

STOCHASTIC MODEL VIEW & FEEDBACK OF QUEUE NETWORK

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ABSTRACT:- In this paper there is discussion about queue network in stochastic model. Jackson [1957, 1963] made a mark in the design of the queue network and initially demonstrated that a solution is product shape for a constant balance equation. Koiengsberg [1958] studied the problem of buffer storage as a cyclic queue system. A cyclic queue is a sort of serial queue in a "circle" where the output of the last node feeds back to the first node. Purple, P.D. [1959] expanded the analysis with feedback on cyclic queues. Jackson networks and items for three distinct queue disciplines were clarified by Basket et al. [1975] about Sharing of the processor (each customer gets a share of and is served simultaneously by a single server), Extensive service and Preventive resume maintenance of the LCFS.

KEYWORDS: Stochastic Model, Cyclic queue, LCFS, feedback network, Average waiting time.

INTRODUCTION:-Mathematicians and operational researchers have done significant work in developing stochastic and simulation models in the analysis of feedback and cyclic queue network. Different authors have introduced different models to encourage theory in realistic circumstances, using various hypothesis situations as their basis. One can observe that, while many such models lack practicability in real life circumstances, the mathematical methods used in the study are truly elegant. Here is a short description of some of the work they choose in a series they have created in order to deal with specific circumstances in their lives. Feedback

queues are those in which a customer once served if his service is failing and is served over and over before his service is successful (Takacs [1963], Kleinrock [1975], Takagi [1987], Singh T.P. [1996]).

For instance a packet transmitted from the source to the destination can be returned in certain real life scenarios as for example a feedback queue and it can proceed as it is until the packet is eventually transmitted. In the last few decades, the university has carried out extensive research in the field of feedback and cyclic queue model (Burke's[1956], Jackson[1957], Finch P.D.[1959], Buchan & Koiengsbeg, Singh T.P.[1994]). In the field of queue networking theory Jackson did a remarkable job. Let us take it for granted that units (customer/message/packages) individually arrive in Poisson at rate β and that the service time is general. Once the service is complete, the device is cycled back into the system with a chance to successfully perform a service apart from the p system or if service is unsuccessful $(1-p)$. This is called input from Bernoulli. It is not likely that all customers, who necessarily join and leave at the same node, or who follow the same route once they enter the customer of the system, can return to the previously visited nodes, miss any of the nodes entirely and choose to stay in the system forever.

Gordan and Newell [1967] consider a network of Markov queues where a fixed and limited number of customers say (K) circulates across the network. There is no outside feedback or departure from the network, unlike Jackson's open network. Considering a closely connected network of " k " nodes, the contribution of the I nodes will be at the next node $i+1$ (1 alternatives to $i-1$), and the last node " k " input will be from the I node and so forth. A cyclic queue such as this is called. A cyclic queue structure consists of several series of service channels, with a closed circle at the centre. T.P. Singh Singh. [1994] broadened the research carried out by various cyclic queue network researchers. Further Singh T.P., Kusum (2010) carried out detailed feedback queue model empirical studies assuming that the service rate was proportional to the queue numbers and heterogeneous feedback channels. This study is another widespread work done by previous researchers in the field of feedback.

TRANSIENT BEHAVIOR OF THREE STAGE FEEDBACK QUEUE SYSTEM

This segment explores a network of three queues in sequence, input from the third server, both arriving and departing poisonously. We assume that the service rate is proportional to the amount of your queue. The equation of differential differences was tested transiently. The mean queue size and other parameters were obtained by means of statistical tools for the queue model. The model is implemented in the banking service structure, administrative installation and decision-making. The transient behaviour of the system was examined and the differential equation, thus developed in the model, solved by using the generating function technique and Lagrange's partial differential equation solution process. The aim of this section is to discuss different queue characteristics such as medium length of the queue system, variance of it, etc., and useful in the decision-making, produce, banking service, etc.

PROBLEM FORMULATION:

Consider a system of three service nodes say S_1, S_2, S_3 . Customers or items arrive in Poisson process at S_1 , and then go through S_2 and S_3 for required service, the service pattern at both the nodes are exponentially distributed. Let " λ " be mean arrival rate at s_1 and $\mu n_1, \mu n_2, \mu n_3$ denotes exponential service parameters at S_1, S_2, S_3 . Customer after service at S_1 go through mode S_2 and then S_3 in a tandem queue while customer at S_3 is either depart with probability p_3 or feed back to server s_1 with probability p_{31} . Such that $p_3 + p_{31} = 1$. It is further assumed that service parameters $\mu n_1, \mu n_2, \mu n_3$ are directly proportional to their respective queue numbers defined by $\mu n_1 = \mu_1 n_1, \mu n_2 = \mu_2 n_2, \mu n_3 = \mu_3 n_3$.

MATHEMATICAL MODELING:

Define P_{n_1, n_2, n_3}^t the probability at time (t), there are n_1, n_2, n_3 items in system in front of nodes S_1, S_2, S_3 ($n_1, n_2, n_3 > 0$),

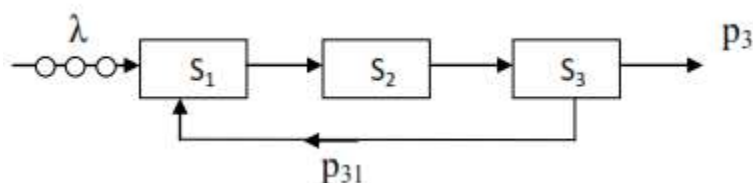


Figure - I

Connecting the various state probabilities at time 't+δt' time dependent differential difference equations for the model are obtained as:

SOLUTION METHODOLOGY:

To solve the system of equations (1) to (5) we apply generating function technique.

For this define g.f.t as:

$$F(x, y, z, t) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \sum_{n_3=0}^{\infty} P_{n_1, n_2, n_3}(t) x^{n_1} y^{n_2} z^{n_3}$$

$$F_{n_2, n_3}(x, t) = \sum_{n_1=0}^{\infty} P_{n_1, n_2, n_3}(t) x^{n_1}$$

$$F_{n_3}(x, y, t) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} P_{n_1, n_2, n_3}(t) x^{n_1} y^{n_2}$$

$$F_{n_3}(x, y, t) = \sum_{n_2=0}^{\infty} F_{n_2, n_3}(x, t) y^{n_2}$$

$$F(x, y, z, t) = \sum_{n_3=0}^{\infty} F_{n_3}(x, y, t) z^{n_3}$$

$$z = C_3 e^{\mu_3 t} + p_3 + p_{31} x$$

Solving above (1) to (5) on the line of Maggu [1981] & Singh T.P. [1984], Using equations simplifying, ultimate solution in transient form can be expressed as :-

$$\frac{\partial F}{\partial t} + \mu_1 x - y \frac{\partial F}{\partial x} + \mu_2 y - z \frac{\partial F}{\partial y} + \mu_3 z - p_3 - p_{31}x \frac{\partial F}{\partial z} = \lambda(x - 1)F$$

(6)

As the equation (6) in 4 independent variable x, y, z and t of the Lagrange auxiliary equations is a linear partial differential equation first order by

$$\frac{dt}{1} = \frac{dx}{\mu_1 x - y} = \frac{dy}{\mu_2 y - z} = \frac{dz}{\mu_3 z - p_3 - p_{31}x} = \frac{dF}{\lambda(x-1)}$$

(7)

Now we proceed to obtain four independent solutions of (7).

For this applying the solution technique of Lagrange's Auxiliary Equation, we have

$$\frac{dt}{1} = \frac{dx}{\mu_1 x - y}$$

On integration we get,

$$C_1 = x - y e^{-\mu_1 t}$$

And $x = C_1 e^{\mu_1 t} + y$

Similarly, $C_2 = y - z e^{-\mu_2 t}$

$$y = C_2 e^{\mu_2 t} + z$$

And $C_3 = z - p_3 - p_{31}x e^{-\mu_3 t}$

$$z = C_3 e^{\mu_3 t} + p_3 + p_{31}x$$

Mean Queue length:

$$L_Q = L_{Q1} + L_{Q2} + L_{Q3}$$

$$L_Q = \frac{\lambda (1 - p_{32}p_{23})}{\mu_1 (1 - p_{12}p_{21} - p_{23} p_{32} + p_{12}p_{31} - \lambda (1 - p_{32}p_{23}))} + \frac{\lambda p_{12}}{\mu_2 (1 - p_{12}p_{21} - p_{23} p_{32} + p_{12}p_{31} - \lambda p_{12})}$$

$$+ \frac{\lambda p_{12}p_{23}}{\mu_3 (1 - p_{12}p_{21} - p_{23} p_{32} + p_{12}p_{31} - \lambda p_{12}p_{23})}$$

WAITING TIME IN QUEUE $(W_q) = \frac{L_Q}{\lambda}$

APPLICATION:

The model finds applications in a variety of circumstances in the fields of manufacturing, office administration, computer networking, banking facilities, administrative installation, industrial enterprise, etc. Consider the products with three steps of manufacturing, and the faulty portion of the production item is lowered at the rear, after operation of the first phase machine and defective items are transferred for further operation to the next step machine. Either item is dropped in the second stage, or it is sent for reprocessing into the first stage, or to the following third stage in succession for operation. For instance, in the soft drink industry of glass cum, we found that the services are performed in different phases, starting from the second phase, bottles of shape and size deficiencies are returned to the first phase, the next phase is replaced with leaked bottles or other non-traceable defects to the second or first phases. In addition, the filled bottles are sent to be corked in the next process or not sufficient to be corked. In an administrative setup file, the files will be sent from one section to the second for comment then to the final testing authority or to the authority for further clarification either to transfer the file back or to the original one. In the formulation and simulation of computers, networking, the banking and other such systems the idea of the network was found to be very useful. The Queuing network acts as a model for a multi-programmed computer and communication system and for certain parts of the production system. Recycling or feedback can occur in any multi-

stage queuing phase. Recycling is common in production processes where quality controls are carried out after certain phases, and reprocessing components which do not meet quality standards. Likewise, a telecommunications network may process messages by a random sequence of nodes, which is likely to require that certain messages often go through the same step. Two models were proposed in this report. The medium length of the whole system allows the decision maker to restructure the queue system to reduce congestion and to avoid the service system from becoming idle.

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