



THE ISOMORPHISM OF THE LEARNING GROUP: A CROSSROAD OF MANY CONCEPTS

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ABSTRACT

This article describes how to explore the concept of group isomorphism in the undergraduate summary algebra course. Despite performing missions on various elements of the same type, we interviewed them to understand the ideas of scientists. Here are some observations that emerged from the interview evaluation: First, students show a strong need for "standard" concrete step-by-step procedures. And they tend to get caught up in having to deal with some range of freedom from their choices. Second, students show different levels of anthropomorphism and localization of the language, such as "you can find a function that takes all the details of G in every element of G ". To send a list of homes, students first choose a dwelling that they find easier; but it turns out that their desires depend on the type of challenge and the type of complexity involved. It is preferably within the responsibility of the company; students are more often syntactically simple dwellings, whereas for tasks involving certain groups, students prefer a hearth that is less computationally complex.

Keywords: Isomorphism, Homomorphism, isomorphic groups, naive isomorphism

1. INTRODUCTION

What do you think of the concept of isomorphism? Experienced mathematicians may have considered it a single concept and actually integrated it as one. However, students in the first study of rare algebra rarely reach this mature idea. For them, isomorphism is a complex and interrelated concept, constructed and linked to many other concepts, but it is only partially understood in itself. For example, to understand group isomorphism, you need to understand group ideas, features, and quantifiers. On the other hand, reading the witches about isomorphism at the right time will help you better understand these related theories. In this article, we'll explore how undergraduates in the first algebra course can learn the concept of group isomorphism. To ask this question, we interviewed a group of students and found their answers to some of the points we found interesting. In that case, the study results were curated. Interviews with students revealed how isomorphic activities interact with ideas such as order (both groups and group objects) and flexibility. Our perception of student work on isomorphic functions always leads to major problems. Isomorphic functions were the only cause. The problem of variance, the degree of freedom in algorithm construction, the opposite of proof, the construction process, mathematical objects. Some of these links are discussed in detail in the Conclusions section (Section 6). The main sections of the article describe the findings (Section 4) and findings (Section 5). Section 3 contains a review of the literature. Section 2 contains historical notes, insights into isomorphic concepts, the psychological processes involved in their understanding, and the various divisions involved. This division is very easy from a legal point of view, but it is important in the teaching / learning process. This includes the difference in the official definition of the same type as "ignorance". The relationship between two groups is an isomorphic object. Processes involved in isomorphic demonstrations compared to processes with atypical symptoms. Processes related to the processing of abstract isomorphisms and isomorphisms in specific situations.

2. THE VARIOUS FACES OF ISOMORPHISM

2.1. The Meaning of Isomorphism

In rare cases, if "same without writing", the two groups are isomorphic. So if you get a group and rename its object and its function, you get an isomorphic copy of the same group. This is exactly what is called "naive isomorphism" and will be discussed in more detail in Section 4. The official definition is far from this exact concept. The isomorphism from group $[G, o]$ to group $[G \sim, o']$ is the work of person f from G to G' that satisfies all $f(AOB) = f(a) o' f(b)$. is. $G a, b$. If there is an isomorphism from one to the other, the two groups are said to be isomorphic. Therefore, in order to (mentally) construct an isomorphism concept according to this formula definition, it is necessary to construct an isomorphism as a functional concept and an object (because we are



measuring this object). The need for this additional structure is reduced by using irrational isomorphisms that can explain the simplicity of the naive version linked to the official version. The deeper meaning of the isomorphism in one of these interpretations is that open communication is a naive or explicit version of the official version of the function, preserving the structure of the group (that is, its size and function). That's why we keep everything in the group. Intangible assets. As an example, here are two isomorphic groups that play an important role in the study reported here. These two groups are equal to \mathbb{Z}_3 . This is a group of all $\{1, 2, \text{and } 3\}$ allowed in relation to the construction performance of the structure. Dihedral group D_3 -A group of all equilateral triangle symmetries, related to the structure of the work. You can confirm that they are isomorphic by adding numbers to the triangle line and looking at each equation as a harmony of these three numbers. This quickly shows that they are naive and isomorphic. To define the formal isomorphism between them, you need to explicitly build the function responsible for each measure of the corresponding consent. (However, there is a problem here, see the newspapers Zazkis and Dubinsky). This example demonstrates the importance and application of isomorphism because it can be used for both analytical capabilities in D_3 and geometric interpretation of \mathbb{Z}_3 . For example, you can see why \mathbb{Z}_3 has three items ("demonstration"):) And order 3 ("rotation").

2.2. Various Distinctions

Isomorphic concepts are rich and multifaceted. In this dialogue, I would like to focus on three differences. These differences are very simple from a formal point of view. However, they are very important in system coaching / knowledge acquisition.

(A) Differences in the relationship between isomorphic companies and isomorphic objects: Looking at the formal definition, this difference is rarely mentioned. But from a mental activity perspective, there are incredible differences. The isomorphic relationship is symmetric and intuitive (in its naive model) and does not require the concept of function for its expertise U . Isomorphic elements are directional, "practical" and much more difficult to recognize. There is no simple model for that. By the way, the term homomorphism also does not allow a naive definition that requires the use of a function, perhaps for the purpose of the same direction.

(B) Proof of isomorphism and proof of non-isomorphism: The first method must show a positive type characteristic match or existence. The latter seems to require proof of non-lifestyle and it is very difficult to do. However, students often appear to avoid this problem based on what is called the "deepest" of the same type. That is, you will find fairly clear evidence of non-isomorphism. Refers to one asset of a company that does not fit the other company.

(C) Abstract vs. Isomorphism in Specific Cases: In an abstract context, it is easy to think that all isomorphic agencies are essentially the same. In other words, abstract organizations are considered to be essentially isomorphic classes. But can unique isomorphic properties also play a role in dealing with a particular example? Some isomorphism's are so important that it is difficult to think of any two of these organizations as "same." For example, real numbers with addition are isomorphic to fantastic real numbers with exponential multiplication, but no one shows that they are equal. (There is another reason why these two systems are not "living" the same thing. Seeing them as a society, given the fact that they have a much richer structure than just a company. Knows only a small part of its nature.)

3. BACKGROUND: DESCRIPTION OF THE RESEARCH

The majority of scholars interviewed in these studies (Group A) come from first-year courses taught in computing using the author at the Technion-Israel Institute of Technology (IIT). Some data objects are taken from Kent State University Daylight Saving Time Time Institute for Excessive On-Duty Teachers (Group B) students and from selected trendy IIT courses (groups) taught by one of the authors. Will be done. C). Introduce all student organizations. Company A and Company B scholars were taught with an unconventional approach of laptop sports (programming at ISETL) and teams working primarily, and realized the lecture method. (For a more complete description of the course and therefore coaching methods, see Cf. Leronand Dubinsky, 1995). The approach used in these instructions to introduce isomorphism is particularly relevant to these studies. It came to be delivered very quickly in the direction, using the "naive isomorphic" idea mentioned in subsection 2.1. This intuitive definition is then widely used throughout the journey. For example, the student discovers that each circulation tissue of order n is



(simply) isomorphic to $[Z, + n]$, and that the quotient groups $Z_{12} / 0.4, 8$ are isomorphic to Z_4 (operation desk). Confirm (by building). A long time later, in the trail, a formal definition of the same type was revealed, referring to a reference to the naive version. The findings are very likely to be motivated by both non-traditional coaching techniques and, therefore, early and intuitive isomorphic creation. In fact, the miles that many "misunderstandings" caused here are expected to continue with even greater force in traditional elegance. However, the impact of pedagogy on student design is not considered here (except for some remote commentary) and is worth considering individually. The essential framework of the study consists of detailed mid-established interviews with 5 of A's 24 students. Interviews were conducted 7 weeks after the final exam and lasted 60-90 minutes each. Students in companies B and C are roughly approached by isomorphism as part of a more general study of the idea of data institutions. All interviews were recorded and transcribed. Interview questions for A and Group C were provided orally to researchers, but were supported by pre-arranged questionnaires (see Figures 1, 2 and 3). In this report, the interview data is translated into English (from Hebrew only), so there is also a survey. The most effective inquiries to B's students were those that could be applied to this study. "Is $\$ 3$ the same type as the Z_6 ?" Behind the interview survey, various ideas are complicated. Question 1 and some (see Figure 1) aim to acquire student expertise in isomorphism within the context of a particular example, from knowledge to novels, paintings to abstract presentations. The tables of some institutions are because the $\$ 3$ and Z_6 companies were conscious of researchers, but previously they did not explicitly show these exact tables. They have also never seen a $Z_2 \times Z_3$ or U_9 agency, except for the general discussion of U, \sim companies. The project of finding isomorphic and non-identical pairs among a given company, whether table type or name type, was unexpected for researchers. They may enjoy classifying a particular society into isomorphic lessons (for example, all circular societies in a particular order are isomorphic, and all societies of the fourth order are classified into isomorphic lessons. And many other societies), they do not have a particular experience of the specified type. Questions 3 and 4 (see Figure 2) are alleged to investigate student know-how about which isomorphism and non-isomorphism's are included in the abstract. Question 3 is expressed to allow slow probing, from the (assumed) simpler "isomorphism" to the isomorphism of larger, stiff objects. Questions 5 and 6 (see Figure 3) relate to the student's understanding of isomorphic mapping versions and invariants. This theme has been changed to be covered in the classroom at some level, but generally with an emphasis on understanding what kind of homes are preserved and why. There wasn't much to edit the list. Invariant.

4. FORMULATION AND RESULT

For a set of absolutely favorite studies and perfectly good topics, our priority is not the conceptual framework of ideas for methodological integrity, but first and foremost some exciting ideas and further research. Was to return some instructions for. Therefore, we have organized this segment around some important phenomena that emerged from the interview. Each subsection relates to at least one of these phenomena and contains summaries, useful data from interviews, and recommended intellectual methods to explain the phenomenon. These phenomena once again reveal the complex nature of the same type. And it's complex, a complex network of relationships with many different principles.

LEARNING ISOMORPHISM

Question1. Here are the three operation tables for the group; the task is to determine which are isomorphic to each other and which are not.

[Answer: G and G ° are isomorphic to \mathbb{Z}_3 and G' is isomorphic to \mathbb{Z}_6 .]

*	a	b	c	d	x	y	
a	d	y	x	a	c	b	
b	c	d	a	b	y	x	
c	b	x	y	c	a	d	G
d	a	b	c	d	x	y	
x	y	c	b	x	d	a	
y	x	a	d	y	b	c	

x	a'	b'	c'	d'	x'	y'	
a'	b'	c'	a'	y'	d'	x'	
b'	c'	a'	b'	x'	y'	d'	
c'	a'	b'	c'	d'	x'	y'	G'
d'	y'	x'	d'	b'	a'	c'	
x'	d'	y'	x'	a'	c'	b'	
y'	x'	d'	y'	c'	b'	a'	

o	a ^o	b ^o	c ^o	d ^o	x ^o	y ^o	
a ^o	y ^o	c ^o	b ^o	x ^o	d ^o	a ^o	
b ^o	x ^o	y ^o	d ^o	c ^o	a ^o	b ^o	
c ^o	d ^o	a ^o	x ^o	b ^o	y ^o	c ^o	G ^o
d ^o	c ^o	x ^o	a ^o	y ^o	b ^o	d ^o	
x ^o	b ^o	d ^o	y ^o	a ^o	c ^o	x ^o	
y ^o	a ^o	b ^o	c ^o	d ^o	x ^o	y ^o	

Question 2. The task now is the same as Question 1, but only describes the group.

[Answer: U9 and G are isomorphic to \mathbb{Z}_6 .]

(i) A symmetric group of \mathbb{Z}_3 , $\{1, 2, 3\}$.

(ii) A group of elements of \mathbb{Z}_9 those are relatively prime to 9 by operating U 9 and multiplication mood 9.

(iii) $G = \{[g_1, g_2] \mid g_1 \in \mathbb{Z}_2, g_2 \in \mathbb{Z}_3\}$ Next operation: $[g_1, g_2] * [h_1, h_2] = [g_1, z_2, h_1, g_2, z_3, h_2]$

Fig. 1. The interview questionnaire, first part.

Nomenon, and contains a description, supporting data from the interviews, and suggested mental processes that may account for the phenomenon. These phenomena demonstrate once more the compound nature of isomorphism, and its complex web of relationships with many other concepts.

Question 3.

(I) Suppose you are given two groups. How can I tell if they are isomorphic? How can I convince someone that they are really isomorphic? How can I prove it?

[If the student does not give a complete answer, including features, the interviewer asks:]

ii) What is the same type? [Now repeat question (i) to see if the answer in (ii) caused any change.]

Question 4. Suppose you are given two groups. How can you tell that they are not isomorphic?

Fig. 2. The interview questionnaire, second part.

4.1. Isomorphism and type of order

One element of business that stands out in student painting is the order of many elements within an organization. When asked if two given institutions are isomorphic, students usually start by calculating the sequence of different factors. For example, an office of the same type group for \$ 3 gets the details of three items for order 1 (identification), order 2, and item for order 3. You can combine these facts into one. A sequence like [1, 2, 2, 2, 3, 3]. This is called an order sort organization. Formally, the "order type" of a set is an increasing sequence of numbers, which is the order of the organizing elements. (For the purposes of this simple document, the term order type has been added here. Researchers do not always know the duration, and all references to the subject asking students are their work and Dialogue). You can use the order type to reveal that the companies are not isomorphic. In fact, when asked if the \$ 3 and the Z6 are isomorphic, Eric (Group B) said that the \$ 3 has three elements of order 2, and the Z6 is the best of these. Given the fact, companies replied that they were not isomorphic. But what about the opposite? If the order types of the two companies are the same, do students tend to conclude that they are isomorphic? According to our interviews, students often make this (unjustified) assumption.

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Question 5.

(i) Do you know what it means for a property to be "saved" (or immutable) under the same type?

[After making sure they know what this means :]

(ii) Give three examples of such invariants.

(iii) Give an example of a property that is not immutable.

Question 6. Two isomorphic groups A and B, and additional

=, Group G. You will be asked to determine which of the following statements is Mac and which is wrong.

(i) If A has an element of order 3, then B has an element of order 3.

(ii) If A is commutative, then B is also commutative.

(iii) If the element of A is a matrix, the element of B is also a matrix.

(iv) If A is a subgroup of G, so is B.

(v) If G is a subgroup of A, then G is a subgroup of B.

(vi) [Optional, for stronger students] If A is a homomorphism image of G, then B is also a homomorphism image of G.

(vii) If the operation of A is a function composition, the operation of B is a function composition.) If A is infinite, B is also infinite.

Fig. 3. The interview questionnaire, third part.

5. CONCLUSION

Ultimately, you may want to provide additional information about the research reported in this article. First, the unique and therefore whole interaction, and second, the function of "naive" isomorphism in the study of abstract algebra. Indeed, the concept of isomorphism itself, however, is a proper expression of



many well-known ideas about similarity and distinction; in particular two different things are comparable under the act of proper abstraction. The concept is often thought of as. Here is another example: First, the phenomenon of scholars sticking to building the selected isomorphism is a peculiar case of the student's desire for formal tactics and the general phenomenon of student fear of free or uncertain methods, or perhaps strategy. Often seen as. Regardless of the degree of freedom. This can also be seen continuously as a peculiar example of the more widespread phenomenon of "fear of freedom." Choosing an option is a pain for some citizens. 'Other. Second, the problem of defining definitions related to isomorphisms could be a simple single sample of the problem of functions and quantifiers. More specifically, the problem of function quantification. This may also be related to more general development issues related to tactics and gadgets related to the construction of relevant mathematical designs. Second, you may want to go beyond the empirical basis of this document and provide a greater favorable view of isomorphic knowledge and education. More precisely, it played an important role through the naive isomorphic concept in the road. A more comprehensive treatment of this issue, which is a component of the paperwork of ongoing research, may be given elsewhere. First, the idea of introducing a sophisticated subject in a naive version can be a unique case of a deep and widespread concept of proficiency through a series of continuous improvements-complex without losing a forest of trees. This is your chance to manage the concept. (This happens within the traditional approach of learning by knowing small pieces of linear continuum). Second, the idea of "isomorphism" can be a much easier concept to formulate and study than the concept of object isomorphism itself. The overall order of coaching ranges from homomorphism and homomorphism (a special type of homomorphism) to connections between companies that are isomorphic (that is, if there is "one isomorphism for a choice"). We agree that a doctrine-appropriate series should be largely contrary to the order of the subjects. In fact, while a naive version of the connection between homologous companies could be invented and cleaned up for researchers to know, it's difficult to find a "naive" variation that matches the isomorphism of an object itself. -Do you think you need good functionality, even for homomorphism's, probably because of their inherently directional (rather than symmetric) nature?

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