'USEFUL' FUZZY DIVERGENCE MEASURE AND ITS PROPERTIES

Saima Manzoor Sofi[1], Safeena Peerzada[2] and M.A.K. Baig[3]

P.G. Department of Statistics, University of Kashmir, Srinagar, India-190006

ABSTRACT

In this paper, we introduce a new measure of 'useful' fuzzy divergence having order α and type β. We check the validity of the proposed measure and also discuss the important properties of this measure. The results are verified by using R Software.

Keywords: *fuzzy sets, 'useful' fuzzy, 'useful' fuzzy divergence.*

1. Introduction

Divergence measures are basically introduced to measure or compare the distance between any two probability distributions. A variety of divergence measures have been introduced which are applicable in a number of fields such as analysis of contingency table, pattern recognition, economics and political science, biology, signal processing, etc. In fuzzy mathematics, the divergence measures are used as fuzzy directed divergence and are also used to obtain the utility of an event that is, how much useful an event is in comparison to other event.

Various fuzzy divergence measures have been introduced by several authors to measure the discrepancy between two fuzzy sets. The first measure of directed divergence was given by Kullback and Leibler [1]. Corresponding to this, Bhandari and Pal [2] gave the measure of fuzzy directed divergence as:

$$I(A,B)=\sum_{i=1}^{n}\left[\mu_A(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}+(1-\mu_A(x_i))\log\frac{1-\mu_A(x_i)}{1-\mu_B(x_i)}\right]$$

Bhandari and Pal [2] also gave the measure of fuzzy symmetric divergence as:

$$J(A,B)=I(A,B)+I(B,A)$$

Hooda and Bajaj [3] defined the following 'useful' fuzzy directed divergence by considering together the concept of fuzziness and probability with utility:

$$I(A,B;P;U)=\frac{\sum_{i=1}^{n}u_i\,p_i\left[\mu_A(x_i)\log\frac{\mu_A(x_i)}{\mu_B(x_i)}+(1-\mu_A(x_i))\log\frac{1-\mu_A(x_i)}{1-\mu_B(x_i)}\right]}{\sum_{i=1}^{n}u_i\,p_i};u_i>0\ \ \forall i$$ Kapur [4], Fan, Ma and Xie [5], Parkash and Sharma [6], Anshu Ohlan [7] etc. have proposed different measures of fuzzy directed divergence.

This paper is divided into four sections. Section 1 corresponds to the introduction where the basic concepts are presented. In section 2, a new measure of 'useful' fuzzy divergence measure is defined and its properties are given in the section 3. Lastly, in section 4 conclusion of the paper is presented.

2. New 'Useful' Fuzzy Divergence Measure

We define a two parametric 'useful' fuzzy divergence measure of order α and type β corresponding to the 'useful' fuzzy information measure given by Saima Manzoor Sofi et al. [8]:

$$I(A,B;U)=-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_B(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_B(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]$$

$$\alpha\geq 0,\alpha\neq 1,\ 0<\beta\leq 1\ \&\ u_i>0 \qquad (1)$$

and 'useful' fuzzy symmetric divergence measure as:

$$J(A,B;U)=I(A,B;U)+I(B,A;U)$$

The above defined measure is a valid measure of 'useful' fuzzy divergence if it satisfies the following properties:

(i) $I(A,B;U)\geq 0.$

(ii) $I(A,B;U)=0$ if $\mu_A(x_i)=\mu_B(x_i)$

(iii) $I(A,B;U)\neq I(B,A;U)$

(iv) $I(A,B;U)$ should remain same even if $\mu_A(x_i)$ is replaced by $1-\mu_A(x_i)$ and $\mu_B(x_i)$ by $1-\mu_B(x_i)$

(v) $I(A,B;U)$ is convex i.e., $\frac{\partial^2 I(A,B;U)}{\partial \mu_A^2(x_i)} > 0$ & $\frac{\partial^2 I(A,B;U)}{\partial \mu_B^2(x_i)} > 0$ for $\alpha \geq 0, \alpha \neq 1,\ 0 < \beta \leq 1 \& u_i > 0$

Example: We verify the above properties in the tables given below by considering two fuzzy sets *A* & *B*:

Table 1

$\mu_A(x_i)$	$\mu_B(x_i)$	u_i	α	β	$I(A,B;U)$
0.65	0.42	1	0.53	0.79	0.2271539
0.23	0.28	2			
0.82	0.05	3			
0.44	0.90	4			
0.97	0.73	5			

Table 2

$\mu_A(x_i)$	$\mu_B(x_i)$	u_i	α	β	$I(A,B;U)$
0.65	0.65	1	0.53	0.79	0.0
0.23	0.23	2			
0.82	0.82	3			
0.44	0.44	4			
0.97	0.97	5			

From Table (1) & (2), it is clear that $I(A,B;U)$ is non-negative (i.e., $I(A,B;U) > 0$) & $I(A,B;U) = 0$ for $\mu_A(x_i) = \mu_B(x_i)$, respectively.

International Journal of Advance Research in Science and Engineering
Volume No.07, Special Issue No.04, March 2018
www.ijarse.com
IJARSE
ISSN: 2319-8354

Table 3

$\mu_A(x_i)$	$\mu_B(x_i)$	u_i	α	β	$I(A,B;U)$	$I(B,A;U)$
0.65	0.42	1				
0.23	0.28	2				
0.82	0.05	3	0.53	0.79	0.2271539	0.2373578
0.44	0.90	4				
0.97	0.73	5				

From Table (3), it is obvious that $I(A,B;U) \neq I(B,A;U)$

Table 4

$\mu_A(x_i)$	$1-\mu_A(x_i)$	$\mu_B(x_i)$	$1-\mu_B(x_i)$	u_i	α	β	$I(A,B;U)$
0.65	0.35	0.42	0.58	1			
0.23	0.77	0.28	0.72	2			
0.82	0.18	0.05	0.95	3	0.53	0.79	0.2271539
0.44	0.56	0.90	0.10	4			
0.97	0.03	0.73	0.27	5			

It is clear from Table (4), that we get the same value for $I(A,B;U)$ if we exchange $\mu_A(x_i)$ by $1-\mu_A(x_i)$ and $\mu_B(x_i)$ by $1-\mu_B(x_i)$.

Table 5

$\mu_A(x_i)$	$\mu_B(x_i)$	u_i	α	β	$\frac{\partial^2 I(A,B;U)}{\partial \mu_A^2(x_i)}$	$\frac{\partial^2 I(A,B;U)}{\partial \mu_B^2(x_i)}$
0.65	0.42	1			0.2391	0.1322
0.23	0.28	2			0.3454	0. 2348
0.82	0.05	3	0.53	0.79	0.9840	5.2471
0.44	0.90	4			0.3291	0.5371
0.97	0.73	5			6.8999	0. 3499

International Journal of Advance Research in Science and Engineering
Volume No.07, Special Issue No.04, March 2018
www.ijarse.com
IJARSE
ISSN: 2319-8354

Table (5) implies the convexity of the proposed measure that is, $\frac{\partial^2 I(A,B;U)}{\partial \mu_A^2(x_i)}>0$ & $\frac{\partial^2 I(A,B;U)}{\partial \mu_B^2(x_i)}>0$.

Hence, we conclude from the results of above tables that the measure defined in (1) satisfies property (1) to (5). Thus, the measure is a valid 'useful' fuzzy divergence measure of order α and type β.

In particular, we have

1. For $\alpha=0$ & $\beta=1, I(A,B;U)=0.$
2. For $\beta=0, I(A,B;U)=0.$
3. For $u_i=1 \forall i=1,2,...,n$, $I(A,B;U)$ tends to the fuzzy divergence measure given by Safeena Peerzada et al. [9].

3. Some More Properties of 'Useful' Fuzzy Divergence Measure

In addition to the above properties, the measure (1) satisfies the following properties:

(a) $I(A\cup B,A;U)+I(A\cap B,A;U)=I(B,A;U)$

Proof: Suppose $X_1 \& X_2$ are two fuzzy sets defined as $X_1=\{x/x\in X_i, \mu_A(x_i)\geq \mu_B(x_i)\}$ & $X_2=\{x/x\in X_i, \mu_B(x_i)>\mu_A(x_i)\}$.

In set X_1, we have

$$\mu_{A\cup B}(x)=\max\{\mu_A(x_i),\mu_B(x_i)\}=\mu_A(x_i) \;\&\; \mu_{A\cap B}(x)=\min\{\mu_A(x_i),\mu_B(x_i)\}=\mu_B(x_i)$$

In set X_2, we have

$$\mu_{A\cup B}(x)=\max\{\mu_A(x_i),\mu_B(x_i)\}=\mu_B(x_i) \;\&\; \mu_{A\cap B}(x)=\min\{\mu_A(x_i),\mu_B(x_i)\}=\mu_A(x_i)$$

We have from (1)

$$I(A,B;U)=-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_B(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_{\mathbf{B}}(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]$$

Now, consider $I(A\cup B,A;U)+I(A\cap B,A;U)=$

$$-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_{A\cup B}(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{A\cup B}(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]+$$

$$\left[-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_{A\cap B}(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{A\cap B}(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\left[\left(\frac{\sum_{X_1}u_i\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right)+\left(\frac{\sum_{X_2}u_i\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right)\right]$$

$$-\frac{\beta}{1-\alpha}\log_D\left[\left(\frac{\sum_{X_1}u_i\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right)+\left(\frac{\sum_{X_2}u_i\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right)\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{X_2}u_i\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]$$

$$-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{X_1}u_i\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]$$

$$= -\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{X_i} u_i\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)} + (1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n} u_i}\right]$$

$$= I(B, A; U)$$

This establishes (a).

(b) $I(\overline{A\cup B};U, \overline{A\cap B};U) = I(\bar{A}\cap\bar{B};U, \bar{A}\cup\bar{B};U)$

Proof: Consider $I(\overline{A\cup B};U, \overline{A\cap B};U) =$

$$-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n} u_i\left\{\mu_{\overline{A\cup B}}(x_i)^{\beta(1-\alpha)}\mu_{\overline{A\cap B}}(x_i)^{1-\beta(1-\alpha)} + (1-\mu_{\overline{A\cup B}}(x_i))^{\beta(1-\alpha)}(1-\mu_{\overline{A\cap B}}(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n} u_i}\right] \quad (2)$$

From R.H.S., we have

$I(\bar{A}\cap\bar{B};U, \bar{A}\cup\bar{B};U) =$

$$-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n} u_i\left\{\mu_{\bar{A}\cap\bar{B}}(x_i)^{\beta(1-\alpha)}\mu_{\bar{A}\cup\bar{B}}(x_i)^{1-\beta(1-\alpha)} + (1-\mu_{\bar{A}\cap\bar{B}}(x_i))^{\beta(1-\alpha)}(1-\mu_{\bar{A}\cup\bar{B}}(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n} u_i}\right]$$

$$= -\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n} u_i\left\{\mu_{\overline{A\cup B}}(x_i)^{\beta(1-\alpha)}\mu_{\overline{A\cap B}}(x_i)^{1-\beta(1-\alpha)} + (1-\mu_{\overline{A\cup B}}(x_i))^{\beta(1-\alpha)}(1-\mu_{\overline{A\cap B}}(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n} u_i}\right] \quad (3)$$

Taking (2) and (3) together, we get (b).

(c) $I(A\cup B, C;U) + I(A\cap B, C;U) = I(A, C;U) + I(B, C;U)$

Proof: L.H.S. =

IJARSE
ISSN: 2319-8354

$$-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_{A\cup B}(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{A\cup B}(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]+$$

$$\left[-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_{A\cap B}(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{A\cap B}(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\left[\left(\frac{\sum_{X_1}u_i\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right)+\left(\frac{\sum_{X_2}u_i\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right)\right]$$

$$-\frac{\beta}{1-\alpha}\log_D\left[\left(\frac{\sum_{X_1}u_i\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right)+\left(\frac{\sum_{X_2}u_i\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right)\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]+$$

$$\left[-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_B(x_i)^{\beta(1-\alpha)}\mu_C(x_i)^{1-\beta(1-\alpha)}+(1-\mu_B(x_i))^{\beta(1-\alpha)}(1-\mu_C(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]\right]$$

$$=I(A,C;U)+I(B,C;U)$$

(d) $I(A,\overline{A};U)=I(\overline{A},A;U)$

Proof: L.H.S.:

$$I(A,\overline{A};U)=-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_{\overline{A}}(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_{\overline{A}}(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_A(x_i)^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]$$

(4)

R.H.S.:

$$I(\overline{A},A;U)=-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_{\overline{A}}(x_i)^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{\overline{A}}(x_i))^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{(1-\mu_A(x_i))^{\beta(1-\alpha)}\mu_A(x_i)^{1-\beta(1-\alpha)}+\mu_A(x_i)^{\beta(1-\alpha)}(1-\mu_A(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]$$

(5)

Comparing (4) and (5), we get L.H.S. = R.H.S.

(e) $I(\overline{A},\ \overline{B};U)=I(A,B;U)$

Proof: We have

$$I(\overline{A},\ \overline{B};U)=-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_{\overline{A}}(x_i)^{\beta(1-\alpha)}\mu_{\overline{B}}(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{\overline{A}}(x_i))^{\beta(1-\alpha)}(1-\mu_{\overline{B}}(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_B(x_i))^{1-\beta(1-\alpha)}+\mu_A(x_i)^{\beta(1-\alpha)}\mu_B(x_i)^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]$$

$$=I(A,B;U)$$

(f) $I(A,\overline{B};U)=I(\overline{A},B;U)$

Proof: We have

$$I(A,\overline{B};U)=-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_A(x_i)^{\beta(1-\alpha)}\mu_{\overline{B}}(x_i)^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}(1-\mu_{\overline{B}}(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_A(x_i)^{\beta(1-\alpha)}(1-\mu_B(x_i))^{1-\beta(1-\alpha)}+(1-\mu_A(x_i))^{\beta(1-\alpha)}\mu_B(x_i)^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right] \quad (6)$$

$$I(\overline{A},B;U)=-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_i\left\{\mu_{\overline{A}}(x_i)^{\beta(1-\alpha)}\mu_B(x_i)^{1-\beta(1-\alpha)}+(1-\mu_{\overline{A}}(x_i))^{\beta(1-\alpha)}(1-\mu_B(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right]$$

$$=-\frac{\beta}{1-\alpha}\log_D\left[\frac{\sum_{i=1}^{n}u_l\left\{(1-\mu_A(x_i))^{\beta(1-\alpha)}\mu_B(x_i)^{1-\beta(1-\alpha)}+\mu_A(x_i)^{\beta(1-\alpha)}(1-\mu_B(x_i))^{1-\beta(1-\alpha)}\right\}}{\sum_{i=1}^{n}u_i}\right] \quad (7)$$

Comparing (6) and (7), we get $I(A,\overline{B};U)=I(\overline{A},B;U)$.

(g) $I(A,B;U)+I(\overline{A},B;U)=I(\overline{A},\overline{B};U)+I(A,\overline{B};U)$.

IJARSE
ISSN: 2319-8354

Proof: It is obvious from (e) and (f) that (g) holds.

4. CONCLUSION

The measure of ‘useful’ fuzzy directed divergence is studied in the present paper. The important properties of this measure are also given. Further, R-Software is used to give numerical illustration.

REFERENCES:

[1] S. Kullback and R.A. Leibler, On Information and Sufficiency, *Annals of Mathematical Statistics, 22,* 1951, 79-86.

[2] D. Bhandari and N.R. Pal, Some New Information Measures For Fuzzy Sets, *Information Sciences, 67,* 1993, 209-228.

[3] D.S. Hooda and R.K. Bajaj, ‘Useful’ Fuzzy Measures of Information, Integrated Ambiguity and Directed Divergence, *International Journal Of General Systems, 39(6),* 2010, 647-658.

[4] J.N. Kapur, *Measures Of Fuzzy Information* Mathematical Sciences Trust Society, New Delhi 1997.

[5] J.L. Fan, Y.L. Ma and W.X. Xie, On Some Properties of Distance Measure, *Fuzzy Sets and Systems, 117,* 2001, 355-361.

[6] O. Parkash and P.K. Sharma, Some New Measures of Fuzzy Directed Divergence and Their Generalization, *Journal of the Korean Society of Mathematical Education Series B, 12,* 2005, 307-315.

[7] Anshu Ohlan, Overview on Development of Fuzzy Information Measures, *International Journal of All Research Education and Scientific Methods, 12(4)*, 2016, 17-21.

[8] Saima Manzoor Sofi, Safeena Peerzada, M.A.K. Baig and A. H. Bhat, A New Generalized ‘Useful’ Fuzzy Information Measure and its Properties. *(communicated)*

[9] S. Peerzada, Saima Manzoor Sofi and M.A.K. Baig, A New Two Parametric Fuzzy Divergence Measure and Its Properties. *(communicated)*