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Higher Order Squeezing in Spontaneous and Stimulated FiveWave-Mixing Process

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ABSTRACT

Higher order squeezing in spontaneous and stimulatedfive-wave mixing optical process has been studied.It is found that squeezing increases non linearly with photon number of the fundamental as well as harmonic mode. It is also shown that selective phase values of the field amplitude give rise to squeezing up to fourth order under short time approximation.

Keywords:Higher order squeezing, Quantum fluctuations, Quantum optics.

1. INTRODUCTION

Squeezed state of electromagnetic field is a non-classical state [1, 2]. Squeezed states are observed theoretically as well as experimentally implemented in various nonlinear optical processes such as harmonic generation [3-5], multi-wave mixing processes [6-8], and optical parametric oscillation [9-11].

Nowadays, the interest to generate higher order amplitude squeezing is mainly due to the rapid development of techniques for making higher-order correlation measurements inquantum information and communication [12, 13] where the field fluctuations in one of the quadrature are reduced below the vacuum noise level, and can be used in overcoming the shot-noise precision restrictions in optical measurements and enhancing the capacity of communication channels [14, 15]. Applications of squeezed states are reported in implementation of continuous cryptography [16], teleportation of coherent states [17] and dense coding [18] etc.

In the present work, we have reported the generation of higher order squeezed state in both spontaneous and stimulated mode using five-wave mixing.

2. HIGHER ORDER SQUEEZING

Higher order squeezing is defined in various ways. Hong and Mandel [19, 20] and Hillery [1] have introduced the notion of higher order squeezing of quantized electromagnetic field as generalization of normal squeezing.Fourth orderamplitudesqueezing is defined in terms of operators Z_1 and Z_2 as

$$Z_1 = \frac{1}{2}\left(A^4 + A^{\dagger 4}\right) \quad \text{and} \quad Z_2 = \frac{i}{2}\left(A^4 - A^{\dagger 4}\right) \tag{1}$$

Where Z_1 and Z_2 are the real and imaginary parts of the fourth order field amplitude, respectively. A and $A^\dagger$ are slowly varying operators defined by $A = ae^{i\omega t}$ and $A^\dagger = a^\dagger e^{-i\omega t}$.

The operators Z_1 and Z_2 obey the commutation relation

$$\left[Z_1, Z_2\right] = i/\ 4\,(16N_{1A}^3\left(t\right) + 24N_{1A}^2\left(t\right) + 56N_{1A}\left(t\right) + 24) \tag{2}$$

This leads to the uncertainty relation

$$\Delta Z_1 \Delta Z_2 \geq 1/\ 4\left\langle \left(16N_{1A}^3\left(t\right) + 24N_{1A}^2\left(t\right) + 56N_{1A}\left(t\right) + 24\right) \right\rangle \tag{3}$$

where N_A is the usual number operator.

Fourth order amplitudesqueezing is said to exist in Z_1 variable

$$(\Delta Z_1)^2 < 1/\ 4\left\langle 16N_{1A}^3\left(t\right) + 24N_{1A}^2\left(t\right) + 56N_{1A}\left(t\right) + 24 \right\rangle \tag{4}$$

Or the squeezing f is

$$f = (\Delta Z_1)^2 - 1/\ 4\left\langle 16N_{1A}^3\left(t\right) + 24N_{1A}^2\left(t\right) + 56N_{1A}\left(t\right) + 24 \right\rangle < 0 \tag{5}$$

3. FIVE-WAVEMIXINGPROCESS

A multiwave mixing process can be viewed in the optics as a process involving multi photon interaction. In this process, the interaction is looked upon as a process which involves the absorption oftwo pump photons, each having frequency ω_1 and emission of twoprobe photons of frequency ω_2,and signal photons of frequency ω_3 where

$2\omega_1 = 2\omega_2 + \omega_3$

The Hamiltonian for this process is given as follows (ħ=1

$$H = \omega_1 a^\dagger a + \omega_2 b^\dagger b + \omega_3 c^\dagger c + g\left(a^2 b^{\dagger 2} c^\dagger + a^{\dagger 2} b^2 c\right) \tag{6}$$

in which g is a coupling constant. A = aexp ($i\omega_1$t), B = bexp ($i\omega_2$t) and C = c exp ($i\omega_3$t) arethe slowly varying operators at frequencies ω_1, ω_2 and ω_3, $a(a^\dagger)$, $b(b^\dagger)$ and $c(c^\dagger)$ are the usual annihilation (creation) operators, respectively. The Heisenberg equation of motion for fundamental mode A is given

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + i\left[H, A\right] \tag{7}$$

By using the short-time approximation technique, we expand $A(t)$ by using Taylor's series expansion and retaining the terms up to g^2t^2 as

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$$A(t) = A - 2igtA^{\dagger}B^{2}C + g^{2}t^{2}\left[2AN_{B}^{2}N_{C} - A^{\dagger}A^{2}N_{B}^{2} - 4A^{\dagger}A^{2}N_{B}N_{C} - 4A^{\dagger}A^{2}N_{B} - 2A^{\dagger}A^{2}N_{C} - 2A^{\dagger}A^{2}\right] \tag{8}$$

Where $N_A = A^{\dagger}A,\ N_B = B^{\dagger}B$ and $N_C = C^{\dagger}C$.

Using Equations (8) number of photons in mode A may be expressed

$$\begin{aligned} N_{1A}(t) &= A^{\dagger}(t)A(t) \\ &= A^{\dagger}A - 2g^{2}t^{2}A^{\dagger 2}A^{2}\left(B^{\dagger 2}B^{2} + 4B^{\dagger}B + 2\right) \end{aligned} \tag{9}$$

$$\begin{aligned} N_{1A}^{2}(t) &= N_{1A}(t)N_{1A}(t) \\ &= A^{\dagger 2}A^{2} + A^{\dagger}A - 4g^{2}t^{2}\left(A^{\dagger 3}A^{3} + 2A^{\dagger 2}A^{2}\right)\left(B^{\dagger 2}B^{2} + 4B^{\dagger}B + 2\right) \end{aligned} \tag{10}$$

$$\begin{aligned} N_{1A}^{3}(t) &= N_{1A}(t)N_{1A}(t) \\ &= A^{\dagger 3}A^{3} + 3A^{\dagger 2}A^{2} + A^{\dagger}A - 6g^{2}t^{2}\left(A^{\dagger 4}A^{4} + 5A^{\dagger 3}A^{3} + 4A^{\dagger 2}A^{2}\right)\left(B^{\dagger 2}B^{2} + 4B^{\dagger}B + 2\right) \end{aligned} \tag{11}$$

3.1 SPONTANEOUS FOURTH ORDER AMPLITUDE SQUEEZING

Initially, we consider the quantum state of the field amplitude as a product of coherent state for the fundamental mode A and the vacuum state for modes B and C

$$|\psi\rangle = |\alpha\rangle_A\, |0\rangle_B\, |0\rangle_C \tag{12}$$

Using Equations (8) and (12) the fourth-order amplitude of the fundamental mode is expressed as

$$\begin{aligned} Z_{1A}(t) &= \frac{1}{2}\left[A^{4}(t) + A^{\dagger 4}(t)\right] \\ &= \frac{1}{2}\left[A^{4} + A^{\dagger 4} - g^{2}t^{2}\left(8A^{\dagger}A^{5} + 12A^{4} + 8A^{\dagger 5}A + 12A^{\dagger 4}\right)\right] \end{aligned} \tag{13}$$

Using Equations (12) and (13), we get the expectation values as

$$\begin{aligned} \left\langle \psi \left| Z_{1A}^{2}(t) \right| \psi \right\rangle = \frac{1}{4}\Big[& \alpha^{8} + \alpha^{*8} + 2|\alpha|^{8} + 16|\alpha|^{6} + 72|\alpha|^{4} + 96|\alpha|^{2} + 24 - g^{2}t^{2}(+56\alpha^{8} + 56\alpha^{*8} \\ & + 16\alpha^{8}|\alpha|^{2} + 16\alpha^{*8}|\alpha|^{2} + 32|\alpha|^{10} + 368|\alpha|^{8} + 2304|\alpha|^{6} + 5568|\alpha|^{4} \\ & + 4224|\alpha|^{2} + 576)\Big] \end{aligned} \tag{14}$$

and

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$$\left\langle\psi\left|Z_{1A}\left(t\right)\right|\psi\right\rangle^{2}=\frac{1}{4}\Big[\alpha^{8}+\alpha^{*8}+2\left|\alpha\right|^{8}-2g^{2}t^{2}(12\alpha^{8}+12\alpha^{*8}+8\alpha^{8}\left|\alpha\right|^{2}+8\alpha^{*8}\left|\alpha\right|^{2}+24\left|\alpha\right|^{8}+16\left|\alpha\right|^{10})\Big] \tag{15}$$

Therefore

$$\left[\Delta Z_{1A}\left(t\right)\right]^{2}=\frac{1}{4}\Big[16\left|\alpha\right|^{6}+72\left|\alpha\right|^{4}+96\left|\alpha\right|^{2}+24-g^{2}t^{2}(32\alpha^{8}+32\alpha^{*8}+320\left|\alpha\right|^{8}+2304\left|\alpha\right|^{6}+5568\left|\alpha\right|^{4}+4224\left|\alpha\right|^{2}+576)\Big] \tag{16}$$

Using Equations (9), (10), (11) and (12) a straightforward but strenuous calculation yields

$$\begin{aligned}&\left[\Delta Z_{1A}\left(t\right)\right]^{2}-\frac{1}{4}\left\langle 16N_{1A}^{3}\left(t\right)+24N_{1A}^{2}\left(t\right)+56N_{1A}\left(t\right)+24\right\rangle\\&=-8g^{2}t^{2}[2\left|\alpha\right|^{8}(\cos 8\theta+2)+36\left|\alpha\right|^{6}+131\left|\alpha\right|^{4}+132\left|\alpha\right|^{2}+18]\end{aligned} \tag{17}$$

The right hand side of Equation (17) is negative, indicating that squeezing occurs in fourth order fieldamplitude in the fundamental mode in five wave mixing.

3.2 STIMULATED FOURTH ORDER AMPLITUDE SQUEEZING

Initially, we consider the quantum state of the field amplitude as a product of coherent state for the fundamental mode A and harmonic mode for B and vacuum state for mode C i.e.

$$\left|\psi\right\rangle=\left|\alpha\right\rangle_{A}\left|\beta\right\rangle_{B}\left|0\right\rangle_{C} \tag{18}$$

Using Equations (8) and (18) the fourth-order amplitude of the fundamental mode is expressed as

$$\begin{aligned}F_{1A}\left(t\right)&=\frac{1}{2}\left[A^{4}\left(t\right)+A^{\dagger 4}\left(t\right)\right]\\&=\frac{1}{2}\left[A^{4}+A^{\dagger 4}-g^{2}t^{2}\left(4A^{\dagger}A^{5}+6A^{4}+4A^{\dagger 5}A+6A^{\dagger 4}\right)\left(B^{\dagger 2}B^{2}+4B^{\dagger}B+2\right)\right]\end{aligned} \tag{19}$$

Using Equations (18) and (19), we get the expectation values as

$$\begin{aligned}\left\langle\psi\left|F_{1A}^{2}\left(t\right)\right|\psi\right\rangle=\frac{1}{4}\Big[&\alpha^{8}+\alpha^{*8}+2\left|\alpha\right|^{8}+16\left|\alpha\right|^{6}+72\left|\alpha\right|^{4}+96\left|\alpha\right|^{2}+24-g^{2}t^{2}(+28\alpha^{8}+28\alpha^{*8}\\&+8\alpha^{8}\left|\alpha\right|^{2}+8\alpha^{*8}\left|\alpha\right|^{2}+16\left|\alpha\right|^{10}+184\left|\alpha\right|^{8}+1152\left|\alpha\right|^{6}+2784\left|\alpha\right|^{4}\\&+2112\left|\alpha\right|^{2}+288)(\left|\beta\right|^{4}+4\left|\beta\right|^{2}+2)\Big]\end{aligned} \tag{20}$$

and

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$$\left\langle \psi \left| F_{1A}(t) \right| \psi \right\rangle^2 = \frac{1}{4}\Big[\alpha^8 + \alpha^{*8} + 2|\alpha|^8 - g^2t^2(12\alpha^8 + 12\alpha^{*8} + 8\alpha^8|\alpha|^2 + 8\alpha^{*8}|\alpha|^2 + 24|\alpha|^8 + 16|\alpha|^{10})(|\beta|^4 + 4|\beta|^2 + 2)\Big] \tag{21}$$

Therefore

$$\left[\Delta F_{1A}(t)\right]^2 = \frac{1}{4}\Big[16|\alpha|^6 + 72|\alpha|^4 + 96|\alpha|^2 + 24 - g^2t^2(16\alpha^8 + 16\alpha^{*8} + 160|\alpha|^8 + 1152|\alpha|^6 + 2784|\alpha|^4 + 2112|\alpha|^2 + 288)(|\beta|^4 + 4|\beta|^2 + 2)\Big] \tag{22}$$

Using Equations (9), (10), (11) and (18) a straightforward but strenuous calculation yields

$$\left[\Delta F_{1A}(t)\right]^2 - \frac{1}{4}\left\langle 16N_{1A}^3(t) + 24N_{1A}^2(t) + 56N_{1A}(t) + 24\right\rangle = -4g^2t^2\Big[2|\alpha|^8(\cos 8\theta + 2) + 36|\alpha|^6 + 131|\alpha|^4 + 132|\alpha|^2 + 18\Big]\Big(|\beta|^4 + 4|\beta|^2 + 2\Big) \tag{23}$$

The right hand side of Equation (23) is negative, indicating that squeezing occurs in fourth order of field amplitude in the fundamental mode in five wave mixing.

4. RESULT

The presence of squeezing in spontaneous and stimulated five-wave mixing is shown in Equations (17) and (23) respectively. Figures 1 and 2 show that squeezingincrease nonlinearly with $|\alpha|^2$ and hence increase the non-classicality of the field amplitude. Further it is shown from Equations (17) and (23) that by varying the values of initial phase of the coherent state (θ), number of photons present in the radiation field prior to the interaction ($|\alpha|^2$) and the interaction time (t) the degree of higher order squeezing present in the system can be tuned.

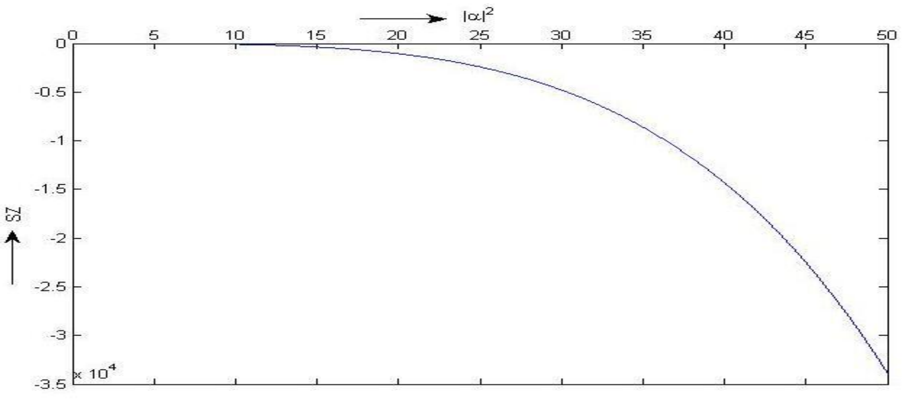

Figure 1. Dependence of spontaneous fourth order amplitudesqueezing on $|\alpha|^2$.

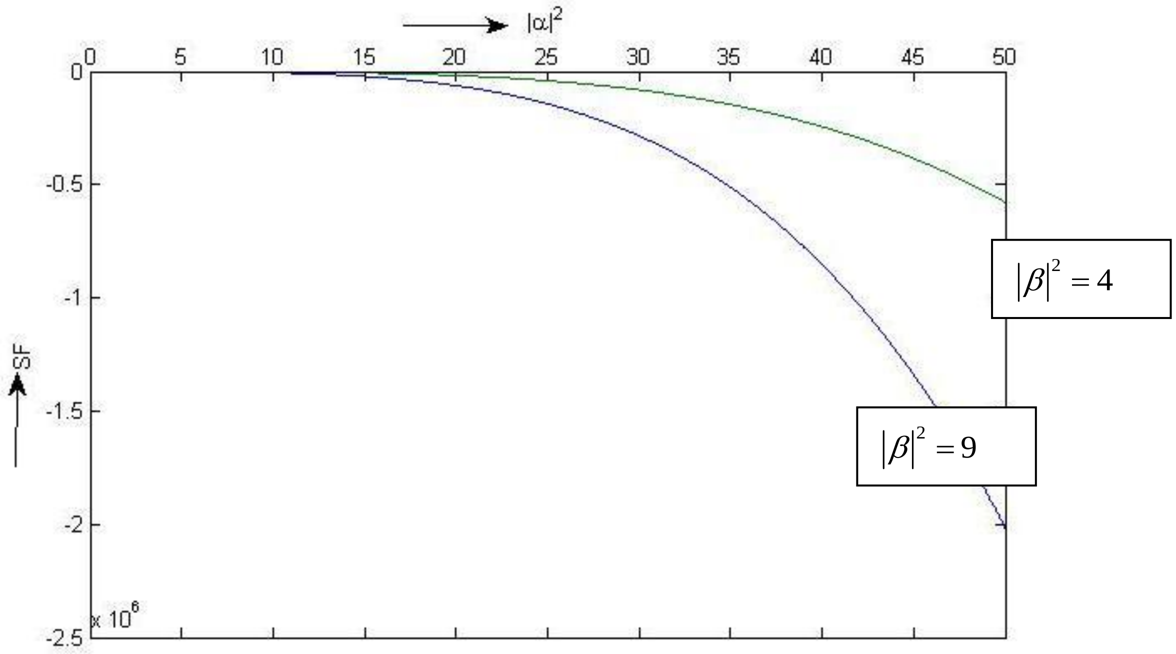

Figure 2. Dependence of stimulated fourth order amplitude squeezing on $|\alpha|^2$ and $|\beta|^2$

6. CONCLUSION

The results show the presence of higher order squeezing in spontaneous and stimulated five-wave mixing process. From figures 1 and 2 we can conclude that the degree of squeezing directly depends upon the photon number of the fundamental mode as well as on the harmonic mode. A comparison between results of spontaneous and stimulated processes shows the occurrence of multiplication factor ($|\beta|^4 + 4|\beta|^2 + 2$). Thus, it implies that squeezing in the fundamental mode in stimulated interaction is greater than corresponding squeezing in spontaneous interaction.

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