# A COMMON FIXED POINT THEOREM FOR WEAKLY COMPATIBLE MAPS SATISFYING COMMON PROPERTY (E.A.) IN INTUITIONISTIC FUZZY METRIC SPACE 

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#### Abstract

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The aim of this paper is to prove new common fixed point theorems for six mapping usingthe concept of the common EA property and weakly compatible mappings on intuitionistic fuzzy metric spaces.


Keywords:fixed point theorem, weakly compatible, E.A. property.

Mathematics Subject Classification: 54H25, 54A40,47H10.

## Introduction:

The study of common fixed point of mappings satisfying contractive type conditions has been a very active field of research. Atanassov [4] introduction and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. All results which hold of fuzzy sets can betransformed Intuitionistic fuzzy sets but converse need not be true. Alacaet al. [2] proved the well-known fixed point theorems of intuitionistic fuzzy metric spaces. Later on, Turkoglu et al.

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[13] proved Jungck's[6] common fixed point theorem in the setting of intuitionistic fuzzy metric space. Aamri and El Moutawakil [1] defined a property (E.A) for self-maps which contained the class of noncompatible maps in metric spaces and proved common fixed point theorems. Subsequently, there are a number of results proved for contraction maps satisfying property (E.A) in different settings such as probabilistic metric spaces [5]; fuzzy metric spaces [8]; intuitionistic fuzzy metric spaces [11]. In 2008, Alaca, Turkoglu and Yildiz [3] proved a common fixed point theorem for continuouscompatible maps on completeintuitionistic fuzzy metric space. In the present paper,westudy a common fixed point theorems for weakly compatible self-mapssatisfyingthe E.A. property in theintuitionistic fuzzy metric space.Our results improve and generalize the result of V. Mandwariya and A. Rajput [15] and V. Mandwariya and F. Qureshi [16].

## 2. PRELIMINARIES

First, we list some important definitions and theorems, which are useful for our main results.
Definition 2.1(10): A binary operation $*:[0,1] \times[0,1] \rightarrow[0,1]$ is a continuoust-norm if $*$ satisfies the following conditions:
(i) $\quad$ is commutative and associative;
(ii) $\quad$ is continuous;
(iii) $\mathrm{a} * 1=\mathrm{a}$ for all $\mathrm{a} \in[0,1]$;
(iv) $\mathrm{a} * \mathrm{~b} \leq \mathrm{c} * \mathrm{~d}$, whenever $\mathrm{a} \leq \mathrm{c}$ and $\mathrm{b} \leq \mathrm{d}$, for all $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d} \in[0,1]$.

Definition 2.2(10): A binary operation $\diamond:[0,1] \times[0,1] \rightarrow[0,1]$ is a continuoust-conorm if $\diamond$ satisfies the following conditions:
(i) $\quad \circ$ is commutative and associative;
(ii) $\Delta$ is continuous;
(iii) $\mathrm{a} \triangleleft 0=\mathrm{a}$ for all $\mathrm{a} \in[0,1]$;
(iv) $\quad \mathrm{a} \diamond \mathrm{b} \leq \mathrm{c} \diamond \mathrm{d}$, whenever $\mathrm{a} \leq \mathrm{c}$ and $\mathrm{b} \leq \mathrm{d}$, for all $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d} \in[0,1]$.

Definition 2.3 (2): A 5-tuple ( $\mathrm{X}, \mathrm{M}, N, *, \diamond$ ) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous T -norm, $\Delta$ is a continuous T -conorm and $\mathrm{M}, \mathrm{N}$ are fuzzy sets on $X^{2} \times(0, \infty)$ satisfying the following conditions, for all $x, y, z \in X, s, t>0$,
(i) $\mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{t})+\mathrm{N}(\mathrm{x}, \mathrm{y}, \mathrm{t}) \leq 1$
(ii) $\mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{t})=\mathrm{o}$
(iii) $\mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{t})=1$ if and only if $\mathrm{x}=\mathrm{y}$;
(iv) $\mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{t})=\mathrm{M}(\mathrm{y}, \mathrm{x}, \mathrm{t}) \neq 0$ for $\mathrm{t} \neq 0$;
(v) $\mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{t}) * \mathrm{M}(\mathrm{y}, \mathrm{z}, \mathrm{s}) \leq \mathrm{M}(\mathrm{x}, \mathrm{z}, \mathrm{t}+\mathrm{s})$;
(vi) $\mathrm{M}(\mathrm{x}, \mathrm{y}, \cdot):[0, \infty) \rightarrow[0,1]$ iscontinuous.
(vii) $\lim _{t \rightarrow \infty} M(x, y, t)=1$
(viii) $\mathrm{N}(\mathrm{x}, \mathrm{y}, 0)=1$
(ix) $N(x, y, t)=0$ if and only if $x=y$;
(x) $N(x, y, t)=N(y, x, t) \neq 0$ for $t \neq 0$;
(xi) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t+s)$;
(xii) $\mathrm{N}(\mathrm{x}, \mathrm{y}, \cdot):[0, \infty) \rightarrow[0,1]$ is continuous.
(xiii) $\lim _{t \rightarrow \infty} N(x, y, t)=0$

Then $(\mathbf{M}, \mathrm{N})$ is called an intuitionistic fuzzy metric on X . The functions $M(x, y, \operatorname{tand} N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t , respectively.

Remark2.1: Every fuzzy metric space $(\mathrm{X}, \mathrm{M}, *)$ is an intuitionistic fuzzy metric space of the form ( $\mathrm{X}, \mathrm{M}, 1-M, *, \diamond$ ) such that $t-$ norm $*$ and $t$ conorm $\diamond$ are associated as $x \diamond y=1-$ $((1-x) *(1-y))$ for all $x, y \in \mathrm{X}$.

Remark 2.2: In an intuitionistic fuzzy metric space ( $\mathrm{X}, \mathrm{M}, N, *, \diamond$ ), $\mathrm{M}(x, y, \cdot)$ is nondecreasing and $\mathrm{N}(x, y, \cdot)$ is non-increasing for all $x, y \in \mathrm{X}$.

Definition 2.4[14]: Let $S$ and $T$ be self mapping of an intuitionistic fuzzy metric space(X, M, $N, *, \diamond)$. Then a pair (S, T) is said to be compatible if
$\lim _{n \rightarrow \infty} M\left(S T x_{n}, T S x_{n}, t\right)=1 \operatorname{andlim}_{n \rightarrow \infty} N\left(S T x_{n}, T S x_{n}, t\right)=0$
For all $t>0$, whenever $\left\{x_{n}\right\}$ is sequence in X such that $\lim _{n \rightarrow \infty} S x_{n}=\lim _{n \rightarrow \infty} T x_{n}=u$ for some $u \in \mathrm{X}$.
Definition 2.5 [7]: Let $S$ and $T$ be self mapping of an intuitionistic fuzzy metric space(X, M, $N, *, \diamond$ ) are said to be weak compatible if $\mathrm{AB} x=\mathrm{BA} x$ when $\mathrm{A} x=\mathrm{B} x$ for some $x \in \mathrm{X}$.

## Definition 2.5: (E. A. Property)

Let $f$ and $g$ be two self-maps of a fuzzy metric space ( $\mathrm{X}, \mathrm{M}, *$ ). We say that $f$ and $g$ satisfy the property E. A. property if there exists a sequence $\left\{x_{n}\right\}$ such that
$\lim _{n \rightarrow \infty} f x_{n}=\lim _{n \rightarrow \infty} g x_{n}=z$ for some $z \in X$.
Note that weakly compatible and property (E.A) are independent to each other (see; [9], Ex. 2.2).

## Definition 2.6: (Common E. A. Property)

Let $A, B, S, T: X \rightarrow X$ where $X$ is a fuzzy metric space, then the pair $\{A, S\}$ and $\{B, T\}$ said to satisfy common E. A. property if there exist two sequences $\left\{x_{n}\right\}$ and $\left\{y_{n}\right\}$ in $X$ such that $\lim _{n \rightarrow \infty} A y_{n}=\lim _{n \rightarrow \infty} B x_{n}=\lim _{n \rightarrow \infty} S y_{n}=\lim _{n \rightarrow \infty} T x_{n}=z$ for somez $\in \mathrm{X}$.

Lemma 2.1 [12]:Let $(X, M, N, *, \diamond)$ be an IFM-space and for all $x, y \in X, t>0$ and if for a number $\mathrm{k} \in(0,1), \mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{kt}) \geq \mathrm{M}(\mathrm{x}, \mathrm{y}, \mathrm{t})$ and $\mathrm{N}(\mathrm{x}, \mathrm{y}, \mathrm{kt}) \leq \mathrm{N}(\mathrm{x}, \mathrm{y}, \mathrm{t})$, then $\mathrm{x}=\mathrm{y}$.
2.1 Theorem:Let $(\mathrm{X}, \mathcal{M}, *)$ be a complete fuzzy metric space. Let $\mathrm{A}, \mathrm{B}, \mathrm{S}, \mathrm{T}, \mathrm{P}$ and Q be six self -mappingssatisfying the following condition:
(i) $\quad \mathrm{P}(\mathrm{X}) \subseteq \mathrm{AB}(\mathrm{X})$ and $\mathrm{Q}(\mathrm{X}) \subseteq \mathrm{ST}(\mathrm{X})$.
(ii) Pair $(\mathrm{P}, \mathrm{AB})$ and $(\mathrm{Q}, \mathrm{ST})$ are weakly compatible and $\mathrm{AB}=\mathrm{BA}, \mathrm{PB}=\mathrm{BP}, \mathrm{SQ}=\mathrm{QS}, \mathrm{QT}$ $=\mathrm{TQ}$ and $\mathrm{ST}=\mathrm{TS}$.
(iii) There exists a number $\mathrm{k} \in(0,1)$ such that
$\mathcal{M}(P x, Q y, A B x, \mathrm{kt}) * \mathcal{M}(A B x, P x, P x, k t)$
$\geq \min \left\{\begin{array}{c}\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathcal{M}(\mathrm{STy}, \mathrm{ABx}, \mathrm{ABx}, \mathrm{t})+\mathrm{r}(\mathrm{t}) \frac{\mathcal{M}(\mathrm{STy}, \mathrm{Px}, \mathrm{Px}, \mathrm{t})}{\mathcal{M}(\mathrm{Px}, \mathrm{ABx}, \mathrm{ABx}, \mathrm{t})}\right), \\ \mathcal{M}(\mathrm{Px}, \mathrm{Px}, \mathrm{Qy}, \mathrm{t}) * \mathcal{M}(\mathrm{Qy}, \mathrm{STy}, \mathrm{STy}, \mathrm{t})]\end{array}\right\}$
for all $\mathrm{x}, \mathrm{y} \in \mathrm{X}$ all $t>0$ and some $\mathrm{k} \in[0,1]$ where $\mathrm{p}, \mathrm{q}, \mathrm{r}: \mathrm{R}^{+} \rightarrow(0,1]$ be three function such that $p(t)+q(t)+r(t)=1$.

If the range of the subspaces $\mathrm{P}(\mathrm{X})$ or $\mathrm{AB}(\mathrm{X})$ or $\mathrm{Q}(\mathrm{X})$ or $\mathrm{ST}(\mathrm{X})$ is complete, then $\mathrm{A}, \mathrm{B}, \mathrm{S}, \mathrm{T}, \mathrm{P}$ and Q have a unique common fixed point in X .

In the next section, we prove a theorem similar to theorem 2.1 satisfyingthe condition Varshaet . al.[16].

## 3. MAIN RESULT

3.1 Theorem: Let $(\mathrm{X}, \mathrm{M}, N, *, \diamond)$ be a intuitionistic fuzzy metric space and $\mathrm{A}, \mathrm{B}, \mathrm{S}, \mathrm{T}, \mathrm{P}$ and Q be six mapping satisfying the following conditions
(iv) $\quad \mathrm{P}(\mathrm{X}) \subseteq \mathrm{AB}(\mathrm{X})$ and $\mathrm{Q}(\mathrm{X}) \subseteq \mathrm{ST}(\mathrm{X})$ and $\mathrm{ST}(\mathrm{X})$ is closed

$$
M(P x, Q y, t) \geq \varphi\left(\min \left\{\begin{array}{c}
\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{M}(\mathrm{ST} x, \mathrm{AB} y, \mathrm{t})+\mathrm{r}(\mathrm{t}) \frac{\mathrm{M}(\mathrm{ST} x, \mathrm{P} x, \mathrm{t})}{\mathrm{M}(\mathrm{P} x, \mathrm{AB} y, \mathrm{t})}\right), \\
\mathrm{M}(\mathrm{P} x, \mathrm{Q} y, \mathrm{t}) * \mathrm{M}(\mathrm{Q} y, \mathrm{ST} x, \mathrm{t})]
\end{array}\right\}\right)
$$

and

$$
N(P x, Q y, t) \leq \varphi\left(\max \left\{\begin{array}{c}
\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{N}(\mathrm{ST} x, \mathrm{AB} y, \mathrm{t})+\mathrm{r}(\mathrm{t}) \frac{\mathrm{N}(\mathrm{ST} x, \mathrm{P} x, \mathrm{t})}{\mathrm{N}(\mathrm{P} x, \mathrm{AB} y, \mathrm{t})}\right)  \tag{3.1.1}\\
\mathrm{N}(\mathrm{P} x, \mathrm{Q} y, \mathrm{t}) \diamond \mathrm{N}(\mathrm{Q} y, \mathrm{ST} x, \mathrm{t})]
\end{array}\right\}\right)
$$

for all $x, y \in \mathrm{X}, t>0$, where $\Phi:[0,1] \rightarrow[0,1]$ is continuous function with $\Phi(s)>s$ whenever 0 $<s<1$ and $\mathrm{p}, \mathrm{q}, \mathrm{r}: \mathrm{R}^{+} \rightarrow(0,1]$ be three function such that $p(t)+q(t)+r(t)=1$. Suppose that the pair ( $\mathrm{P}, \mathrm{ST}$ ) and ( $\mathrm{Q}, \mathrm{AB}$ ) satisfies CommonE.A. property and ( $\mathrm{P}, \mathrm{ST}$ ) and ( $\mathrm{Q}, \mathrm{AB}$ ) are weakly compatible. Also suppose that $S T(X)$ and $A B(X)$ is a closed subset of $X$. Then $A, B, S, T, P$ and Q have a unique fixed point in X .

Proof:Since ( $\mathrm{P}, \mathrm{ST}$ ) and ( $\mathrm{Q}, \mathrm{AB}$ ) satisfies Common E.A. property
Therefore there exist a sequence $\left\{x_{n}\right\}$ and $\left\{y_{n}\right\}$ in $X$ such that

$$
\begin{aligned}
& \lim _{n \rightarrow \infty} M\left(P y_{n}, z, t\right)=\lim _{n \rightarrow \infty} M\left(Q x_{n}, z, t\right)=\lim _{n \rightarrow \infty} M\left(S T y_{n}, z, t\right)=\lim _{n \rightarrow \infty} M\left(A B x_{n}, z, t\right)=1 \\
& \lim _{n \rightarrow \infty} P y_{n}=\lim _{n \rightarrow \infty} Q x_{n}=\lim _{n \rightarrow \infty} A B x_{n}=\lim _{n \rightarrow \infty} S T y_{n}=z
\end{aligned}
$$

For some $z \in X$ and every $t>0$
Suppose that $\mathrm{ST}(\mathrm{X})$ is closed subset of X so there exist $u \in \mathrm{X}$ such that $\mathrm{ST} u=z$.
Let $\mathrm{P} u=z$, if not then
Put $x=u$ and $y=x_{n}$ in equation (3.1.1), we have
$M\left(P u, Q x_{2 n}, t\right) \geq \varphi\left(\min \left\{\begin{array}{c}\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{M}\left(\mathrm{ST} u, \mathrm{AB} x_{n}, \mathrm{t}\right)+\mathrm{r}(\mathrm{t}) \frac{\mathrm{M}(\mathrm{ST} u, \mathrm{Pu}, \mathrm{t})}{\mathrm{M}\left(\mathrm{Pu}, \mathrm{AB} x_{n}, \mathrm{t}\right)}\right) \\ \left.\mathrm{M}\left(\mathrm{Pu}, \mathrm{Q} x_{n}, \mathrm{t}\right) * \mathrm{M}\left(\mathrm{Q} x_{n}, \mathrm{ST} u, \mathrm{t}\right)\right]\end{array}\right\}\right)$
Taking limit $n \rightarrow \infty$ we have
$M(P u, z, t) \geq \varphi\left(\min \left\{\begin{array}{c}\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{M}(\mathrm{z}, \mathrm{z}, \mathrm{t})+\mathrm{r}(\mathrm{t}) \frac{\mathrm{M}(\mathrm{z}, \mathrm{Pu}, \mathrm{t})}{\mathrm{M}(\mathrm{Pu}, \mathrm{z}, \mathrm{t})}\right), \\ \mathrm{M}(\mathrm{Pu}, \mathrm{z}, \mathrm{t}) * \mathrm{M}(\mathrm{z}, \mathrm{z}, \mathrm{t})]\end{array}\right\}\right)$
$M(P u, z, t) \geq \varphi\left(\min \left\{\begin{array}{c}(\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})+\mathrm{r}(\mathrm{t})), \\ \mathrm{M}(\mathrm{Pu}, \mathrm{z}, \mathrm{t}) * 1\end{array}\right\}\right)$
$M(P u, z, t) \geq \varphi\left(\min \left\{\begin{array}{c}1, \\ M(\mathrm{Pu}, \mathrm{z}, \mathrm{t})\end{array}\right\}\right)$
$M(P u, z, t) \geq \varphi(M(P u, z, t))$
$M(P u, z, t)>M(P u, z, t)$
and

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$N\left(P u, Q x_{2 n}, t\right) \leq \varphi\left(\max \left\{\begin{array}{c}\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{N}\left(\mathrm{ST} u, \mathrm{AB} x_{n}, \mathrm{t}\right)+\mathrm{r}(\mathrm{t}) \frac{\mathrm{N}(\mathrm{ST} u, \mathrm{Pu}, \mathrm{t})}{\mathrm{N}\left(\mathrm{Pu}, \mathrm{AB} x_{n}, \mathrm{t}\right)}\right) \\ \left.\mathrm{N}\left(\mathrm{Pu}, \mathrm{Q} x_{n}, \mathrm{t}\right) \otimes \mathrm{N}\left(\mathrm{Q} x_{n}, \mathrm{ST} u, \mathrm{t}\right)\right]\end{array}\right\}\right)$
$N(P u, z, t) \leq \varphi\left(\max \left\{\begin{array}{c}\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{N}(\mathrm{z}, \mathrm{z}, \mathrm{t})+\mathrm{r}(\mathrm{t}) \frac{\mathrm{N}(\mathrm{z}, \mathrm{Pu}, \mathrm{t})}{\mathrm{N}(\mathrm{Pu}, \mathrm{z}, \mathrm{t})}\right) \\ \mathrm{N}(\mathrm{Pu}, \mathrm{z}, \mathrm{t}) \diamond \mathrm{N}(\mathrm{z}, \mathrm{z}, \mathrm{t})]\end{array}\right\}\right)$
$N(P u, z, t) \leq \varphi\left(\max \left\{\begin{array}{c}(\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})+\mathrm{r}(\mathrm{t})), \\ \mathrm{N}(\mathrm{Pu}, \mathrm{z}, \mathrm{t}) \bullet 0\end{array}\right\}\right)$
$N(P u, z, t) \leq \varphi\left(\max \left\{\begin{array}{c}1, \\ \mathrm{~N}(\mathrm{Pu}, \mathrm{z}, \mathrm{t})\end{array}\right\}\right)$
$N(P u, z, t) \leq \varphi(N(P u, z, t))$
$N(P u, z, t)<N(P u, z, t)$
This is contradiction. Thus $\mathrm{P} u=z$
Hence $P u=S T u=z$
Suppose that $\mathrm{AB}(\mathrm{X})$ is a closed subset of X so there exist $v \in \mathrm{X}$ such that $\mathrm{AB} v=z$
Let $\mathrm{Q} v=z$ if not then
Put $x=y_{n}$ and $y=v$ in equation (3.1.1), we have
$M\left(P y_{n}, Q v, t\right) \geq \varphi\left(\min \left\{\begin{array}{c}\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{M}\left(\mathrm{ST} y_{n}, \mathrm{AB} v, \mathrm{t}\right)+\mathrm{r}(\mathrm{t}) \frac{\mathrm{M}\left(\mathrm{ST} y_{n}, \mathrm{P} y_{n}, \mathrm{t}\right)}{\mathrm{M}\left(\mathrm{P} y_{n}, \mathrm{AB}, \mathrm{t}\right)}\right), \\ \left.\mathrm{M}\left(\mathrm{P} y_{n}, \mathrm{Q} v, \mathrm{t}\right) * \mathrm{M}\left(\mathrm{Q} v, \mathrm{ST} y_{n}, \mathrm{t}\right)\right]\end{array}\right\}\right)$
Taking limit as $\rightarrow \infty$, we have
$M(z, Q v, t) \geq \varphi\left(\min \left\{\begin{array}{c}\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{M}(\mathrm{z}, \mathrm{z}, \mathrm{t})+\mathrm{r}(\mathrm{t}) \frac{\mathrm{M}(\mathrm{z}, \mathrm{z}, \mathrm{t})}{\mathrm{M}(\mathrm{z}, \mathrm{t})}\right), \\ \mathrm{M}(\mathrm{z}, \mathrm{Q} v, \mathrm{t}) * \mathrm{M}(\mathrm{Q} v, \mathrm{z}, \mathrm{t})]\end{array}\right\}\right)$
$M(z, Q v, t) \geq \varphi\left(\min \left\{\begin{array}{c}(\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})+\mathrm{r}(\mathrm{t})), \\ \mathrm{M}(\mathrm{z}, \mathrm{Q} v, \mathrm{t}) * \mathrm{M}(\mathrm{Qv}, \mathrm{z}, \mathrm{t})]\end{array}\right\}\right)$
$M(z, Q v, t) \geq \varphi\left(\min \left\{\begin{array}{c}1, \\ M(\mathrm{z}, \mathrm{Q} v, \mathrm{t})\end{array}\right\}\right)$
$M(Q v, z, t) \geq \varphi(M(Q v, \mathrm{z}, \mathrm{t}))$
$M(Q v, z, t)>M(Q v, z, t)$
and

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$N\left(P y_{n}, Q v, t\right) \geq \varphi\left(\min \left\{\begin{array}{c}\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{N}\left(\mathrm{ST} y_{n}, \mathrm{AB} v, \mathrm{t}\right)+\mathrm{r}(\mathrm{t}) \frac{\mathrm{N}\left(\mathrm{ST} y_{n}, \mathrm{P} y_{n}, \mathrm{t}\right)}{\mathrm{N}\left(\mathrm{P} y_{n}, \mathrm{ABv}, \mathrm{t}\right)}\right), \\ \left.\mathrm{N}\left(\mathrm{P} y_{n}, \mathrm{Q} v, \mathrm{t}\right) \otimes \mathrm{N}\left(\mathrm{Q} v, \mathrm{ST} y_{n}, \mathrm{t}\right)\right]\end{array}\right\}\right)$
Taking limit as $n \rightarrow \infty$, we have
$N(z, Q v, t) \geq \varphi\left(\max \left\{\begin{array}{c}\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{M}(\mathrm{z}, \mathrm{z}, \mathrm{t})+\mathrm{r}(\mathrm{t}) \frac{\mathrm{N}(\mathrm{z}, \mathrm{z}, \mathrm{t})}{\mathrm{N}(\mathrm{z}, \mathrm{z}, \mathrm{t})}\right) \\ \mathrm{N}(\mathrm{z}, \mathrm{Q} v, \mathrm{t}) \circ \mathrm{N}(\mathrm{Q} v, \mathrm{z}, \mathrm{t})]\end{array}\right\}\right)$
$N(z, Q v, t) \geq \varphi\left(\max \left\{\begin{array}{c}(\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})+\mathrm{r}(\mathrm{t})), \\ \mathrm{N}(\mathrm{z}, \mathrm{Q} v, \mathrm{t}) \diamond \mathrm{N}(\mathrm{Q} v, \mathrm{z}, \mathrm{t})\end{array}\right\}\right)$,
$N(z, Q v, t) \geq \varphi\left(\max \left\{\begin{array}{c}1, \\ \mathrm{~N}(\mathrm{z}, \mathrm{Q} v, \mathrm{t})\end{array}\right\}\right)$
$N(Q v, z, t) \geq \varphi(\mathrm{N}(Q v, \mathrm{z}, \mathrm{t}))$
$N(Q v, z, t)>\mathrm{N}(Q v, \mathrm{z}, \mathrm{t})$
This is contradiction. Thus $Q v=z$
Hence $Q v=A B v=z$
Suppose $u, v$ are the coincidence point of $(\mathrm{P}, \mathrm{ST})$ and $(\mathrm{Q}, \mathrm{AB})$ respectively. Since $(\mathrm{P}, \mathrm{ST})$ and $(\mathrm{Q}, \mathrm{AB})$ are weakly compatible then PSTu $=S T P u$ and $Q A B v=A B Q v$

This gives $P z=S T z$ and $Q z=A B z$.

Now we show that $z$ is common fixed point of P and ST if $P z \neq z$ using (3.1.1), we obtain
$M(P u, z, t) \geq \varphi\left(\min \left\{\begin{array}{c}\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{M}(\mathrm{z}, \mathrm{z}, \mathrm{t})+\mathrm{r}(\mathrm{t}) \frac{\mathrm{M}(\mathrm{z}, \mathrm{Pu}, \mathrm{t})}{\mathrm{M}(\mathrm{Pu}, \mathrm{z}, \mathrm{t})}\right) \\ \mathrm{M}(\mathrm{Pu}, \mathrm{z}, \mathrm{t}) * \mathrm{M}(\mathrm{z}, \mathrm{z}, \mathrm{t})]\end{array}\right\}\right)$
$M(P u, z, t) \geq \varphi\left(\min \left\{\begin{array}{c}(\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})+\mathrm{r}(\mathrm{t})), \\ \mathrm{M}(\mathrm{Pu}, \mathrm{z}, \mathrm{t}) * 1\end{array}\right\}\right)$
$M(P u, z, t) \geq \varphi\left(\min \left\{\begin{array}{c}1 \\ M(\mathrm{Pu}, \mathrm{z}, \mathrm{t})\end{array}\right\}\right)$
$M(P u, z, t) \geq \varphi(M(P u, z, t)))$
$M(P u, z, t)>M(P u, z, t)$
and

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$N(P u, z, t) \geq \varphi\left(\max \left\{\begin{array}{c}\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{N}(\mathrm{z}, \mathrm{z}, \mathrm{t})+\mathrm{r}(\mathrm{t}) \frac{\mathrm{N}(\mathrm{z}, \mathrm{Pu}, \mathrm{t})}{\mathrm{N}(\mathrm{Pu}, \mathrm{z}, \mathrm{t})}\right), \\ \mathrm{N}(\mathrm{Pu}, \mathrm{z}, \mathrm{t}) \diamond \mathrm{N}(\mathrm{z}, \mathrm{z}, \mathrm{t})]\end{array}\right\}\right)$
$N(P u, z, t) \geq \varphi\left(\max \left\{\begin{array}{c}(\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})+\mathrm{r}(\mathrm{t})), \\ \mathrm{N}(\mathrm{Pu}, \mathrm{z}, \mathrm{t}) \diamond 0]\end{array}\right\}\right)$
$N(P u, z, t) \geq \varphi\left(\max \left\{\begin{array}{c}1, \\ N(\mathrm{Pu}, \mathrm{z}, \mathrm{t})\end{array}\right\}\right)$
$N(P u, z, t) \geq \varphi(\mathrm{N}(P u, \mathrm{z}, \mathrm{t}))$
$N(Q v, z, t)>N(P u, z, t)$
This is contradiction
Hence $\quad P z=S T z=z$
Similarly we can show that $\mathrm{Q} z=z$
Thus $P z=S T z=Q z=A B z=z$
Hence $z$ is a common fixed point of A, B, S, T, P and Q.

## Uniqueness-

Let $w$ be any other fixed point of $\mathrm{A}, \mathrm{B}, \mathrm{S}, \mathrm{T}, \mathrm{P}$ and Q such that $w \neq z$.
$M(w, z, t) \geq \varphi\left(\min \left\{\begin{array}{c}\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{M}(\mathrm{STw}, \mathrm{ABz}, \mathrm{t})+\mathrm{r}(\mathrm{t}) \frac{\mathrm{M}(\mathrm{STw}, \mathrm{Pw}, \mathrm{t})}{\mathrm{M}(\mathrm{Pw}, \mathrm{ABz}, \mathrm{t})}\right), \\ \mathrm{M}(\mathrm{Pw}, \mathrm{Qz}, \mathrm{t}) * \mathrm{M}(\mathrm{Qz}, \mathrm{STw}, \mathrm{t})]\end{array}\right\}\right)$
$M(w, z, t) \geq \varphi\left(\min \left\{\begin{array}{c}{[\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})+\mathrm{r}(\mathrm{t})] \mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{t}),} \\ \mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{t})\end{array}\right\}\right)$
$M(w, z, t) \geq \varphi\left(\min \left\{\begin{array}{c}\mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{t}), \\ \mathrm{M}(\mathrm{w}, \mathrm{z}, \mathrm{t})\end{array}\right\}\right)$
$M(w, z, t) \geq \varphi(\mathrm{M}(w, \mathrm{z}, \mathrm{t}))$
$M(w, z, t)>M(w, \mathrm{z}, \mathrm{t})$
and
$N(w, z, t) \leq \varphi\left(\max \left\{\begin{array}{c}\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{N}(\mathrm{ST} w, \mathrm{AB} z, \mathrm{t})+\mathrm{r}(\mathrm{t}) \frac{\mathrm{N}(\mathrm{ST} w, \mathrm{Pw}, \mathrm{t})}{\mathrm{N}(\mathrm{Pw}, \mathrm{AB} z, \mathrm{t})}\right), \\ \mathrm{N}(\mathrm{Pw}, \mathrm{Q} z, \mathrm{t}) \diamond \mathrm{N}(\mathrm{Q} z, \mathrm{ST} w, \mathrm{t})]\end{array}\right\}\right)$
$N(w, z, t) \leq \varphi\left(\max \left\{\begin{array}{c}{[\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})+\mathrm{r}(\mathrm{t})] \mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{t}),} \\ \mathrm{N}(\mathrm{Pu}, \mathrm{z}, \mathrm{t}) \diamond 0\end{array}\right\}\right)$
$N(w, z, t) \leq \varphi\left(\max \left\{\begin{array}{c}\mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{t}), \\ \mathrm{N}(\mathrm{w}, \mathrm{z}, \mathrm{t})\end{array}\right\}\right)$
$N(w, z, t) \leq \varphi(N(w, z, t))$
$N(w, z, t)<N(w, z, t)$

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This is contradiction. Therefore $w=z$
Thus $z$ is unique fixed common point of $\mathrm{A}, \mathrm{B}, \mathrm{S}, \mathrm{T}, \mathrm{P}$ and Q .
3.2 Corollary: Let (X, M, N, *, ॰) be a intuitionistic fuzzy metric space and A, B, S and T, be four mapping satisfying the following conditions
(i) $\quad A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$ and $S(X)$ is closed
(ii)

$$
M(A x, B y, t) \geq \varphi\left(\min \left\{\begin{array}{c}
\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{M}(\mathrm{~S} x, \mathrm{~T} y, \mathrm{t})+\mathrm{r}(\mathrm{t}) \frac{\mathrm{M}(\mathrm{~S} x, \mathrm{~A} x, \mathrm{t})}{\mathrm{M}(\mathrm{~A} x, \mathrm{~T} y, \mathrm{t})}\right), \\
\mathrm{M}(\mathrm{~A} x, \mathrm{~B} y, \mathrm{t}) * \mathrm{M}(\mathrm{~B} y, \mathrm{~S} x, \mathrm{t})]
\end{array}\right\}\right)
$$

and

$$
N(A x, B y, t) \leq \varphi\left(\max \left\{\begin{array}{c}
\left([\mathrm{p}(\mathrm{t})+\mathrm{q}(\mathrm{t})] \mathrm{N}(\mathrm{~S} x, \mathrm{~T} y, \mathrm{t})+\mathrm{r}(\mathrm{t}) \frac{\mathrm{N}(\mathrm{~S} x, \mathrm{~A} x, \mathrm{t})}{\mathrm{N}(\mathrm{~A} x, \mathrm{~T} y, \mathrm{t})}\right) \\
\mathrm{N}(\mathrm{~A} x, \mathrm{~B} y, \mathrm{t}) \diamond \mathrm{N}(\mathrm{~B} y, \mathrm{~S} x, \mathrm{t})]
\end{array}\right\}\right)
$$

for all $x, y \in \mathrm{X}, t>0$, where $\Phi:[0,1] \rightarrow[0,1]$ is continuous function with $\Phi(s)>s$ whenever 0 $<s<1$ and $\mathrm{p}, \mathrm{q}, \mathrm{r}: \mathrm{R}^{+} \rightarrow(0,1]$ be three function such that $p(t)+q(t)+r(t)=1$. Suppose that the pair (A, S) and (B,T) satisfies CommonE.A. property and (A, S) and (B, T) are weakly compatible. Also suppose that $S(X)$ and $T(X)$ is a closed subset of X. Then A, B, S, and T have a unique fixed point in X .

Proof: Proof involves the same technique as in Theorem 3.1.

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