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COMMON FIXED POINT THEOREMS IN $\mathcal{M}$-FUZZY METRIC SPACE

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Abstract: In this paper we prove common fixed point theorem on $\mathcal{M}$- fuzzy metric space under the condition of compatibility on $\mathcal{M}$- fuzzy metric spaces.Our results generalize and modify several comparable results in the literature.

Keywords: Common fixed point theorem, $\mathcal{M}$- fuzzy metric spaces, compatible mapping.

Introduction:

In 1965, Zadeh [19] introduced the famous theory of fuzzy sets and used it as a tool for dealing with uncertainty arising out of lack of information about certain complex system. Deng [2], Erceg [4], Kaleva and Seikkala[7] and Kramosil and Michalek [8] introduced the concepts of fuzzy metric spaces in different ways. George and veeramani [5] modified the concept of fuzzy metric spaces due to Kramosil and Michalek [8] and defined the Hausdorff topology of fuzzy metric spaces. Many authors [1, 6, 9, 10, 11, 13, 14, 17] have studied the fixed point theory in fuzzy metric spaces.Vashuki [18] obtained the fuzzy version of common fixed point theorem for using extra conditions. There have been number of generalization of Metric spaces. One such generalization is generalized metric space on D-metric space initiated by Dhage [3] in 1992.

He proved some regulation fixed points for a self map satisfying a contraction for complete and bounded D- metric spaces. Rhoades [12] generalized Dhage's contractive condition by increasing the number of factors an proved the existence of unique fixed point f self- map in D-metric space.

Sedgi and Shobe [15] modified the definition of D- metric space and introduced D*-metric space and proved some basic properties in D*-metric space .Using the concept of D*-metric Sedgi and Shobe[15] defined the $\mathcal{M}$- fuzzy metric space and proved a common fixed point theorem in $\mathcal{M}$-fuzzy metric spaces. In this paper we prove common fixed point theorem using compatible mapping on $\mathcal{M}$-fuzzy metric space

2. PRELIMINARIES

2.1 Definition: A binary operation$*$: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if $*$ satisfies the following conditions:

(i) * is commutative and associative,

(ii) * is continuous,

(iii)$a * 1 = a$ for all $a \in [0, 1]$,

(iv)$a * b \leq c * d$, whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0, 1]$.

Example of t -norm are $a * b = a\, b$ and $a * b = \min\{a, b\}$

2.2Definition:A 3-tuple (X, $\mathcal{M}$, $*$) is said to be $\mathcal{M}$ - fuzzy metric space if X is an arbitrary set, $*$ is a continuous$t$ -norm and $\mathcal{M}$ is fuzzy sets in $X^3 \times (0, \infty)$ satisfying the following conditions, for all $x, y, z \in X, s, t > 0$.

(FM-1) $\mathcal{M}(x, y, z, t) > 0$,

(FM-2) $\mathcal{M}(x, y, z, t) = 1$ for allt > 0 if and only if $x = y = z$,

(FM-3) $\mathcal{M}(x, y, z, t) = \mathcal{M}(p\{x, y, z\}, t)$ (symmetry), where p is a permutation function,

(FM-4) $\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, z, s) \leq \mathcal{M}(x,y,z, t + s)$,

(FM-5) $\mathcal{M}(x, y, z, \cdot)$: $(0,\infty) \to [0, 1]$ is continuous.

2.3 Example: Let (X, d) be a metric space. Denote $a * b = a.b$for all $a, b \in [0, 1]$. For each$t \in]0, \infty[$, let $\mathcal{M}(x, y, zt) = \frac{t}{t + D(x,\ y,\ z)}$

where $D\ (x, y, z)$ = max $\{d(x,y), d(y,z), d(x,z)\}$ for all x, y, z∈X. Then (X, $\mathcal{M}$, $*$) is a $\mathcal{M}$-fuzzy metric space. We call the $\mathcal{M}$-fuzzy metric $\mathcal{M}$, induced by the metric d, as the standard $\mathcal{M}$-fuzzy metric metric space.

2.4 Definition: A sequence $\{x_n\}$ in a $\mathcal{M}$-fuzzy metric space (X, $\mathcal{M}$, $*$) is called Cauchy sequenceif and only if for each $0 < \varepsilon < 1$ and t> 0, there exits n_0∈ N such that M(x_n, x_n, x_m, t)> 1- ε for all n, $m \geq n_0$.

The sequence $\{x_n\}$ is said to converges to a point x in X (i.e.$\lim_{n\to\infty} x_n = x$) if $\lim\limits_{n\to\infty} \mathcal{M}(x, x, x_n$, t) = 1for all t> 0.

A $\mathcal{M}$-fuzzy metric space (X, $\mathcal{M}$, $*$) is said to be complete if every Cauchy sequence in it converges to a point in it.

2.5 Definition: Let (X, $\mathcal{M}$, $*$) be a $\mathcal{M}$-fuzzy metric space. $\mathcal{M}$is said to be continuous function on $X^3 \times (0, \infty)$ if $\lim_{n\to\infty} \mathcal{M}(x_n,\ y_n,\ z_n,\ t_n) = \mathcal{M}\ (x, y, zt)$

Whenever a sequence $\{(x_n, y_n, z_n, t_n)\}$ in $X^3 \times (0, \infty)$ converges to a point $(x,\ y,\ z, t)$ in $X^3 \times (0, \infty)$, i.e. $\lim_{n\to\infty} x_n = x$,$\lim_{n\to\infty} y_n = y$,$\lim_{n\to\infty} z_n = z$ and $\lim_{n\to\infty} \mathcal{M}(x,\ y,\ z,\ t_n) = \mathcal{M}\ (x, y, zt)$.

2.6 Definition: Let S and T be self mapping of$\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}$,$*$) are said to be weakly compatible if they commute at a coincidence point, i.e. Sx = Tx implies that S$Tx = STx$

2.7 Definition: Let S and T be self mapping of$\mathcal{M}$- fuzzy metric space (X, $\mathcal{M}$,$*$) are said to be compatible if for all t> 0

$\lim_{n\to\infty} \mathcal{M}\ (STx_n, TSx_n, TSx_n,\ t) = 1$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = x$ for some x in X

2.8Lemma: Let(X, $\mathcal{M}$, $*$) be a fuzzy metric space. For any x, y∈X and y> 0, we have

(i) $\mathcal{M}(x, x, y, t = \mathcal{M}(x, y, y, t)$.

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(ii) $\mathcal{M}(x, y, z, .)$ = is nondecreasing.

Proof: (i) Let $\varepsilon > 0$. For by (FM-4) we have

(1.1) $\mathcal{M}(x, x, y, \varepsilon + t) \geq \mathcal{M}(x, x, x, \varepsilon) * \mathcal{M}(x, y, y, t) = \mathcal{M}(x, y, y, t)$,

(1.2) $\mathcal{M}(y, y, x, \varepsilon + t) \geq \mathcal{M}(y, y, y, \varepsilon) * \mathcal{M}(y, x, x, t) = \mathcal{M}(y, x, x, t)$.

By taking limit $\varepsilon \to 0$ in (1.1) and (1.2), we get $\mathcal{M}(x, x, y, t) = \mathcal{M}(x, y, y, t)$.

(ii) By (FM-4) we have $\mathcal{M}(x, y, a, t) * \mathcal{M}(a, z, z, s) \leq \mathcal{M}(x, y, z, t + s)$ for any $z, a \in X$ and $s, t > 0$. Let $a = z$, then we have $\mathcal{M}(x, y, z, t) * \mathcal{M}(z, z, z, s) \leq \mathcal{M}(x, y, z, t + s)$ so that $\mathcal{M}(x, y, z, t + s) \geq \mathcal{M}(x, y, z, t)$.

In the following example, we know that both d -metric and fuzzy metric induce a $\mathcal{M}$-fuzzy metric.

2.9 Lemma: Let (X, $\mathcal{M}$, $*$)be a $\mathcal{M}$- fuzzy metric space. If there exist k ∈ (0, 1) such that for all$x, y \in X$

$\mathcal{M}(x, y, y, kt) \geq \mathcal{M}(x, y, y, t)$ for all $t > 0$ then $x = y$.

3. MAIN RESULT

3.1 Theorem: Let(X, $\mathcal{M}$, $*$) be a complete fuzzy metric space. Let A, B, S, T, P and Q be six self –mappingssatisfying the following condition:

(i) P(X) ⊆ AB(X) and Q(X) ⊆ ST(X).

(ii) Pair (P, AB) and (Q, ST) are weakly compatible and AB = BA, PB = BP, SQ = QS, QT = TQ and ST = TS.

(iii) There exists a number k∈ (0, 1) such that

$\mathcal{M}(Px, Qy, ABx, kt) * \mathcal{M}(ABx, Px, Px, kt)$

$$\geq \min\left\{\begin{matrix}\left([\,p(t)+q(t)]\mathcal{M}(STy,\ ABx,\ ABx,\ t) + r(t)\frac{\mathcal{M}(STy,\ Px,\ Px, t)}{\mathcal{M}(Px,\ ABx,\ ABx,\ t)}\right), \\ \mathcal{M}(Px,\ Px,\ Qy,\ t) * \mathcal{M}(Qy,\ STy,\ STy,\ t)]\end{matrix}\right\}$$

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for all x, y ∈ X all $t > 0$ and some k ∈ [0, 1] where p, q, r : $R^+ \to (0, 1]$ be three function such that $p(t) + q(t) + r(t) = 1$.

If the range of the subspaces P(X) or AB(X) or Q(X) or ST(X) is complete, then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: By @, $\{y_n\}$ is a Cauchy sequence in X. Since X is complete, so $\{y_n\}$ converges to a point $z \in$ X. Since $\{Px_{2n}\}$, $\{Qx_{2n+1}\}$, $\{ABx_{2n+1}\}$ and $\{STx_{2n+1}\}$ are subsequences of $\{y_n\}$, they also converges to the same point z.

Since P(X) ⊂ AB(X), there exists a point $w \in$ X such that $ABw = z$

Put $x = w$ and $y = x_{2n+1}$in inequality (iii), we get

$$\mathcal{M}(Pw, Qx_{2n+1}, ABw, \mathrm{k}t) * \mathcal{M}(ABw, Pw, Pw, kt)$$

$$\geq \min\left\{\begin{array}{c}\left([\,p(t)+q(t)]\mathcal{M}(STx_{2n+1},\ ABw,\ ABw,\ t)+ r(t)\frac{\mathcal{M}(STx_{2n+1},\ Pw,\ Pw,\ t)}{\mathcal{M}(Pw,\ ABw,\ ABw,\ t)}\right),\\ \mathcal{M}(Pw,\ Pw,\ Qx_{2n+1},\ t) * \mathcal{M}(Qx_{2n+1},\ STx_{2n+1},\ STx_{2n+1},\ t)]\end{array}\right\}$$

Taking limit as $\to \infty$, we have

$$\mathcal{M}(Pw, z, z, \mathrm{k}t) * \mathcal{M}(z, Pw, Pw, kt) \geq \min\left\{\begin{array}{c}\left([\,p(t)+q(t)]\mathcal{M}(z,\ z,\ z,\ t)+ r(t)\frac{\mathcal{M}(z,\ Pw,\ Pw,\ t)}{\mathcal{M}(Pw,\ z,\ z,\ t)}\right),\\ \mathcal{M}(Pw,\ Pw,\ z,\ t) * \mathcal{M}(z,\ z,\ z,\ t)]\end{array}\right\}$$

$$\mathcal{M}(Pw, z, Pw, 2kt) \geq \min\left\{\begin{array}{c}\left([\,p(t)+q(t)]\mathcal{M}(z,\ z,\ z,\ t)+ r(t)\frac{\mathcal{M}(Pw, z, z, t)}{\mathcal{M}(Pw, z, z, t)}\right),\\ \mathcal{M}(Pw, z, z, t) * \mathcal{M}(z, z, z, t)]\end{array}\right\}$$

$$\mathcal{M}(Pw, z, Pw, \mathrm{k}t) \geq \min\left\{\begin{array}{c}\left([\,p(t)+q(t)]\mathcal{M}(z,\ z,\ z,\ t)+ r(t)\frac{\mathcal{M}(Pw, z, z, t)}{\mathcal{M}(Pw, z, z, t)}\right),\\ \mathcal{M}(Pw, z, z, t) * \mathcal{M}(z, z, z, t)]\end{array}\right\}$$

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$\mathcal{M}(Pw, z, z, kt) \geq \min\left\{\begin{matrix}(p(t)+q(t)+r(t)), \\ \mathcal{M}(Pw, z, z, t) * 1]\end{matrix}\right\}$

$\mathcal{M}(Pw, z, z, kt) \geq \min\left\{\begin{matrix} \\ 1, \mathcal{M}(Pw, z, z, t)\end{matrix}\right\}$

$\mathcal{M}(Pw, z, z, kt) \geq \mathcal{M}\ (Pw, z, z, t)$

By lemma (2.9), $Pw = z$

Therefore $ABw = Pw = z$

Since Q(X) ⊂ ST(X),there exists a point $v \in X$ such that $STv = z$

Put $x = w$ and $y = v$in inequality (iii), we get

$$\mathcal{M}(Pw, Qv, ABw, kt) * \mathcal{M}\ (ABw, Pw, Pw, kt)$$

$$\geq \min\left\{\begin{matrix}\left([\ p(t)+q(t)]\mathcal{M}(STv,\ ABw,\ ABw,\ t)+ r(t)\frac{\mathcal{M}(STv,\ Pw,\ Pw,\ t)}{\mathcal{M}(Pw,\ ABw,\ ABw,\ t)}\right), \\ \mathcal{M}(Pw,\ Pw,\ Qv,\ t) * \mathcal{M}(Qv,\ STv,\ STv,\ t)]\end{matrix}\right\}$$

Taking limit as $\rightarrow \infty$, we have

$$\mathcal{M}(z, Qv, z, kt) * \mathcal{M}\ (z,\ z, z, kt) \geq \min\left\{\begin{matrix}\left([\ p(t)+q(t)]\mathcal{M}(z, z, z, t)+ r(t)\frac{\mathcal{M}(z, z, z, t)}{\mathcal{M}(z, z, z, t)}\right), \\ \mathcal{M}(z, z, Qv, t) * \mathcal{M}(Qv, z, z, t)]\end{matrix}\right\}$$

$$\mathcal{M}(Qv, z,\ z, kt) * 1 \geq \min\left\{\begin{matrix}\left([\ p(t)+q(t)]\mathcal{M}(z, z, z, t)+ r(t)\frac{\mathcal{M}(z, z, z, t)}{\mathcal{M}(z, z, z, t)}\right), \\ \mathcal{M}(z, z, z, t)\end{matrix}\right\}$$

$$\mathcal{M}(Qv, z,\ z, kt) \geq \min\left\{\begin{matrix}(p(t)+q(t)+ r(t)), \\ 1\end{matrix}\right\}$$

$\mathcal{M}(Qv, z,\ z, kt) \geq \min\{1, 1\}$

$\mathcal{M}(Qv, z,\ z, kt) \geq 1$

$STv = Qv = z$

Therefore $ABw = Pw = STv = Qv = z$

Since the pair (P, AB) is weakly compatible, therefore $ABw = Pw$ implies that $PABw = ABPw$

i.e. $Pz = ABz$

Now we show that z is a fixed point of P.

Putting $x = z$ and $y = v$ in inequality (iii), we have

$$\mathcal{M}(Pz, Qv, ABz, kt) * \mathcal{M}(ABz, Pz, Pz, kt)$$

$$\geq \min\left\{ \begin{array}{c} \left([\,p(t)+q(t)]\mathcal{M}(STv, ABz, ABz, t) + r(t)\frac{\mathcal{M}(STv, Pz, Pz, t)}{\mathcal{M}(Pz, ABz, ABz, t)} \right), \\ [\mathcal{M}(Pz, Pz, Qv, t) * \mathcal{M}(Qv, STv, STv, t)] \end{array} \right\}$$

$$\mathcal{M}(Pz, z, Pz, kt) * \mathcal{M}(Pz, Pz, Pz, kt) \geq \min\left\{ \begin{array}{c} \left([\,p(t)+q(t)]\mathcal{M}(z, Pz, Pz, t) + r(t)\frac{\mathcal{M}(z, Pz, Pz, t)}{\mathcal{M}(Pz, Pz, Pz, t)} \right), \\ [\mathcal{M}(Pz, Pz, z, t) * \mathcal{M}(z, z, z, t)] \end{array} \right\}$$

$$\mathcal{M}(Pz, z, Pz, kt) * 1 \geq \min\left\{ \begin{array}{c} \left(p(t)+q(t)+r(t) \right)\mathcal{M}(z, Pz, Pz, t), \\ [\mathcal{M}(Pz, z, z, t) * 1] \end{array} \right\}$$

$$\mathcal{M}(Pz, z, z, kt) \geq \min\left\{ \begin{array}{c} \mathcal{M}(Pz, z, z, t), \\ \mathcal{M}(Pz, z, z, t) \end{array} \right\}$$

$$\mathcal{M}(Pz, z, z, kt) \geq \mathcal{M}(Pz, z, z, t)$$

By lemma (2.9), $Pz = z$

Therefore $ABz = Pz = z$

Similarly, pair of map (Q, ST) is weakly compatible, we have

$Qz = STz = z$

Now we show that $Bz = z$, by putting $x = Bz$ and $y = x_{2n+1}$ in inequality (iii), we have

$$\mathcal{M}(PBz, Qx_{2n+1}, AB(Bz), kt) * \mathcal{M}(AB(Bz), PBz, PBz, kt)$$

$$\geq \min\left\{ \begin{array}{c} \left([\,p(t)+q(t)]\mathcal{M}(STx_{2n+1}, AB(Bz), AB(Bz), t) + r(t)\frac{\mathcal{M}(STx_{2n+1}, PBz, PBz, t)}{\mathcal{M}(Pz, AB(Bz), AB(Bz), t)} \right), \\ [\mathcal{M}(PBz, PBz, Qx_{2n+1}, t) * \mathcal{M}(Qx_{2n+1}, STx_{2n+1}, STx_{2n+1}, t)] \end{array} \right\}$$

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Taking limit as $n \to \infty$ and using lemma 2.2, we have

$Bz = z$

Since $ABz = z$, therefore, $ABz = Bz = STz = Pz = Qz = z$

Finally we show that $Tz = z$, by putting $x = z$ and $y = Tz$ in inequality (iii), we have

$\mathcal{M}(Pz, QTz, ABz, kt) * \mathcal{M}(ABz, Pz, Pz, kt)$

$$\geq \min\left\{\begin{array}{c}\left([\,p(t)+q(t)]\mathcal{M}(ST(Tz),\ ABz,\ ABz,\ t)+ r(t)\frac{\mathcal{M}(ST(Tz),\ Pz,\ Pz,\ t)}{\mathcal{M}(Pz,\ ABz,\ ABz,\ t)}\right),\\ \mathcal{M}(Pz,\ Pz,\ QTz,\ t) * \mathcal{M}(QTz,\ ST(Tz),\ ST(Tz),\ t)]\end{array}\right\}$$

$$\mathcal{M}(z, Tz, z, kt) * \mathcal{M}(z, z, z, kt) \geq \min\left\{\begin{array}{c}\left([\,p(t)+q(t)]\mathcal{M}(Tz, z, z, t)+ r(t)\frac{\mathcal{M}(Tz,\ z,\ z,\ t)}{\mathcal{M}(z,\ z,\ z,\ t)}\right),\\ \mathcal{M}(z, z, Tz,\ t) * \mathcal{M}(Tz, Tz, Tz, t)]\end{array}\right\}$$

$$\mathcal{M}(z, Tz, z, kt) * 1 \geq \min\left\{\begin{array}{c}\left(p(t)+q(t)+ r(t)\right)\mathcal{M}(Tz, z, z, t),\\ \mathcal{M}(z, z, Tz,\ t) * 1]\end{array}\right\}$$

$$\mathcal{M}(Tz,\ z, z,\ kt) \geq \min\left\{\begin{array}{c}\mathcal{M}(Tz, z, z, t),\\ \mathcal{M}(z, z, Tz,\ t)\end{array}\right\}$$

$\mathcal{M}(Tz,\ z, z,\ kt) \geq \mathcal{M}(Tz,\ z, z,\ kt)$

Therefore $Tz = z$

Hence$ABz = Bz = STz = Pz = Qz = z$

Uniqueness: Finally we prove that A, B, S, T, P and Qhave a unique common fixed point.

Let r be another common fixed point of A, B, S, T, P and Q.

Then $ABr = STr = Pr = Qr = r$

Put $x = r$ and $y = z$ in inequality (iii), we have

$\mathcal{M}(Pr, Qz, ABr, kt) * \mathcal{M}(ABr, Pr, Pr, kt)$

$$\geq \min\left\{\begin{array}{c}\left([\,p(t)+q(t)]\mathcal{M}(STz,\ ABr,\ ABr,\ t)+ r(t)\frac{\mathcal{M}M(STz,\ Pr,\ Pr, t)}{\mathcal{M}(Pr,\ ABr,\ ABr,\ t)}\right),\\ \mathcal{M}(Pr,\ Pr,\ Qz,\ t) * \mathcal{M}(Qz,\ STz,\ STz,\ t)]\end{array}\right\}$$

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$$\mathcal{M}(r, z, r, kt) * \mathcal{M}(r, r, r, kt) \geq \min\left\{\begin{matrix}\left([p(t)+q(t)]\mathcal{M}(z, r, r, t) + r(t)\frac{\mathcal{M}(z, r, r, t)}{\mathcal{M}(r, r, r, t)}\right), \\ \mathcal{M}(r, r, z, t) * \mathcal{M}(z, z, z, t)]\end{matrix}\right\}$$

$$\mathcal{M}(r, z, r, kt) * 1 \geq \min\left\{\begin{matrix}\left(p(t)+q(t)+r(t)\right)\mathcal{M}(z, r, r, t), \\ \mathcal{M}(r, r, z, t) * 1]\end{matrix}\right\}$$

$$\mathcal{M}(z, r, r, kt) \geq \min\left\{\begin{matrix}\left(p(t)+q(t)+r(t)\right)\mathcal{M}(z, r, r, t), \\ \mathcal{M}(z, r, r, t)\end{matrix}\right\}$$

$$\mathcal{M}(z, r, r, kt) \geq \min\left\{\begin{matrix}\mathcal{M}(z, r, r, t), \\ \mathcal{M}(z, r, r, t)\end{matrix}\right\}$$

$$\mathcal{M}(z, r, r, kt) \geq \mathcal{M}(z, r, r, t)$$

Hence, by lemma 2.9, we have $z = r$

This completes the proof of theorem.

3.1.2 Corollary: Let(X, $\mathcal{M}$, $*$) be a complete fuzzy metric space. Let A, S, P and Q be six self – mappingssatisfying the following condition:

(i) $P(X) \subseteq A(X)$ and $Q(X) \subseteq S(X)$.

(ii) Pair (P, S) and (Q, A) are weakly compatible.

(iii) There exists a number $k \in (0, 1)$ such that

$$\mathcal{M}(Px, Qy, Ax, kt) * \mathcal{M}(Ax, Px, Px, kt)$$
$$\geq \min\left\{\begin{matrix}\left([p(t)+q(t)]\mathcal{M}(Sy, Ax, Ax, t) + r(t)\frac{\mathcal{M}(Sy, Px, Px, t)}{\mathcal{M}(Px, Ax, Ax, t)}\right), \\ \mathcal{M}(Px, Px, Qy, t) * \mathcal{M}(Qy, Sy, Sy, t)]\end{matrix}\right\}$$

for all $x, y \in X$ all $t > 0$ and some $k \in (0, 1)$ where $p, q, r : R^{+} \rightarrow (0, 1]$ be three function such that $p(t) + q(t) + r(t) = 1$.

If the range of the one subspaces complete, then A, S, P and Q have a unique common fixed point in X.

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