

The Art of Mathematical Modeling

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We all learn mathematics at different levels starting from school to college. We are taught the basic techniques of mathematics at all stages. In this process the mathematical techniques take the primary importance and the mathematical modeling takes the secondary part. We are going to discuss here how there is a need for Mathematical modeling to take primary importance and the mathematical techniques or tools to take a secondary importance.

What is Mathematical modeling ?

For example if we say the height of a person y is 2 times the height of a person x then we take the help of the mathematical technique to model the data by writing the equation $y = 2x$. Suppose we have to find the width of a river. We are not going to actually cross the river for finding its width. We use simple Trigonometry to find its width. We try to express the width of the river in terms of some distances and angles measured from our side. In physics we get the equations of motion connecting the initial and final velocities u and v with the distance covered as S during the time t as mentioned below:

$$V = u + at, \quad v^2 = u^2 + 2as, \quad S = ut + \frac{1}{2}(at^2) .$$

We may try to find the weight of the earth. We cannot be using a balance to find the weight of the earth. We use the available data to model the situation and get the weight of the earth. Mathematical modeling basically consists of expressing the real world problems in terms of mathematical problems and solving them to interpret the results in the language of the real world.

Sometimes it is very difficult to model the real world problems. Most often we face non-linear equations. All problems of mechanical engineering are always non-linear. From the data obtained by the performance of an experiment we try to construct a model to get the expected theoretical values. The non-linear equations are linearized in a small interval to get the expected values. At a small interval of our interested study we assume the data are on a straight line. Thus we get the line of best fit. For larger intervals we assume a parabolic fit. Sometimes we use an exponential fit. These expected theoretical values are compared with the actual experimental values. Depending upon the difference between these two values we try to minimize the error using the method of least squares. The lesser the difference between the expected values and the actual values the greater the accuracy of the model.

We also have the technique of using the numerical methods for some complicated equations. The most important fact here is to locate the approximate root of the given equation. For example when somebody wants to reach a particular location in a particular city. The first step is to travel in the right direction to reach the city. This is the first step of iteration towards the solution. Then the second step is to find out the direction for reaching a land mark in that particular location. Reaching this land mark is the second step of iteration. From this land mark reaching the particular location by taking many right and left turns is the third step. This is the third step of iteration. Our particular location is the required solution here. In each step we are going nearer and nearer to the solution. This is how the numerical technique works. The more the number of iterations the more accurate the solution would be.

Mathematical modeling also plays a predominant role in political science also. They use Graph theory in the form of tree diagrams for the result analysis. Win-win situations, win-lose situations are all modeled by weighted graphs. Combinatorics is used for modeling and predicting the results of outcome. Theory of clusters is an important modern technique to predict the success of candidates contesting from different groups. The principle of inclusion and exclusion is used for finding out the number of viewers of the T.V programs in different channels. Computer networking also use the Pigeon-hole principle. Putting it in the simplest form “If there are three pigeons and there are only two pigeon holes then at least one pigeon hole must accommodate more than one pigeon.” We illustrate an example where there are thirteen children in a group. At least two of them have their birthdays in the same month. The twelve months correspond to the twelve pigeon holes. The thirteen children correspond to the thirteen pigeons. Hence at least two of them will have their birthdays in the same month.

The differential equations play a vital role in mathematical modeling. The simple harmonic motion is governed by an ordinary differential equation. Diffusion of glucose or medicine in the blood stream are all governed by ordinary differential equations. The structural engineers often study the deflection of beams using the fourth order differential equation. The Laplace transformations make the solving of differential equations easier. The Sturm-Liouville differential equations help to solve the eigen-value problems. The differential equations play a significant role in finding the shortest distance between two space curves, finding a closed curve with a given perimeter and enclosing the maximum area, finding the time of shortest descent by a bead along a space curve are all examples cited from the Calculus of variations.

Mathematical modeling unifies the different subjects of Applied mathematics. The subjects may be different but the model may be common for them. The mathematical model for one field may be equally valid for another field. But care must be taken not to thrust a model on a field unless it is really applicable there. Though mathematical models give theoretical results only those models are acceptable which can explain, predict or control situations. Some times we have to keep on improving the models to get best possible results. Ultimately it requires lot of experience and a deep insight in the subject and situation which is to be modeled.

Though there are many more examples of real world problems which can be modeled mathematically only a very few are cited here briefly.

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