

Study of Symmetry in Ring Interconnection Network

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ABSTRACT

In this paper, Ring architecture is taken as an example of Interconnection Network. We have derived Adjacency Matrix of Ring and used it to study connection between nodes by applying logical AND operation among nodes.

Keywords: Adjacency Matrix, Interconnection Network, Logical AND, Parallel and Distributed Architecture, Ring architecture.

1. INTRODUCTION

1.1. Interconnection Network Architecture:

Interconnection network is a network in which nodes are connected with each other, here nodes may be a single processor or set of processors, to other nodes. We can categorize Interconnection networks on the basis of the way they are connected to each other i.e., on their topology. Topology is the way of connection in which one node is connected to other nodes. [3] Interconnection networks are from a group of high-speed computer networks. They are composed of processing elements at one end of the network and memory elements at the other end, they are connected by switching elements. They are highly used in parallel computing [11]

1.2. Ring Interconnection Network:

It is a type of interconnection network. It is one of the greenest modes of connecting nodes with each other. The nodes are connected with each other in such a way that they form a ring. For communication from one node to other node, it sends the messages to its adjacent node. Therefore, the data or message passes through in sequence from one node to other and reach the destination node [3]. Following is the graphical representation of Ring Interconnection Network:

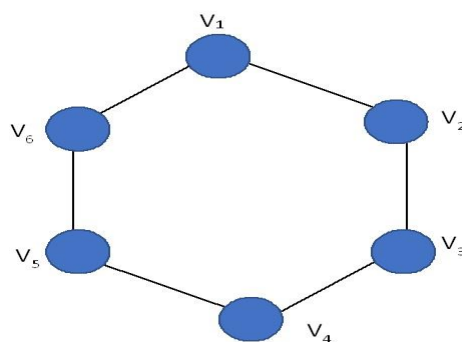


Figure 1. Ring Interconnection Network

1.3. Adjacency Matrix:

In Computer Science, an Adjacency Matrix is a square matrix. a finite graph is represented by an Adjacency Matrix. The elements of the Adjacency matrix signifies 1 if the pairs of vertices are adjacent to each other or signifies 0 if the pairs of vertices are not adjacent to each other in the graph. Since the elements of Matrix is either 0 or 1, therefore it is also known as BIT Matrix or Boolean Matrix. It is a special case of a finite simple graph in which the adjacency matrix is a (0,1)-matrix. If the graph is undirected (i.e. all of its edges are bidirectional), the adjacency matrix is symmetric.[7]

Definition:For a simple graph with vertex set $U = \{u_1, \dots, u_n\}$, the adjacency matrix is a square $n \times n$ matrix A such that its element A_{ij} is one when there is an edge from vertex u_i to vertex u_j , and zero when there is no edge. [7]

1.4. Boolean Operation:

The fundamental operations of Boolean algebra are AND [conjunction], OR [disjunction], and NOT [negation]. These Boolean operations are articulated by operators \wedge [AND], \vee [or] and \neg [NOT]. They are collectively referred to as Boolean operators.

The basic Boolean operations on variables x and y are defined as follows:[4][10]

Table 1. Boolean Operators

Boolean operation	Boolean Operator	Boolean Expression	Definition
Conjunction	AND	$X \wedge Y$	$X \wedge Y = 1$ if $X = Y = 1, X \wedge Y = 0$ otherwise
Disjunction	OR	$X \vee Y$	$X \vee Y = 0$ if $X = Y = 0, X \vee Y = 1$ otherwise
Negation	NOT	$\neg X$	$\neg X = 0$ if $X = 1, \neg X = 1$ if $X = 0$

2. ADJACENCY MATRIX OF RING

We have derived Adjacency Matrix of Ring from Fig. 1 and is shown below in Fig. 2. We are also representing Matrix in Tabular form below in Table 2:

$$R = \begin{bmatrix} 1 & 1 & 00 & 0 & 1 \\ 1 & 1 & 10 & 0 & 0 \\ 0 & 1 & 11 & 0 & 0 \\ 0 & 0 & 11 & 1 & 0 \\ 0 & 0 & 01 & 1 & 1 \\ 1 & 0 & 00 & 1 & 1 \end{bmatrix}$$

Figure 2. Adjacency Matrix of Ring Interconnection Network

Table 2. Tabular Form of Adjacency Matrix of Ring

	V1	V2	V3	V4	V5	V6
V1	1	1	0	0	0	1
V2	1	1	1	0	0	0
V3	0	1	1	1	0	0
V4	0	0	1	1	1	0
V5	0	0	0	1	1	1
V6	1	0	0	0	1	1

3. APPLYING LOGICAL ‘AND’ OPERATION AMONG NODES

In order to study the properties of ring interconnection, we have taken Adjacency Matrix of it and now we will apply Logical AND operation between each pair of nodes. This will give us an idea of connectivity between nodes and will help us in proving various graphical properties of Ring. Here, in the following section we have shown table in which each column represents AND operation between all pairs of nodes of Ring.

3.1. Applying AND operation between V1 and other nodes:

Table 3. AND logical operation between V1 and other nodes

$V1 \wedge V1$	$V1 \wedge V2$	$V1 \wedge V3$	$V1 \wedge V4$	$V1 \wedge V5$	$V1 \wedge V6$
1	1	0	0	0	1
1	1	1	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
1	0	0	0	1	1

3.2. Applying AND operation between V2 and other nodes:

Table 4. AND logical operation between V2 and other nodes

$V2 \wedge V1$	$V2 \wedge V2$	$V2 \wedge V3$	$V2 \wedge V4$	$V2 \wedge V5$	$V2 \wedge V6$
1	1	0	0	0	1
1	1	1	0	0	0
0	1	1	1	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

3.3. Applying AND operation between V3 and other nodes:

Table 5 AND logical operation between V3 and other nodes

$V3 \wedge V1$	$V3 \wedge V2$	$V3 \wedge V3$	$V3 \wedge V4$	$V3 \wedge V5$	$V3 \wedge V6$
0	0	0	0	0	0
1	1	1	0	0	0
0	1	1	1	0	0
0	0	1	1	1	0
0	0	0	0	0	0
0	0	0	0	0	0

3.4. Applying AND operation between V4 and other nodes:

Table 6. AND logical operation between V4 and other nodes

$V4 \wedge V1$	$V4 \wedge V2$	$V4 \wedge V3$	$V4 \wedge V4$	$V4 \wedge V5$	$V4 \wedge V6$
0	0	0	0	0	0
0	0	0	0	0	0
0	1	1	1	0	0
0	0	1	1	1	0
0	0	0	1	1	1
0	0	0	0	0	0

3.5. Applying AND operation between V5 and other nodes:

Table 7. AND logical operation between V5 and other nodes

$V5 \wedge V1$	$V5 \wedge V2$	$V5 \wedge V3$	$V5 \wedge V4$	$V5 \wedge V5$	$V5 \wedge V6$
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	1	0
0	0	0	1	1	1
1	0	0	0	1	1

3.6. Applying AND operation between V6 and other nodes:

Table 8. AND logical operation between V6 and other nodes

$V6 \wedge V1$	$V6 \wedge V2$	$V6 \wedge V3$	$V6 \wedge V4$	$V6 \wedge V5$	$V6 \wedge V6$
1	1	0	0	0	1
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	1	1	1
1	0	0	0	1	1

4. ANALYSING SYMMETRY PROPERTY OF RING

Symmetry is a type of invariance: the property that a mathematical object remains unchanged under a set of operations or transformations.[2] Let the objects here are nodes of Ring and operation here is logical AND.

Thus, by the definition of Symmetry, if we apply logical AND operation between any two nodes such as Node1 AND Node2 then the result should be equal to the result of Node2 AND Node1. Mathematically it represented as:

$$A \wedge B = B \wedge A,$$

where A and B are nodes of Ring and R represents Logical AND operation.

This will satisfy the symmetrical property of Ring architecture. To prove the symmetrical property of Ring by using Adjacency Matrix and logical operation, we will apply AND operation among nodes as shown below:

Table 9. AND operation between V1 AND V2

V1	V2	$V1 \wedge V2$	$V2 \wedge V1$
1	1	1	1
1	1	1	1
0	1	0	0
0	0	0	0
0	0	0	0
1	0	0	0

From Table 9, we can conclude that V1 AND V2 and V2 AND V1 has same result. Thus, by tautology it is proved that they are symmetrical in nature.

Similarly, we have derived results for other pair of nodes also. They are shown below:

Table 10. AND operation between V2 AND V3

V2	V3	$V2 \wedge V3$	$V3 \wedge V2$
1	1	1	1
1	1	1	1
0	1	0	0
0	0	0	0
0	0	0	0

1	0	0	0
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Table 11. AND operation between V3 AND V4

V3	V4	V3 [^] V4	V4 [^] V3
1	1	1	1
1	1	1	1
0	1	0	0
0	0	0	0
0	0	0	0
1	0	0	0

Table 12. AND operation between V4 AND V5

V4	V5	V4 [^] V5	V5 [^] V4
1	1	1	1
1	1	1	1
0	1	0	0
0	0	0	0
0	0	0	0
1	0	0	0

Table 13. AND operation between V5 AND V6

V5	V6	V5 [^] V6	V6 [^] V5
1	1	1	1
1	1	1	1
0	1	0	0
0	0	0	0
0	0	0	0
1	0	0	0

Table 14. AND operation between V6 AND V1

V6	V1	V6 [^] V1	V1 [^] V6
1	1	1	1
1	1	1	1
0	1	0	0
0	0	0	0
0	0	0	0



1	0	0	0
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Thus, from above results we can conclude that the Ring Interconnection Network is symmetrical in nature.

5. CONCLUSION

We have used logical operation on Adjacency matrix of Ring Interconnection Network to prove that it is symmetrical. This concludes that we can prove the symmetry of interconnection networks by using logical operation and adjacency matrix also. We can apply this method on other interconnection networks also. We can also use this method in future to prove other properties of interconnection network also.

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