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MATRICES IN ROBOTICS AND COMPUTER GRAPHICS

Supervised by

Mrs. S. Savitha, AP/ MATHS

Presented by

D. Arisrada, G. Rohini,V.E. Nandini

- 1st year CSE students

Builders Engineering College, Nathakadaiyur

ABSTRACT

Computer graphics is a computing field that involves the creating, storing and processing of image content via computer. The usefulness of a matrices in computer graphics is its ability to convert geometric data into different coordinate systems. A matrix is of elements arranged in rows and columns. In this computer graphics column matrices can be used to represent points in 2D or 3D, while matrices of dimension 2×n and 3×n can be used to represent sets of points in 2D or 3D. Matrices allow arbitrary linear transformations to be represented in a consistent format (T(x)=AX for some 2×n (or 3×n) matrix A, called transformation matrix of T), suitable for computation.

Keywords: Matrix, Application, Robotics, Automatics, Graphics,Homogeneous transformation

INTRODUCTION

Arthur Cayley (1821-1895) a great French mathematician, discovered matrices in the year of 1860.

MATRIX:

A elements arranged in a rectangular arrangement of rows and bounded by brackets [] or () is called as matrix.

$$A=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

A is a matrix of order m × n

m ⟶ Rows

n ⟶ Columns

m*n numbers are called an element of matrix.The matrix A is denoted by $[a_{ij}]$

ORDER OF MATRIX

The order of a matrix is denoted by the number of rows and columns.

TYPES OFMATRICES:

- Row matrix
- Column matrix
- Square matrix
- Null or Zero matrix
- Diagonal matrix
- Scalar matrix
- Unit matrix or Identity matrix
- Upper triangular matrix
- Lower triangular matrix
- Symmetric matrix
- Anti-symmetric matrix

IMPORTANT OPERATIONS ON MATRIX

1. ADDITION OF MATRICES

$A=\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{pmatrix}$

$B=\begin{pmatrix} \mathbf{5} & \mathbf{6} \\ \mathbf{7} & \mathbf{8} \end{pmatrix}$

$A+B=\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{pmatrix}\begin{pmatrix} \mathbf{5} & \mathbf{6} \\ \mathbf{7} & \mathbf{8} \end{pmatrix}$

$=\begin{pmatrix} \mathbf{1+5} & \mathbf{2+6} \\ \mathbf{3+7} & \mathbf{4+8} \end{pmatrix}$

$=\begin{pmatrix} \mathbf{6} & \mathbf{8} \\ \mathbf{10} & \mathbf{12} \end{pmatrix}$

$A+B=\begin{pmatrix} \mathbf{6} & \mathbf{8} \\ \mathbf{10} & \mathbf{12} \end{pmatrix}$

If A and B be two matrices of the same order,then their sum A+B is defined as the matrix each element of which is the sum of the corresponding elements of A and B.

2. MULTIPLICATION OF MATRIX

Two matrices can be multiplied only when the number of columns in first is equal to the number of rows in the second.

$A=\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{4} & \mathbf{5} \end{pmatrix}$ $B=\begin{pmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} \end{pmatrix}$

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$$AB=\begin{pmatrix}1 & 2\\4 & 5\end{pmatrix}\begin{pmatrix}2 & 1\\1 & 2\end{pmatrix}$$

$$=\begin{pmatrix}2+2 & 1+4\\8+5 & 4+10\end{pmatrix}$$

$$AB=\begin{pmatrix}4 & 5\\13 & 14\end{pmatrix}$$

APPLICATION OF MATRIX

Matrices have many applications in diverse fields of science,commerce and social science.

Matrix are used in:

- Computer graphics
- Optics
- Cryptography
- Economics
- Chemistry
- Geology
- Robotics and Animation
- Wireless communication and signal processing
- Finance ices
- Mathematics

MATRIX IN ROBOTICS AND AUTOMATIONS

- For checking robot movements
- Controlling the robot

1.Matrix in robotics and automation.By checking robot movements.Controlling the robot

2.Homogenious transformation matrices transformation matrices are the 4*4 matrices that describes the rotation and translation with respect to something else.It is used in robotics as it helps in the movement and automation of the parts of the robots.It is very useful for examining rigid body position and orientation of a sequence of robotic links and joint frames.

3.Transformation matrices can be used to describe that at what angle the servos need to be to reach the desired position in space or may be an underwater autonomous vehicle needs to reach or align itself with several different obstacles inside the water.In other words,transformation helps us to determine the movement of the parts of the objects or the robots with respect to one another.

HOMOGENIOUS TRANFORMATION MATRICES

- Transformation matrices are the 4*4 matrices that describes the rotation and translation with respect to something else.
- It is used in robotics as it helps in the movement and automation of the parts of the robots.
- It is very useful for examining rigid body position and orientation of a sequence of robotic links and joint frames.

STRUCTURE OF HOMOGENOUS TRANSFORMATION

AGlobal=A1*A2*A3*A4

Rotation matrix ↑ Vector(origin to origin) ↑

$$AGlobal=\begin{bmatrix} r11 & r12 & r13 & Ox \\ r21 & r22 & r23 & Oy \\ r31 & r32 & r33 & Oz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scale ↓

Here a represent, translation,rotation,stretching or shrinking and perspective transformation.

ORIENTATION AND POSITION REPRESENTATION

- The columns of the rotation matrix from a sub-orientation matrix while vector is the frame's origin offset.
- Now here column 1 is A vector that orients the framesaxis relative to the base x,y,z axes respectively.Similar interpretations are made for the frame's y and z axes represented by column 2 and 3.
- Also,origin vector with three components represents the frames origin relative to the reference axis.

FRAME INTERPRETATION OF TRANSFORMATION

- Here we have been given with the vector u and its transformation
- Is represented by
- V=Hu
- Now this vector has components as Ux,Uy,Uz in a column and it has to be expanded to 4*1

INTERPRETATION OF HT

$$V=\begin{bmatrix} R & P \\ 0^\wedge T & 1 \end{bmatrix}\begin{bmatrix} Ux \\ Uy \\ Uz \\ 1 \end{bmatrix}=Ru+p$$

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Now in this 1 is added to frame origin the rotation vector (3*3) resolves the vector u in the base frame.

APPLICATION OF MATRICESIN COMPUTER GRAPHICS:

AFFINE SPACE:

An affine space is nothing more than a vector space whose origin we try to forget about, by adding translations to the linear maps. Therefore,we have scalar, vectors and point. More generally, an affine space (Euclidean spaces are specific affine spaces), that is, a function which maps an affine space onto itself while preserving both the dimension of any affine subspaces and the ratios of the lengths of parallel line segments. Consequently, sets of parallel affine subspaces remain parallel after an affine transformation. An affine transformation does not necessarily preserve angles between lines or distances between points, though it does preserve ratios of distances between points lying on a straight line.

AFFINE TRANSFORMATION:

In geometry, an affine transformation can be represented as the composition of a linear transformation plus a transformation. If we want to do any affine transformation in 3D space, we can extend our vectors to fourdimension and using 4×4 matrix to transform them.

HOMOGENEOUS COORDINATE SYSTEM:

The process of implementing several successive transformations is, by using Cartesian coordinates, computationally more extensive. Namely, it can be noticed that to perform translation, matrix addition is carried out, while to perform other transformations, only matrix multiplication is carried out. In order to treat all transformations in the same manner,i.e.to reduce them to matrix multiplication, homogeneous coordinates are introduced. Thus, each Cartesian point T(x,y) can be converted into homogeneous coordinates T(x,y,w), and a coordinate system with this point representation is called a homogeneous coordinate system. Although triples of coordinates commonly display a point in space, here they represent a point in the plane or 2D point. Two sets of homogeneous coordinates, (x,y,w) and (x',y',z'), represent the same point if one is a multiple of the other. For example, coordinates (1,2,3) and (2,4,6) represents the same point. It can be concluded that one point in homogeneous coordinates can be displayed in infinitely many ways, that is, one whole class of triples of coordinates represents the same point in the plane. This class of triples of coordinates which represent one point in the plane (t_x,t_y,t_w), where $t \neq 0$, actually represents a line in the 3D space. It should be noted that at least one homogeneous coordinate must be different from zero, meaning that the point (0,0,0) is not allowed. If the coordinate $w \neq 0$, each of them can be divided by w, which is why the point T can be written as: $T(x,y,w)=T(x/w,y/w,1)=(x_w,y_w,1)$

All such points form a plane with the equation w = 1 in the 3D space. In the case of w = 0, it is a point in infinity, which will not be considered in more detail. The usage of homogeneous coordinates is of great importance in computer graphics. Homogeneous coordinates provide a

uniform mathematical tool for all transformations, where matrix transformation through homogeneous coordinates must be in the 3×3 form[8].

In that sense, homogeneous translation matrix becomes:

$$M_{tr}=\begin{bmatrix}1 & 0 & tx\\ 0 & 1 & ty\\ 0 & 0 & 1\end{bmatrix},$$

Where translation is reduced to matrix multiplication:

$$\begin{pmatrix}x'\\ y'\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}+\begin{pmatrix}tx\\ ty\end{pmatrix}\rightarrow\begin{pmatrix}x'\\ y'\\ 1\end{pmatrix}=\begin{pmatrix}1 & 0 & tx\\ 0 & 1 & ty\\ 0 & 0 & 1\end{pmatrix}\cdot\begin{pmatrix}x\\ y\\ 1\end{pmatrix}$$

Furthermore, the homogeneous translation matrix is:

$$M_{tr}=\begin{bmatrix}1 & 0 & tx\\ 0 & 1 & ty\\ 0 & 0 & 1\end{bmatrix}.$$

Homogeneous rotation matrix:

$$M_{ro}=\begin{bmatrix}cos\emptyset & -sin\emptyset & 0\\ sin\emptyset & cos\emptyset & 0\\ 0 & 0 & 1\end{bmatrix}.$$

Homogeneous scaling matrix:

$$M_{sc}=\begin{bmatrix}Sx & 0 & 0\\ 0 & Sy & 0\\ 0 & 0 & 1\end{bmatrix}$$

And homogeneous shear matrix:

$$M_{sh}=\begin{bmatrix}1 & tan\beta & 0\\ tan\alpha & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.$$

In order to conduct several sequences of the transforms, all these transformations must be compound. After the implementation of all transformations, for example scaling, rotation and translation over the point T in this original order, and the coordinates of the point T' could be calculated in the following way:

$T' = M_{tr}.\ M_{ro}.\ M_{sc}.\ T,$

i.e. they could be calculated by multiplying the homogeneous scaling matrix, homogeneous rotation matrix and homogeneous translation matrix, respectively, into a single matrix.

CONCLUSION:

Computer graphics is an unavoidable part of everyday life. As it is widely known, computer graphics is present in computer games, films, television commercials, as well as in various branches of the industry. In this paper, authors tried to get closer to the students who are dealing with computer science, especially computer graphics, the mathematical background of computer graphics in which the term matrix is in the center. In our presentation, we come to know that matrices are applied in various applications which are very useful to our day-to-day life.

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