERS WITH VACATIONS IN M/M/2 QUEUEING MODEL

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ABSTRACT

The examination on multi-worker queuing frameworks by and large expects the servers to be homogeneous in which the individual assistance rates are something very similar for every one of the servers in the framework. This suspicion might be substantial just when the assistance interaction is exceptionally precisely or electronically controlled. In a queuing framework with human servers, the above suspicion can scarcely be figured it out. It is entirely expected to notice servers delivering administration to indistinguishable positions at various help rates. This reality prompts demonstrating such multi-worker holding up lines with heterogeneous help rates. The examination of a queuing framework with heterogeneous servers and servers' vacations assists with considering the queuing frameworks in which the job of optional positions is considered. As noted before, the investigation of get-away queuing models join the optional positions in the demonstrating. The examination of queuing frameworks with heterogeneous servers and servers' vacations assists with considering the effect of optional positions on the assurance of framework execution. In this paper we examine a Monrovian line with two heterogeneous servers and servers' vacations.

Keyword: homogeneous, excursion queueing frameworks, multi-worker, servers,

INTRODUCTION

The huge improvements in queueing models with server get-away have prompted another part of queueing frameworks, specifically, "excursion queueing frameworks". Server excursions might be because of absence of work, server disappointment, or one more errand being doled out to the server, which happen in applications like PC support and testing, preventive upkeep occupations in a creation framework, need lines, and so forth [1]. In these frameworks, the server isn't generally accessible to serve its essential clients. Multi-worker lines without worker vacations have been concentrated by various creators. Perhaps the most punctual work can be found in Morse (1958) in which the framework has two equal help channels with various assistance rates and just one of them is dynamic at some random time. A showing up client, when the framework is inactive, joins both of the two channels and the framework gets enacted. No other new client is permitted to enter the framework until the continuous assistance is finished at the involved branch. The consistent state brings about the instances of (I) no line is permitted, and, (2) a boundless line is permitted before the help office have been acquired. The unequivocal articulations for state probabilities for a two-worker Markovian line have been gotten in Saaty (1961). The investigation has been reached out to the instance of n free heterogeneous servers in Gupta and Goel (1962). Krishnamurthy (1963) has examined a two-worker model with various help rates in which clients are conceded to the individual assistance stations as indicated by not really settled probabilities. Godini (1965) has additionally examined a M/M/S queueing framework with heterogeneous servers.

REVIEW LITERATURE

Krishna and Lee conducted a research on two-phase services (1990). Doshi (1991) examines a two-phase queueing system with standard service times. Choudhury & Deka investigate a single server general service queue with two stages of heterogeneous service and an unstable server (2012). Choudhury et al. (2009) evaluate a single server, bulk arrival with a general service queue, as well as an optional second phase and an unreliable server. Under N-policy, this system consists of a breakdown and a wait period, assuming that the server is functioning with any phase of service and might break down at any time. Julia Rose Mary et al. (2011) looked at a single vacation queueing system with a second optional service channel that operated under a bi-level control strategy.

Park and colleagues (2010) investigated a single server, two-phase queuing system with a set batch size policy. Under the Bernoulli schedule vacation, a batch arrival queueing system with server provides two phases of heterogeneous service to the incoming batch of clients, one after the other (2004). Nobel and Tijms (1999) investigate two service modes for a single server architecture with batch Poisson input queues. Wu et al (2009) examined BMAP/G/1 G-queues with a second optional service and several vacations using extra variables and a censoring approach. Choi and Kim (2007) investigated a two-phase queueing system with vacations and consumer Bernoulli feedback. Ke et al (2013) examined an infinite-capacity multi-server queueing system with a second phase of optional service using a matrix analytical technique.

Levy and Yechiali (1976) have introduced the vacation policy in a multi-server Markovian queue. They consider a model with s homogeneous servers and exponentially distributed vacation times. Using partial generating function technique, the Probability Generating Function (PGF) of the system size has been obtained and explicit results are deduced for two-homogeneous server vacation queuing model. This chapter is devoted to study a two-heterogeneous sever Markovian queue with server dependent vacations. This type of study can be used in real situations wilt two heterogeneous servers. For example, in machine repairman problems with two heterogeneous servers (the machines), the secondary jobs like maintenance of machine parts depends on the type of machine, which leads to the consideration of two different vacation time distributions. * The impact of the secondary jobs on the system performance measures helps to understand and plan more realistically a real situation than the case without giving due consideration to the secondary jobs.

RESEARCH OBJECTIVES

1. To identify heterogeneous servers and vacations, use the m/m/2 queueing model, which uses an exponential distribution during vacation.

MATHEMATICAL MODEL

Consider an excursion queuing framework with two heterogeneous servers as follows: Customers show up at the framework as indicated by a Poisson interaction with rate boundary X. The showing up clients joins a solitary holding up line. Toward the finish of the excursion he gets back to the framework and starts administration in case there Is somewhere around one client in the line; or on the other hand, if the line is vacant, takes the following get-away, etc. This holds useful for both the servers.

The PGF of framework size is acquired utilizing the halfway producing capacities.

Let X(t) denote the number of customers present in the system at time t. Let the server with service rate |j.i (faster sever) be designated as the first server.

Let

Poj = P[X(t) = j when both the servers are on vacations / X(0) = 0], j > 0

Pi j = P[X (t) = j and the first server is busy In the system while the second erver is on vacation / X (0) = 0], j > 1

P2j = P[X(t) = j and the first server is on vacation while the second server s busy in the system / X(0) = 0], j > 1

P3j = P[X(t) = j and both the servers are busy in the system / X(0) = 0], j > 2

Using the Chapman - Kolmogorov forward equations approach we have,

$$p_{00}(t + \Delta t) = p_{00}(t)\,[\,1 - \lambda\Delta t + o(\Delta t)\,] + p_{01}(t)\,[\,\theta_1\Delta t + o(\Delta t)\,]\,[\,\theta_2\Delta t + o(\Delta t)\,]$$
$$+ p_{11}(t)\,[\,\mu_1\Delta t + o(\Delta t)\,]\,[1 - \lambda\Delta t + o(\Delta t)\,]$$
$$+ p_{21}(t)\,[\,\mu_2\Delta t + o(\Delta t)\,]\,[\,1 - \lambda\Delta t + o(\Delta t)\,] + o(\Delta t)$$

By rearranging the terms, we have,

$$\frac{p_{00}(t + \Delta t) - p_{00}(t)}{\Delta t} = -\lambda\, p_{00}(t) + \mu_1\, p_{11}(t) + \mu_2\, p_{21}(t)$$

As At tends to zero, we obtain in the limit,

$$(d/dt)\ p_{00}(t) = -\lambda\, p_{00}(t) + \mu_1\, p_{11}(t) + \mu_2\, p_{21}(t) \qquad \ldots(1)$$

As t tends to infinity, poo(t) tends to poo, (independent of t), and the equation (1) becomes

$$\lambda\, p_{00} = \mu_1\, p_{11} + \mu_2\, p_{21} \qquad \ldots(2)$$

Similarly, we can derive the following equations:

$$[\lambda + \theta_1 + \theta_2]\, p_{0j} = \lambda\, p_{0,j-i}, \quad j \geq 1 \qquad \ldots(3)$$

$$[\lambda + \mu_1]\, p_{11} = \mu_1\, p_{12} + \mu_2\, p_{32} + \theta_1\, p_{01} \qquad \ldots(4)$$

$$[\lambda + \mu_1 + \theta_2]\, p_{1j} = \lambda\, p_{1,j-i} + \mu_1\, p_{1,j+1} + \theta_1\, p_{0j}, \quad j \geq 2 \qquad \ldots(5)$$

$$[\lambda + \mu_2]\, p_{21} = \mu_2\, p_{22} + \mu_1\, p_{32} + \theta_2\, p_{01} \qquad \ldots(6)$$

$$[\lambda + \mu_2 + \theta_1]\, p_{2j} = \lambda\, p_{2,j-1} + \mu_2\, p_{2,j+1} + \theta_2\, p_{0j}, \ j \geq 2 \qquad \ldots(7)$$

$$[\lambda + \mu_1 + \mu_2]\, p_{32} = [\mu_1 + \mu_2]\, p_{22} + \theta_1\, p_{22} + \theta_2\, p_{12} \qquad \ldots(8)$$

$$[\lambda + \mu_1 + \mu_2]\, p_{3j} = \lambda\, p_{3,j-1} + [\mu_1 + \mu_2]\, p_{3,j+1} + \theta_1\, p_{2j} + \theta_2\, p_{1j} \quad j \geq 2 \ ..(9)$$

Define the partial generating functions:

$$G_0(z) = \sum_{j=0}^{\infty} p_{0j}\, z^j$$

$$G_1(z) = \sum_{j=1}^{\infty} p_{1j}\, z^j$$

$$G_2(z) = \sum_{j=1}^{\infty} p_{2j}\, z^j$$

$$G_3(z) = \sum_{j=2}^{\infty} p_{3j}\, z^j$$

And,

Multiplying the equation (3) by z' and summing over j = 1, 2, 3 we get

$$(\lambda + \theta_1 + \theta_2) [G_0(z) - p_{00}] = \lambda z G_0(z)$$

$$\text{i.e., } (\lambda + \theta_1 + \theta_2 - \lambda z) G_0(z) = \lambda p_{00} + (\theta_1 + \theta_2) p_{00}$$

$$\text{i.e., } [\lambda(1- z) + (\theta_1 + \theta_2)] G_0(z) = \mu_1 p_{11} + \mu_2 p_{21} + (\theta_1 + \theta_2) p_{00} \quad \ldots(10)$$

where we have used the equation (2) similarly, using the equations (4) to (9) we obtain

$$[\lambda z(1- z) - \mu_1(1-z) + \theta_2 z] G_1(z) - \theta_1 z G_0(z) =$$
$$[\mu_2 p_{32} + \theta_2 p_{11}] z^2 - [\mu_1 p_{11} + \theta_1 p_{00}] z \quad \ldots(11)$$

$$[\lambda z(1- z) - \mu_2(1-z) + \theta_1 z] G_2(z) - \theta_2 z G_0(z) =$$
$$[\mu_1 p_{32} + \theta_1 p_{21}] z^2 - [\mu_2 p_{21} + \theta_2 p_{00}] z \quad \ldots(12)$$

$$[\lambda z(1-z) - (\mu_1 + \mu_2) (1-z)] G_3(z) - [\theta_1 G_2(z) + \theta_2 G_1(z)] z =$$
$$- [\theta_1 p_{21} + \theta_2 p_{11} + (\mu_1 + \mu_2) p_{32}] z^2. \quad \ldots(13)$$

$$\left.\begin{aligned} \text{Let,} \quad & G_0(1) = p_{0.} = \sum_{j=0}^{\infty} p_{0j} \\ & G_0(1) = p_{1.} = \sum_{j=1}^{\infty} p_{1j} \\ & G_0(1) = p_{2.} = \sum_{j=1}^{\infty} p_{2j} \\ \text{and,} \quad & G_0(1) = p_{3.} = \sum_{j=2}^{\infty} p_{3j} \end{aligned}\right\} \quad \ldots(14)$$

Evaluating the equations (10) to (13) at z = 1, we get

$$(\theta_1 + \theta_2)(p_{0.} - p_{00}) = \mu_1 p_{11} + \mu_2 p_{21} \quad \ldots(15)$$

$$\theta_2 (p_{1.} - p_{11}) - \theta_1 (p_{0.} - p_{00}) = \mu_2 p_{32} - \mu_1 p_{11} \quad \ldots(16)$$

$$\theta_1 (p_{2.} - p_{21}) - \theta_2 (p_{0.} - p_{00}) = \mu_1 p_{32} - \mu_2 p_{21} \quad \ldots(17)$$

$$\theta_1 (p_{2.} - p_{21}) + \theta_2 (p_{1.} - p_{11}) = (\mu_1 + \mu_2) p_{32} \quad \ldots(18)$$

Using the equation (15) in the equation (10) we get,

$$[\lambda(1-z) + \theta_1 + \theta_2] G_0(z) = (\theta_1 + \theta_2) p_{0.} \quad \ldots(19)$$

Using the equation (16) in the equation (11) we get,

$$[\lambda z(1-z) - \mu_1(1-z) + \theta_2 z] G_1(z) - \theta_1 G_0(z) = (\theta_2 p_{1.} - \theta_1 p_{0.}) z^2 - (\mu_1 p_{11} + \theta_1 p_{00}) z (1-z) \quad \ldots(20)$$

Using the equation (17) in the equation (12) we get,

$$[\lambda z(1-z) - \mu_2(1-z) + \theta_1 z] G_2(z) - \theta_2 G_0(z) = (\theta_1 p_{2.} - \theta_2 p_{0.}) z^2 - (\mu_2 p_{21} + \theta_2 p_{00}) z (1-z) \quad \ldots(21)$$

Using the equation (18) in the equation (13) we get,

$$[\lambda z(1-z) - (\mu_1 + \mu_2)(1-z)] G_3(z) - [\theta_1 G_2(z) + \theta_2 G_1(z)] = -(\theta_1 p_{2.} + \theta_2 p_{1.}) z^2 \quad \ldots(22)$$

The equations (19) to (22) give Gi(z) for i = 0, 1, 2, 3 recursively, provided we can determine the unknown quantities poo, Pn, P21, P32, Po., Pi.. P2, and p3. towards which we now proceed. The equations (15) to (18) are not linearly independent, since, the addition of the equations (15) to (17) lead to the equation (18) and hence we need five more equations to determine the unknowns which are obtained as follows. The first of the further required set of equations that is obvious is,

$$p_{0.} + p_{1.} + p_{2.} + p_{3.} = 1 \quad \ldots(23)$$

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Adding the equations (10), (11), and (12) we find,

$$+ [\ \mu_1 p_{11} + \mu_2 p_{21} + (\ \theta_1 + \theta_2\ )\ p_{00}\ ]$$

$$= - [\ \lambda z(1\text{-}z) - (\ \mu_1 + \mu_2\ )\ (1\text{-}z)\ ]\ G_3(z)$$

$$+ [\ \theta_1 G_2(z) + \ \theta_2 G_1(z)\ ]\ z$$

$$- (1\text{-}z)\ [\ \mu_1 p_{11} + \mu_2 p_{21} + (\ \theta_1 + \theta_2\ )\ p_{00}\ ]$$

where we have used the equation (13). Simplifying and dividing by (1-z) we have,

$$\lambda[\ G_0(z) + z\ (\ G_1(z) + G_2(z) + G_3(z)\ )\ ] - [\ \mu_1 G_1(z) + \mu_2 G_2(z) + (\ \mu_1 + \mu_2\ ) G_3(z)\ ]$$

$$+ (\ \theta_1 + \theta_2\ ) G_0(z) = \mu_1 p_{11} + \mu_2 p_{21} + (\ \theta_1 + \theta_2\ )\ p_{00} \qquad \ldots(24)$$

At z =1, the equation (24) along with the equation (23) gives,

$$\lambda - [\ \mu_1 p_{1\cdot} + \mu_2 p_{2\cdot} + (\ \mu_1 + \mu_2\ )\ p_{3\cdot}] = 0 \qquad \ldots(25)$$

Following the technique used in Levy and Yechiali (1976) we get two more equations as

$$\text{Let,} \quad \left.\begin{aligned} f_0(z) &= \lambda z(1\text{-}z), \\ h_1(z) &= \theta_1 z, \\ h_2(z) &= \theta_2 z, \\ f_1(z) &= \lambda z(1\text{-}z) - \mu_1(1\text{-}z) \\ f_2(z) &= \lambda z(1\text{-}z) - \mu_2(1\text{-}z), \\ f_3(z) &= \lambda z(1\text{-}z) - (\ \mu_1 + \mu_2\ )\ (1\text{-}z) \end{aligned}\right\} \qquad \ldots(26)$$

follows: Using the equation (26), the equations (19) to (22) can be written in the matrix form as

$$A(z)\ \bar{g}(z) = \bar{b}(z)$$

Where

$$A(z) = \begin{pmatrix} f_0(z) + h_1(z) + h_2(z) & 0 & 0 & 0 \\ - h_1(z) & f_1(z) + h_2(z) & 0 & 0 \\ - h_2(z) & 0 & f_2(z) + h_1(z) & 0 \\ 0 & - h_2(z) & - h_1(z) & f_3(z) \end{pmatrix}$$

$$\bar{g}(z) = [\, G_0(z) \quad G_1(z) \quad G_2(z) \quad G_3(z) \,]^T$$

And

$$\bar{b}(z) = \begin{pmatrix} (\theta_1 + \theta_2)\, p_{0.} \\ (\theta_2 p_{1.} - \theta_1 p_{0.})\, z^2 - (\mu_1 p_{11} + \theta_1 p_{00})z(1-z) \\ (\theta_1 p_{2.} - \theta_2 p_{0.})\, z^2 - (\mu_2 p_{21} + \theta_2 p_{00})z\,(1-z) \\ -(\theta_1 p_{2.} + \theta_2 p_{1.})z^2 \end{pmatrix}$$

The equation (27) is a system of four linear equations in Gj(z), i = 0, 1, 2, 3 for all real value of z for which A(z) is nonsingular. Using Cramer's rule we have.

$$|A(z)|\; G_k(z) = |A_k(z)|, \quad k = 0, 1, 2, 3 \qquad \text{...(28)}$$

Where I Ak(z) | is the determinant obtained from the I A(z) I by replacing the (k+1)-st column by the vector in b(z).

Since I A (z) | is a continues function of z, the equation (27) is true for every z in the region I z | < 1. As, Gk (z) > 0, 0 < z < 1, we note that z is also a root of | A k(z) I. We prove in the following lemma that I A(z) I has only two distinct roots in the interval (0, 1). The method is similar to the analysis provided in Levy and Yechiali (1976).

Lemma

The polynomial I A(z) | has exactly two distinct roots in the intervel (0, 1). Proof:

$$|A(z)| = [\, h_0(z) + f_0(z) \,]\,[\, h_2(z) + f_1(z) \,]\,[\, h_1(z) + b_2(z) \,]\,[\, f_3(z) \,] \qquad \text{...(29)}$$

Since each factor on the right hand side is a quadratic in z, | A(z) | is a polynomial of degree 8.

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Let {Zj(k)}, i =1, 2 be the roots of | A(z) | for k = 0, 1, 2, 3

Consider k = 0

$$\text{Now,} \qquad h_0(z) + f_0(z) = 0 \qquad \ldots(30)$$

The roots are : zi(0) and Z2(0)

The equation (27) is >.z(1-z) + (Oi + ()2)z = 0 or, $z[\ \lambda(1-z) + \theta_1 + \theta_2\ ] = 0$

Hence, z = 0 or z = 1+ (Oi+ O2) / A,, which is > 1 and hence z g (0, 1).

Similarly, for k = 3 we find,

$$\lambda z(1 - z) - (\ \mu_1 + \mu_2\ )\ (1 - z) = 0$$

$$(1 - z)\ [\ \lambda z - (\ \mu_1 + \mu_2\ )\ ] = 0$$

Hence, z = 1 or z = (m + f.12) / X., >1 and so z « (0, 1)

Fork=1 ,

$$h_2(z) + f_1(z) = 0$$

$$\text{or,} \quad \lambda z(1 - z) - \mu_1(1 - z) + \theta_2 z = 0$$

$$\text{i.e.,} \quad \lambda z^2 - (\lambda + \mu_1 + \theta_2)z + \mu_1 = 0$$

Solving for z, we find

$$z = [\ (\lambda + \mu_1 + \theta_2\ ) \pm \sqrt{(\ \lambda + \mu_1 + \theta_2)^2 - 4\lambda\mu_1}\ \ ]\ \ / 2\lambda \qquad \ldots(31)$$

Let

$$Q_1 = (\lambda + \mu_1 + \theta_2)^2 - 4\lambda\mu_1$$

$$=(\lambda + \mu_1)^2 - 4\lambda\mu_1 + \theta_2^2 + 2\theta_2(\lambda + \mu_1)$$

$$= (\lambda - \mu_1)^2 + \theta_2^2 + 2\theta_2(\lambda + \mu_1) . \qquad ...(32)$$

$$= (\lambda - \mu_1 + \theta_2)^2 - 2\theta_2(\lambda - \mu_1) + 2\theta_2(\lambda + \mu_1)$$

$$= (\lambda - \mu_1 + \theta_2)^2 + 4\theta_2\mu_1$$

$$> (\lambda - \mu_1 + \theta_2)^2, \text{ as } \theta_2 \text{ and } \mu_1 \text{ are both} > 0 \qquad ...(33)$$

From the equation (31) we find,

$$Q1 = (\mu_1 - \lambda)^2 + \theta_2^2 + 2\theta_2(\mu_1 + \lambda)$$

$$= (\mu_1 - \lambda + \theta_2)^2 - 2\theta_2(\mu_1 - \lambda) + 2\theta_2(\mu_1 + \lambda)$$

$$= (\mu_1 - \lambda + \theta_2)^2 + 4\lambda\theta_2$$

$$> (\mu_1 - \lambda + \theta_2)^2, \text{ as } \lambda \text{ and } \theta_2 \text{ are} > 0 \qquad ...(34)$$

Directly, from the equation (30), we get

$$Q_1 < (\lambda + \mu_1 + \theta_2) \qquad ...(35)$$

Let Z2(1) and zi(1) correspond to the plus and minus signs respectively in the equation (31).

$$z_2(1) = [(\lambda + \mu_1 + \theta_2) + \sqrt{Q_1}] / 2\lambda$$

$$> (\lambda + \mu_1 + \theta_2 + \lambda - \mu_1 + \theta_2) / 2\lambda$$

$$= [1 + \theta_2 / \lambda] > 1.$$

From the equation (33) By using the equation (34)

$$z_1(1) = [(\lambda + \mu_1 + \theta_2) - Q_1] / 2\lambda$$

$$< [(\lambda + \mu_1 + \theta_2) - (\mu_1 - \lambda + \theta_2)] / 2\lambda$$

$$= 1$$

And, from thd equation (35), we have,

$$z_1(1) = [(\lambda + \mu_1 + \theta_2) - Q_1] / 2\lambda$$
$$> [(\lambda + \mu_1 + \theta_2) - (\lambda + \mu_1 + \theta_2)] / 2\lambda$$
$$= 0$$

Thus, for k = 1 there is only one root in the interval (0, 1). In a similar manner, we can show that for k = 2 also there is only one root in the interval (0, 1). Thus we have shown that |A(Z)| has exactly two roots in the open interval (0, 1). Also, it is evident from the corresponding expressions that zi(1) and Z2(1) are distinct. Since there is no further necessity to distinguish the roots, we write

$$z_1(1) = z_1 \text{ and } z_2(1) = z_2.$$

$$[f_0(z_1) + h_1(z_1) + h_2(z_1)] b_1(z_1) + b_0(z_1) h_1(z_1) = 0$$

$$\text{Or, } [f_0(z_1) + h_1(z_1) + h_2(z_1)] [(\theta_2 p_{1.} - \theta_1 p_{0.})z_1^2 - (\mu_1 p_{11} + \theta_1 p_{00})z_1(1-z_1)] + (\theta_1 + \theta_2)p_{0.} \theta_1 z_1^2 = 0$$

$$\text{Or, } [f_0(z_1) + h_1(z_1) + h_2(z_1)] [(\theta_2 p_{1.} - \theta_1 p_{0.})z_1 - (\mu_1 p_{11} + \theta_1 p_{00}) (1-z_1)] + (\theta_1 + \theta_2)p_{0.} \theta_1 z_1 = 0 \quad ...(36)$$

Similarly, for Z2 we can obtain,

$$[f_0(z_2) + h_1(z_2) + h_2(z_2)] [(\theta_1 p_2 - \theta_2 p_{0.})z_2 - (\mu_2 p_{21} + \theta_2 p_{00})(1-z_2)] + (\theta_1 + \theta_2)p_{0.} \theta_2 z_2 = 0 \quad ...(37)$$

Solving equations (2), (15), (16), (17), (18), (23), (25), (36) and (37) we get:

$$p_{1.} = L_3 L_5 / (L_1 L_5 + L_2 L_4) \quad ...(38)$$

$$p_{2.} = L_3 L_4 / (L_1 L_5 + L_2 L_4) \quad ...(39)$$

$$p_{00} = (K_1 p_{1.} + K_2 p_{2.}) / K_3 \quad ...(40)$$

$$p_{11} = (A_8 p_{1.} - C p_{00}) / A_9 \quad ...(41)$$

$$p_{21} = (\lambda p_{00} - \mu_1 p_{11}) / \mu_2 \quad ...(42)$$

$$p_{0.} = (\lambda + \theta_1 + \theta_2) p_{00} / (\theta_1 + \theta_2) \quad ...(43)$$

$$p_{3.} = (\lambda - \mu_1 p_{1.} - \mu_2 p_{2.}) / (\mu_1 + \mu_2) \quad ...(44)$$

$$p_{32} = [\theta_1 (p_{2.} - p_{21}) + \theta_2 (p_{1.} - p_{11})] / (\mu_1 + \mu_2) \quad ...(45)$$

Thus, the determination of Gi(z), for i = 0, 1, 2, 3 is now complete.

THE SYSTEM PERFORMANCE MEASURES

Here we derive the first two moments of the system size for the present vacation model in steady state. Let random variable X denote the system size in steady state and G(z) be the PGF of X. Then we have the result

$$G(z) = \sum_{K=0}^{\infty} \sum_{i=0}^{3} p_{ik} z^k$$

$$= \sum_{i=0}^{3} G_i(z) \quad ...(46)$$

Differentiating the equation (46) witli respect to z at z = 0, we get

$$E(X) = G'(1) = \sum_{i=0}^{3} G_i'(1) \quad ...(47)$$

Similarly, by differentiating the equation (46) with respect to z and evaluating at z = 0 we obtain,

$$E(X^2) = G''(1) = \sum_{i=0}^{3} [G_i''(1) + G_i'(1)] \quad ...(48)$$

The variance of X is obtained from;

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$$Var(X) = E(X^2) - E^2(X) \quad ...(49)$$

Differentiating the equations (19), (20), (21) and (22) twice and evaluating the derivatives at z = 1, we get, after some algebraic manipulations,

$$G_0'(1) = \lambda p_{0.} / (\theta_1 + \theta_2) \quad ...(50)$$

$$G_0''(1) = 2\lambda G_0'(1) / (\theta_1 + \theta_2) \quad ...(51)$$

$$G_1'(1) = [1/\theta_2][(\lambda - \mu_1 + \theta_2)p_{1.} + \theta_1 G_0'(1) - \theta_1 p_{0.} + \mu_1 p_{11} + \theta_1 p_{00}] \quad ...(52)$$

$$G_1''(1) = [1/\theta_2][2(\lambda - \mu_1)G_1'(1) + 2\mu_1 p_{1.} + \theta_1 G_0''(1)] \quad ...(53)$$

$$G_2'(1) = [1/\theta_1][(\lambda - \mu_2 + \theta_1)p_{2.} + \theta_2 G_0'(1) - \theta_2 p_{0.} + \mu_2 p_{21} + \theta_2 p_{00}] \quad ...(54)$$

$$G_2''(1) = [1/\theta_1][2(\lambda - \mu_2)G_2'(1) + 2\mu_2 p_{2.} + \theta_2 G_0''(1)] \quad ...(55)$$

$$G_3'(1) = [1/(\mu_1 + \mu_2 - \lambda)][\lambda + \lambda^2 p_{0.}(\theta_1 + \theta_2)^{-1} - (\mu_1 - \lambda)G_1'(1) - (\mu_2 - \lambda)G_2'(1)] \quad ...(56$$

$$G_3''(1) = [1/(\mu_1 + \mu_2 - \lambda)]\{2\lambda G_3'(1) + 2\lambda^3(\theta_1 + \theta_2)^{-2} p_{0.} + 2\lambda^2(\theta_1 + \theta_2)^{-1} p_{0.} + 2\lambda[G_1'(1) + G_2'(1)] - (\mu_1 - \lambda)G_1''(1) - (\mu_2 - \lambda)G_2''(1)\} \quad ...(57)$$

The equations (47) to (57) determine the basic queue characteristics of the present vacation queuing model.

Figure - 1 and Figure - 2 for dramatic excursion time for M/M/2 heterogeneous line.

Mean system size

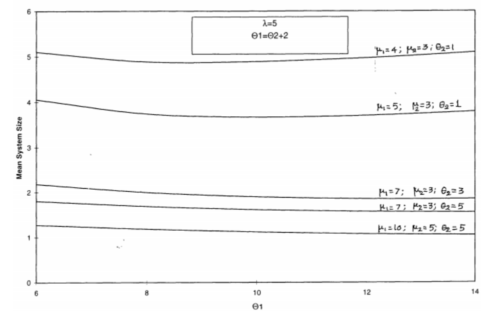

Figure 1

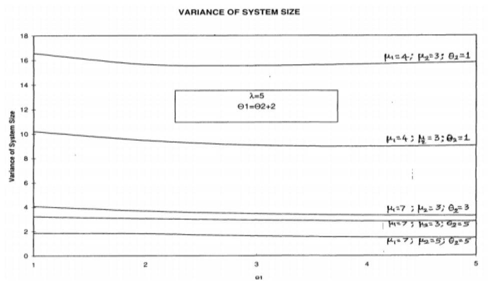

Figure 2

CONCLUSION

In this paper, we concentrated on a M/M/2 queueing model with heterogeneous servers. One server follows different get-away strategy. Yet, this server offers administration at a lower rate during get-away in case clients show up. The other server stays in the framework in any event, when it is empty. This sort of demonstrating can be utilized to consider queuing frameworks with two heterogeneous servers in which the inactive occasions of the servers are used for performing optional positions. We have inferred articulations for the initial two snapshots of the framework size in steady state. These outcomes are promptly versatile to mathematical calculations.

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