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Study of Logics on Architecture in Parallel and Distributed System

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ABSTRACT

In this paper, we have taken anarchitecture of interconnection network in Parallel and Distributed System and derivedits Incidence Matrix. We have also applied logic operations on Incidence matrices to study them. We have also calculated communication complexity of network from Matrices after applying logical operations on them.

Keywords—*Communication Complexity, Logic Operations, Incidence Matrix, Interconnection Network, Parallel and Distributed Network,Topology.*

1. INTRODUCTION

1.1 Parallel and Distributed System

It is a system of network in which computers/host or devices are at different sub-networks communicated with each other. They communicate through some communication links. Various networks get connected to each other for better services. Each network comprises of their own architecture. They have their own interconnection network. There are many interconnection networkarchitectures like Hypercube, Mesh etc. For Parallel and distributed computing, these architectures get connected to each other through some communication links. Even though, Parallel computing and distributed computing meaning overlaps, we can differentiate on the basis that: in Parallel computing hosts/devices share common memory while in distributed computing each host/devices have their own memory [1].

We will study the architecture of interconnection networks here, by implementing logical operations and by deriving connectivity diagram and connectivity states from incidence matrices of architecture.

1.2 Incidence Matrix of Interconnection Network Architecture

We can derive an Incidence Matrix of architecture of any interconnection network by using following method[3]:

(1) $A_{ij} = \begin{cases} 1 \; ifnodeisconnected \\ 0 \; else \end{cases},$

where A represents a matrix with i (number of rows) and j (number of columns).

The Incidence Matrix so derived is always a square sparse matrix since number of nodes for both column and rows are equal and most of the values are 0.

2. STUDY OF CONNECTIVITY DIAGRAM

Let consider an interconnection network with four nodes, the following Fig. 1 shows it graphically:

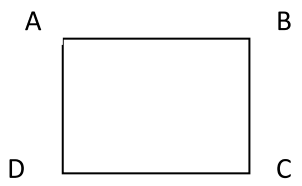

Figure 1. Interconnection Networkarchitecture with 4 nodes

Now,we will derive its Incidence Matrix by using equation (1) as shown above [2]:

$$M_{ij} = \begin{vmatrix} 1 & 10 & 1 \\ 1 & 11 & 0 \\ 0 & 11 & 1 \\ 1 & 01 & 1 \end{vmatrix}$$

Following is the table representation of given matrix:

Table 1. Table Representation of M_{ij} Matrix.

	A	B	C	D
A	1	1	0	1
B	1	1	1	0
C	0	1	1	1
D	1	0	1	1

2.1. Connectivity Diagram

Following is the Connectivity Diagram of above matrix[3]:

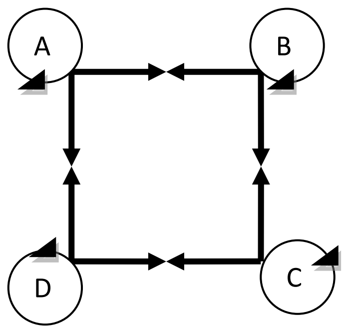

Figure 2. Connectivity Diagram of A_{ij} Matrix.

From the above diagram and table, following connectivity states are derived:

1. 1101 This represents connectivity of A node with other nodes
2. 1110 This represents connectivity of B node with other nodes
3. 0111 This represents connectivity of C node with other nodes
4. 1011 This represents connectivity of D node with other nodes

Following table will illustrate the connectivity state. Let take first connectivity state i.e, 1101:

Table 2. Connectivity State Values.

Element	Represents
1	Shows self-loop of node A
1	Shows A and B are connected
0	Shows A is not connected to C
1	Shows A is connected to D

Similarly, for other connectivity states, value of 1 shows connectivity and 0 shows no connectivity.We can also represent the above-mentioned connectivity states graphically as shown below:

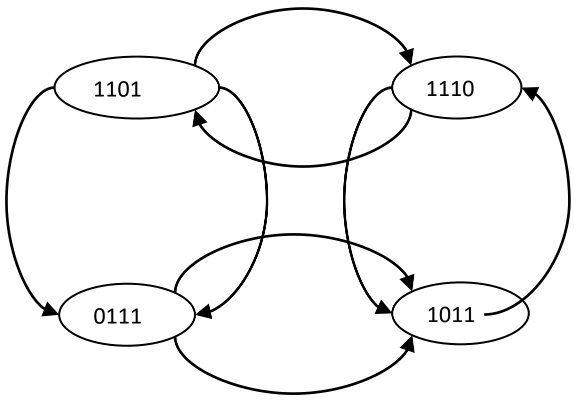

Figure 3. Connectivity State Diagram of Table 1.

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Incidence Matrix of architectures and their connectivity diagrams gives us clear idea about how one node communicates with other in architecture. We can further extend this to study other properties of connectivity of nodes in network.

3. STUDY OF LOGICAL OPERATIONS ON INCIDENCE MATRIX

With the help of Incidence Matrix, we are trying to study architecture with respect to matrices and their properties. We are trying a new approach of studying architecture through matrices and applying logical operations on them. Here, in this paper we have taken a simple architecture with four nodes as shown in Fig.1. Wewill try to apply logical operations like AND, OR, XOR and implication on it and will see the result. Here, at first,we will apply AND operation between nodes of network showed in Fig. 1 with the help of following function:

(2) $M^{AND}{}_{ij} = \{b_i \; AND \; b_j \,,$

Where $M^{AND}{}_{ij}$ is a matrix obtained after applying AND operation on values of i (rows) and j (columns) of matrix M_{ij}.

Assume that every row represents by i{0,1,2,3} and column by j{0,1,2,3}. Following are the table shows total number of rows and columns for AND operation matrix [refer Table 1].Possible conditions for i and j are:

Table 4. List of rows and columns for AND operation matrix.

i	**j**	**i**	**j**	**i**	**j**	**i**	**j**
A	A	B	A	C	A	D	A
A	B	B	B	C	B	D	B
A	C	B	C	C	C	D	C
A	D	B	D	C	D	D	D

We will apply AND operation between values of i and j with the help of equation 2.

The following matrix (let say them AND matrices) will represents the result of AND operation between values offirst row i=A= {1,1,0,1} andfirst columnj=A={1,1,0,1}:

$$M_{AA} = \begin{vmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{vmatrix}$$

Table 5. Tabular Form of M_{AA}[AND operation matrix by applying AND operation on first row and first column]

AND(.)	1	1	0	1

1	1	1	0	1
1	1	1	0	1
0	0	0	0	0
1	1	1	0	1

The following matrix will represent the result of AND operation between values of first row i= A= {1,1,0,1} and second column j= B= {1,1,1,0}:

$$M_{AB} = \begin{vmatrix} 1 & 10 & 1 \\ 1 & 10 & 1 \\ 1 & 10 & 1 \\ 0 & 00 & 0 \end{vmatrix}$$

Table 6. Tabular Form of M_{AB} [AND operation matrix by applying AND operation on first row and second column]

AND(.)	1	1	0	1
1	1	1	0	1
1	1	1	0	1
1	1	1	0	1
0	0	0	0	0

The following matrix will represent the result of AND operation between values of first row i= {1,1,0,1} and third column j= {0,1,1,1}:

$$M_{AC} = \begin{vmatrix} 0 & 00 & 0 \\ 1 & 10 & 1 \\ 1 & 10 & 1 \\ 1 & 10 & 1 \end{vmatrix}$$

Table 7. Tabular Form of M_{AC}[AND operation matrix by applying AND operation on first row and third column]

AND(.)	1	1	0	1
0	0	0	0	0
1	1	1	0	1
1	1	1	0	1
1	1	1	0	1

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The following matrix will represent the result of AND operation between values of first row i= {1,1,0,1} and fourth column j= {1,0,1,1}:

$$M_{AD} = \begin{vmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{vmatrix}$$

Table 8. Tabular Form of M_{AD}[AND operation matrix by applying AND operation on first row and fourth column]

AND(.)	1	1	0	1
1	1	1	0	1
0	0	0	0	0
1	1	1	0	1
1	1	1	0	1

Similarly, when we apply AND operation on other rows and columns we will get, following 16 matrices (including above matrices from AND operation):

01 . M_{AA}
02 . M_{AB}
03 . M_{AC}
04 . M_{AD}
05 . M_{BA}
06 . M_{BB}
07 . M_{BC}
08 . M_{BD}
09 . M_{CA}
10 . M_{CB}
11 . M_{CC}
12 . M_{CD}
13 . M_{DA}
14 . M_{DB}
15 . M_{DC}
16 . M_{DD}

When we will calculate Transpose of each matrix given above, we will get a new matrix. Let they are:

01 . $M^T_{AA}= M_{AA}$

02 $M^T_{AB} = M_{BA}$

03 . $M^T_{AC} = M_{CA}$

04 . $M^T_{AD} = M_{DA}$

05 . $M^T_{BA} = M_{AB}$

06 . $M^T_{BB} = M_{BB}$

07 . $M^T_{BC} = M_{CB}$

08 . $M^T_{BD} = M_{DB}$

09 . $M^T_{CA} = M_{AC}$

10 . $M^T_{CB} = M_{BC}$

11 . $M^T_{CC} = M_{CC}$

12 . $M^T_{CD} = M_{DC}$

13 . $M^T_{DA} = M_{AD}$

14 . $M^T_{DB} = M_{BD}$

15 . $M^T_{DC} = M_{CD}$

16 . $M^T_{DD} = M_{DD}$

If we put these matrices together as values for a combined matrix say M then we will get:

$$M = \begin{bmatrix} M_{AA} & M_{BA} & M_{CA} & M_{DA} \\ M_{AB} & M_{BB} & M_{CB} & M_{DB} \\ M_{AC} & M_{BC} & M_{CC} & M_{DC} \\ M_{AD} & M_{BD} & M_{CD} & M_{DD} \end{bmatrix}$$

$$M^T = \begin{bmatrix} M_{AA} & M_{AB} & M_{AC} & M_{AD} \\ M_{BA} & M_{BB} & M_{BC} & M_{BD} \\ M_{CA} & M_{CB} & M_{CC} & M_{CD} \\ M_{DA} & M_{DB} & M_{DC} & M_{DD} \end{bmatrix}$$

Similarly, with the help of Incidence Matrix, we can apply OR,XOR and Implication operation between nodes with the help of following functions:

(3) $C_{ij} = \{c_i \; OR \; c_j \; where, C_{ij} \; is \; a \; incidence \; matrix \; of \; interconnectionnetwork\}$

(4) $D_{ij} = \{d_i \; XOR \; d_j \; ,where \; D_{ij} \; is \; a \; incidence \; matrix \; of \; interconnection \; network\}$

(5) $E_{ij} = \{e_i \rightarrow e_j \}, where \; E_{ij} \; is \; a \; incidence \; matrix \; of \; interconnection \; network\}$

We can see that, on applying AND operation on Incidence Matrix, we got patterns of strings of {0,1}, repeating itself in systematic and similar manner throughout the matrices.Also, total number of 1's (that shows connectivity of a node with other nodes) are same for all 16 AND matrices. Similarly, total number

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of 1s are same in all 16 matrices of OR and so on. But different for AND, OR, XOR and implication matrices. We can use this further for studying architecture ofinterconnection network.

4. COMMUNICATION COMPLEXITY OF NETWORK

Communication Complexity of network in Parallel and Distributed Computingmeans nodes communicating to how many other nodes at a time i.e., amount or degree of communication for nodes in a network. Here we have tried to find out communication complexities for nodes in AND, OR, XOR and implication matrices.

For AND

(6) $CC_{AND} = n^2 - (2n - 1) = 9$, where$n = 4$ (number of nodes).

If we consider that diagonal nodes are not communicating then,

(7) $CC_{AND} = n^2 - (2n - 1) - 2 = 7$, where n=4 (number of nodes).

For OR

(8) $CC_{OR} = n^2 - 1 = 15$ where$n = 4$ (number of nodes).

If we consider that diagonal nodes are not communicating then,

(9) $CC_{OR} = n^2 - [(2n - 1) - 1] = 10$, where$n = 4$ (number of nodes).

For XOR

(10) $CC_{XOR} = [n^2 - (n - 1)^2] - 1 = 6$, where $n = 4$(number of nodes).

If we consider that diagonal nodes are not communicating then,

(11) $CC_{XOR} = [n^2 - (n - 1)^2] - 3 = 4$, where $n = 4$ (number of nodes).

For Implication

(12) $CC_{imp} = n^2 - (n - 1) = 13$, where$n = 4$ (number of nodes).

This results that, for all AND matrices amount of communication for nodes is 9: for all OR operation is 15: for all XOR is 6 and for implication is 13. That means, on applying a logic operation between any row and column it results in same amount of communication complexity. We have summarized it in the following table:

Table 9. Amount of communication for node in AND, OR, XOR and implication matrices

Logical Matrix	Amount of Communication (Communication Complexity)
AND Matrix	9
OR Matrix	15
XOR Matrix	6
Implication Matrix	13

5. CONCLUSION

We can apply these methods on other Interconnection architectures.This will help us in studying property of robustness between nodes for different interconnections network architectures.Connectivity diagrams give us broad idea about connections of nodes in architecture. Implementation of logical operations between connections of nodes gave us transpose of connections.We can derive complexity of connections also with the help of above matrices.We can derive permutation matrices from these matrices.

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