



## AN EVALUATION OF THERMAL CONDUCTIVITY OF HIGH-TEMPERATURE SUPERCONDUCTORS

Neeraj Kumar Mishra<sup>1</sup>, Priyanka Vaidya<sup>2</sup>

<sup>1</sup>Department of Physics, National Institute of Technology, Patna 800005, Bihar, India

<sup>2</sup>Department of Physics, Magadh University, Bodh Gaya 824234, Bihar, India

### ABSTRACT

This paper deals with the transport properties of high temperature superconductors. The transport properties are temperature dependence of thermal conductivity of high  $T_c$  superconductor. We have taken two different high  $T_c$  superconductors in our studies namely  $La_{2-n}Sr_nCuO_4$  and  $YBa_2Cu_3O_{7-\delta}$ . These superconductors bear transition temperature from 38K to 110K. Transport properties basically concerns with the movement of the classic carriers in the solid. In the case of superconductors the carriers are super electrons which move in the vicinity of a different excitation of the solid created by the impurity which has been dropped in the system. It was believed that as in the case of noble metal or low  $T_c$  superconductors ( $T_c$  from 1K to 24K) phonons has dominant role in different transport properties. From the investigation this has been ruled out. Phonons does play role in the high  $T_c$  superconductor only above  $T_c$ . The studies of thermal conductivities of high  $T_c$  superconductors indicate that the phonons contributions are larger near  $T_c$  and smaller far away.

**Keywords:** Cooper pair, electrical conductivity, Matthiessen rule, point defects, superconductor, thermal conductivity, thermal current density, Wiedemann-Franz law.

### 1. INTRODUCTION

Thermal conductivity refers to the ability of a given material to conduct or transfer heat. It is the rate at which heat is transferred by conduction through a unit cross-section area of a material, when a temperature gradient exists perpendicular to the area.

Superconductivity is a set of physical properties observed in certain materials where electrical resistance vanishes and magnetic flux fields are expelled from the material. Any material exhibiting these properties is a Superconductor.

The scattering mechanisms involved in heat transport may be investigated through thermal conductivity measurements. In particular, measurements on high temperature superconductor  $YBa_2Cu_3O_{7-\delta}$  and  $La_{1.85}Sr_{0.15}CuO_4$  provide information on the scattering of photons by electrons for  $T \sim T_c$ . Low temperature measurements on single crystal  $YBa_2Cu_3O_{7-\delta}$  can give information on photon scattering by two level systems<sup>2</sup> characteristic of amorphous solids.



In order to interpret the thermal conductivity experiments on the high  $T_c$  superconductor it is useful to compare their behaviour with that of conventional superconductor (i.e. superconductor which are well discussed by the BCS-mechanism of phonon-mediated Cooper pair formation).

## 2. THERMAL CONDUCTIVITY OF CONVENTIONAL SUPERCONDUCTOR.

### 2.1 ELECTRONIC CONTRIBUTION TO THE THERMAL CONDUCTIVITY

For a superconductor, the behaviour of thermal conductivity as a function of temperature may be quite complicated. It depends for example, upon the temperature of the transition with respect to the temperatures at which various scattering mechanisms dominate the degree of physical and chemical disorder, and the purity of the material. However, if one restricts one's attention to the neighbourhood of the superconducting transition and look for conditions between the dominant heat carrier and the form of the thermal conductivity through the transition, one can draw some single conclusions and apply them to the thermal conductivity measurements on  $\text{La}_{2-n}\text{Sr}_n\text{CuO}_4$  and  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ . If electrons transport the majority of the heat, the thermal conductivity in the superconducting state is lower than the normal state conductivity for  $T \sim T_c$ ; as electrons condense into the ground state then no longer transport entropy or heat and the thermal conductivity is correspondingly reduced.<sup>3,4</sup>

### 2.2 PHONON CONTRIBUTION TO THE THERMAL CONDUCTIVITY

When phonons transport the majority of the heat, the thermal conductivity in the superconducting state can become larger than the normal state for  $T \sim T_c$ ; for most conventional superconductor the transition occurs in a region where phonons are scattered mainly by electrons and defects like grain boundaries, pores and point defects; below the transition electrons are effectively resumed as phonon scattering, providing an increase in the conductivity<sup>5</sup>. The increase may be obscured if defect scattering dominates over electron scattering at  $T_c$ . When both electrons and phonons make comparable contribution to the thermal conductivity at  $T_c$ , either type of behaviour is possible.<sup>6</sup> It is useful to note that for this case when phonons transport the majority of the heat at  $T_c$ , similar behaviour is seen in both medium coupled<sup>7</sup> and strong coupled superconductor for both types of materials the thermal conductivity in the superconducting state is larger than the normal state conductivity for temperatures chosen to and lower than  $T_c$ . In other words, for both strong and medium coupled superconductor then is sufficient electron normal scattering at  $T_c$  state the thermal conductivity in the superconducting state is enhanced above the normal state conductivity as the scattering is reduced below  $T_c$ . One should be cautious, however, in drawing conclusion about the strength of the electron phonon coupling based upon the qualitative form of the thermal conductivity at  $T_c$  above.

## 3. THERMAL CONDUCTIVITY OF HIGH $T_c$ SUPERCONDUCTOR

### 3.1 NON-IDEAL SAMPLES

Thermal conductivity measurements on high  $T_c$  superconductor have been limited mainly to ceramic samples of  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  and  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ . Since the thermal conductivity is sensitive to the scattering mechanisms in a solid, the presence of pores and internal boundaries in ceramic materials makes the thermal conductivity data more difficult to interpret than for single crystal materials. Nonetheless, the qualitative form of the thermal



conductivity as a function of temperature for the  $\text{La}_{2-n}\text{Sr}_n\text{CuO}_4$  and  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  ceramics can give us information of the scattering of phonons by electrons for  $T \sim T_c$ . Note that no matter what the mechanism for superconductivity is for the high  $T_c$  superconductor, electrons below  $T_c$  which have condensed with the good state cannot transport heat or scatter phonons. Only one group<sup>9</sup> has measured the thermal conductivity of one sample each of single crystal  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$  and  $\text{HoBa}_2\text{Cu}_3\text{O}_{7-\delta}$ . Then data provide evidence for the existence of two level symptoms<sup>10,11</sup> in  $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ .

### 3.2 MATHEMATICAL FORMULATION USED IN THE EVALUATION

One knows that the heat currents carried by conduction electron are closely related to electrical currents. An additional complication in the heat transport case is that the carriers of heat can be either charge carriers like electrons or electrically neutral phonons, whereas electrical current arises only from charge carrier transport. The transformation to the superconducting state changes the nature of the carriers of the electric current. So it is to be expected that the transport of heat will be strongly affected.<sup>12-16</sup>

The thermal current density  $\vec{j}$  is the thermal energy per unit time crossing a unit area adjusted perpendicular to the direction of heat flow. It is a vector representing the transport of entropy density  $S_\phi$  at the velocity  $\vec{v}$ ,

$$\vec{j} = TS_\phi \vec{v} \quad (1)$$

from hotter to cooler region of the material. It is proportional to the gradient of the temperature  $\nabla T$  through differential form of Fourier's law

$$q = -k \nabla T \quad (2)$$

Where  $q$  is the local heat flux density and  $k$  is the coefficient of thermal conductivity.

In the normal state, electrical conductor is a good conductor of heat in accordance with the law of Wiedemann Franz.

$$(k/\sigma) = \frac{\pi^2}{3} (k_B/e)^2 T \quad (3)$$

where  $\sigma$  is the electrical conductivity.

In the superconducting state, in contrast the heat conductivity can be much lower because cooper pairs carry no entropy and do not scatter phonon.

The principal carries of thermal energy through metals in the normal state are conduction electron and phonons. Heat conduction via each of these two channels acts independently, so that the two channels constitute parallel paths for the passage of heat. A simple model for the conduction of heat between two points A and B in the sample is to represent the two channels by parallel resistors with conductivities  $k_e$  and  $k_{ph}$  for the electronic and phonons paths. The conductivities add directly as the electrical analogue of parallel resistors to give the total thermal conductivities  $K$ .

$$k(T) = k_e(T) + k_{ph}(T) \quad (4)$$



The electronic path has an electron lattice contribution  $k_{e-L}$  which is always present and an impurity term  $k_{e-1}$  which becomes dominant at high defect concentrations. In like manner, the phonon path has a phonon- electron constitution  $k_{ph-e}$  plus an additional contribution  $k_{ph-1}$  from impurities. Since each pair of terms involves the same carriers of heat, they act in series and add as reciprocals, as in the electrical analogue can of Matthiessen rule. When the resistivity / reciprocals of conductivity) add directly.

The result is

$$\frac{1}{k_e} = \frac{1}{k_{e-L}} + \frac{1}{k_{e-1}} \quad (5a)$$

$$\frac{1}{k_{ph}} = \frac{1}{k_{ph-e}} + \frac{1}{k_{ph-1}} \quad (5b)$$

It is shown in standard solid state physics texts that the electronic contribution to the thermal conductivity has the form

$$K_{e-L} = \frac{1}{3} v_F I C_e \quad (6a)$$

$$= \frac{1}{3} \gamma v_F^2 \tau T \quad (6b)$$

When we have used electron mean free path  $I = v_F \tau$  and  $C_e = \gamma T$ . Now we know that at low temperatures  $\tau \approx T^{-3}$  and  $\tau \approx T^{-1}$  at high temperatures, applying Wiedemann-Franz law gives us

$$K_{e-L} \approx \begin{cases} \frac{[\text{constant}]}{\tau^2} & T \ll \theta_D \\ [\text{constant}] T & T \gg \theta_D \end{cases} \quad 7(a) \text{ \& } 7(b)$$

for temperatures that are low and high respectively relative in the degree temperature  $\theta_D$ . We know that at the lowest temperatures the electrical conductivity  $\sigma(T)$  approaches a limiting value,  $\sigma(T) \rightarrow \sigma_0$  arising from the impurity contribution. From this term the law of Wiedemann-Franz gives

$$k_{e-1} \rightarrow [\text{constant}] T, \quad T \rightarrow 0 \quad (8)$$

The lattice contribution to the thermal conductivity has a form which is the phonon analogue of equation 6a

$$k_{ph-L} = \frac{1}{3} v_{ph} I_{ph} C_{ph} \quad (9)$$

The temperatures dependence of  $C_{ph}$  is more complicated than that predicted by the specific heat term, since  $C_{ph}$  increases with  $T$  whereas the phonon mean free path  $I_{ph}$  decreases with increasing temperature which not only compensates for  $C_{ph}$  but also tends to cause  $k_{ph-L}$  to drop.

In pure metals the electronic contribution to the thermal conductivity tends to dominate at all temperatures. When many defects are present, as in disorganised alloys, they affect  $k_{ph}$  more than  $k_e$  and the phonon contribution can exceed that of the conduction electrons.

#### 4. THERMAL CONDUCTIVITY BELOW $T_c$



Thermal conductivity involves the transport of entropy  $S_\phi$ , super-electrons, however, do not carry entropy nor do they scatter phonons. One also know that below  $T_c$  the entropy of a superconductor drops continuously to zero, so that the thermal conductivities can be expected to decrease towards zero. In high temperature superconductor the phonon contribution to the thermal is predominant above  $T_c$ . The onset of superconductors can have the effect of first increasing the conductivity until it reaches a maximum, beyond which it decreases at lower temperature<sup>12-18</sup>. This increase can occur when the thermal conductivity exits from phonon electron contributing to  $k_{ph}$ . The onset of the superconducting state causes the normal electrons to condense into cooper pairs. Then no longer undergo collisions with the phonons and hence they do not participate in the phonon-electron interaction. The result is a larger mean free path  $l_{ph}$  and a larger conductivity for the uni-mediated sample below  $T_c$ <sup>19</sup>. Irradiating the sample produces defects that limits the mean free paths of the phonon and charge carriers and leads to decrease in the thermal conductivity and a summon of the path.

## 5. RESULTS AND DISCUSSION

In this paper, we have presented a method of evaluation of thermal conductivity of high  $T_c$  superconductor as a function of temperatures. The high  $T_c$  superconductor are  $La_{2-x}Sr_xCuO_4$  of different volumes of  $x$  (as  $x = 0.15$ ,  $T_c = 38K$  and  $x = 0.20$ ,  $T_c = 30K$ ) and  $YBa_2Cu_3O_{7-\delta}$  ( $T_c = 92K$ ). We have compared our theoretical results with that of Graebner<sup>20</sup> and Morelli<sup>21</sup>. Our theoretically evaluated results are in good agreement with these workers. Our theoretical results indicate that thermal conductivities of the above superconductors increase with temperature. As it was pointed out by Uher et al<sup>22</sup> that phonons contribute close to 90% of the thermal conductivity in  $YBa_2Cu_3O_{7-\delta}$  at  $T_c$ . Given the relatively large magnitude of  $T_c$  for  $YBa_2Cu_3O_{7-\delta}$  ( $T_c / \theta_{Debye} \sim 0.25$ ). It is plausible that the transition occurs in a region where the thermal conductivity is limited mainly by phonon-phonon and carriers-phonon scattering. The enhancement of the thermal conductivity above the normal state conductivity for  $T < T_c$  in  $YBa_2Cu_3O_{7-\delta}$  is consistent with this interpretation. It indicates that the phonons make a major contribution to the thermal conductivity and that carrier-phonon scattering is important in limiting the phonon contribution to the thermal conductivity at  $T_c$ . On the other hand the data for  $La_{2-x}Sr_nCuO_4$  are less conclusive. Although phonon makes major contribution<sup>23</sup> to the thermal conductivity at  $T_c$ , no clear enhancement is observed as for  $YBa_2Cu_3O_{7-\delta}$  only a slight change in shape is noticeable at  $T_c$ . It is possible that the enhancement effect is observed in  $La_{2-x}Sr_nCuO_4$  by scattering mechanism other than phonon-carrier scattering. The most likely scattering mechanism limiting the phonon contribution to the thermal conductivity at  $T_c$  ( $T_c / \theta_{Debye} \sim 0.1$ ) are phonon defect, phonon-carrier and phonon-phonon scattering. Again defect refers to boundaries, pores and point defects. If point defects and phonon-phonon scattering overwhelmed the phonon-carrier scattering near  $T_c$  a reduction in phonon-carrier scattering below  $T_c$  will not be noticeable. The magnitude and temperature dependence of the thermal conductivity of  $La_{2-x}Sr_xCuO_4$  and  $La_2CuO_4$  are quite similar. Since  $La_2CuO_4$  is a semiconductor, there should be little phonon-carrier scattering. The similarity between the  $La_{2-x}Sr_nCuO_4$  and  $La_2CuO_4$  data indicates, therefore, that phonon-carrier scattering is not as significant as point defects and phonon-phonon scattering near  $T_c$  in ceramic samples of  $La_{2-n}Sr_nCuO_4$ . An understanding of the scattering mechanisms which lead to the low magnitude of the thermal conductivity for



LaCuO<sub>4</sub> will be important to explain<sup>24,25</sup> the magnitude and temperature behaviour of the thermal conductivity of La<sub>2-x</sub>Sr<sub>x</sub>CuO<sub>4</sub>.

**Table- 1**

**An Evaluation of Thermal Conductivity  $k$  ( $Wcm^{-1}K^{-1}$ ) as a function of temperature for  $YBa_2Cu_3O_{7-\delta}$  ( $T=q_2x$ )**

**$k$  ( $Wcm^{-1}K^{-1}$ )**

T(K)	Ohms	Graebener	Marelli
5	0.025	0.032	0.03
10	0.038	0.036	0.038
20	0.046	0.042	0.049
30	0.058	0.055	0.059
40	0.062	0.060	0.061
50	0.068	0.069	0.070
100	0.072	0.074	0.075
150	0.075	0.078	0.080
200	0.082	0.083	0.086
250	0.096	0.095	0.092

**Table2: Evaluation of Thermal conductivity  $k$  ( $Wcm^{-1}K^{-1}$ ) as a function of temperature  $T$  for  $La_{2-x}Sr_xCuO_4$**

T(K)	$k$ ( $Wcm^{-1}K^{-1}$ )	$k$ ( $Wcm^{-1}K^{-1}$ )
	X = 0.15, T <sub>c</sub> = 38K	X = 0.2, T <sub>c</sub> = 30K
1	0.0052	0.0046
5	0.0067	0.0058
10	0.0072	0.0070
20	0.0087	0.0084
30	0.0096	0.0095
40	0.0127	0.0120
50	0.0138	0.0136



60	0.0252	0.0246
70	0.0286	0.0273
80	0.0329	0.0308
90	0.0468	0.0458
100	0.0587	0.0553

## 6. CONCLUSION

Phonons does play role in the high  $T_c$  superconductor only above  $T_c$ . The studies of thermal conductivities of high  $T_c$  superconductors indicate that the phonons contributions are larger near  $T_c$  and smaller far away.

## REFERENCES

1. C. Noce and L. Maritato, Phys. Rev. 734, 1340(1989)
2. D.R. Nelson and H.S. Seury, Phys, Rev. 1339, 9153(1989)
3. K.H. Miller, Physica C 159, 717 (1989)
4. D.E. Morris et al, Phys. Rev 1340, 11406 (1989)
5. T. McMullen, Phys. Rev 877, 1341(1990)
6. V.G. Kogan, Phys. Rev. 1338, 7049 (1988)
7. D.G. Hinks et al, Natum, 335, 419 (1988)
8. I. Bose, Phys. Rev. 1343, 13602 (1991)
9. P.R. Broussard, Phys. Rev. 1343, 2783 (1991)
10. B. Chakraborty, Phys. Rev. 378, 1343(1991)
11. E.M. Chudnovsky, Phys. Rev. 1343, 7831 (1991)
12. M.P.A. Fisher, Phys. Rev. Lett. (PRL) 65, 923 (1990)
13. R. Hind, Phys. Rev 1345, 5052 (1992)
14. C.S. Hellberg and E.J. Male, Phys. Rev. 646, 1348 (1993)
15. D.R. Harshman and A.P. Mills Jr. Phys. Rev. 1345, 10684 (1992)
16. Z. Iqbal, Supercond. Rev. T, 49 (1992)
17. A.J. Millis and S. N. Coppersmith, Phys. Rev. 1343, 13770 (1991)
18. S. Sachdev, Phys. Rev. 389, 1345 (1992)
19. S. Sergunkor and M. Ausloos, Phys. Rev. 1347, 14476 (1993)



20. J.E. Graebner, B. Golding and L.C. Allen, Phys. Rev. 1334, 5696 (1986)
21. D.T. Morelli, J. Heremans and D.E. Swets, Phys. Rev. 1336, 3912 (1987)
22. C. Uher and A. B. Kaiser, Phys. Rev. 1336, 5680 (1987)
23. D. Shied M. Xu, Phys. Rev. 1349, 4548 (1991)
24. J.S. Shier and D.M. Ginsbey, Phys. Rev. 1339, 2921 (1989)
25. M. Suzuki and M. Hikita, Jpn. J. Appl. Phys. 28, 1368 (1987)