International Journal of Advance Research in Science and Engineering
Volume No.07, Special Issue No.04, March 2018
www.ijarse.com
IJARSE
ISSN: 2319-8354

Statistical Properties and Applications of Generalized Poisson-Lindley Distribution

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ABSTRACT

A Generalized Two parameter Poisson Lindley Distribution has been obtained by Shanker and Mishra (2014) on compounding the Poisson distribution with the Lindley distribution. This class of distributions contains several distributions such as negative binomial, one parameter Poisson- Lindley and Geometric distribution as special cases. In this paper an attempt has been made to discuss various statistical properties viz; raw and central moments, skewness, kurtosis and index of dispersion of the model. The expression for probability generating function and moment generating function has also been derived. Further estimation of the parameters is discussed using the method of moments and maximum likelihood estimators. The usefulness of the new model is illustrated by means of two real data sets. Examples are given by fitting of this distribution to data, and the fit is compared with that obtained by using other distributions.

Keywords: Generalized Poisson-Lindley Distribution; Gamma distribution, Method of estimation, Moment generating function, Skewness and Kurtosis.

I INTRODUCTION

In the recent past, in order to model count data researchers proposed many distributions. The interested variable in many branches like Insurance, Economics, Engineering, Biometrices etc. is the count variable. Though traditional models like Poisson, negative binomial, Geometric and their generalizations were utilized to measure count data but it is found that these models fails to exhibit the property of right tail behaviour of the data set.Lindley (1958) introduced one parameter Lindley distribution in the context of fiducial distribution and Bayesian statistics. Sankaran (1970) introduced discrete Poisson- Lindley distribution by assuming the Poisson parameter to follow a Lindley distribution. Borah and Begum (2000) studied certain properties of Poisson-Lindley and its derived distributions. Borah and DekaNath (2001) studied the inflated Poisson- Lindley distribution. Ghitany, et. al (2008a, 2008b) studied Lindley and Zero-truncated Poisson- Lindley distribution. Ghitanyet. al (2008) studied some statistical properties of Lindley distribution and showed that it is much more flexible than exponential distribution and thus it maybe concluded that Lindley distribution is better than exponential distribution. Shaker et al (2012) introduced the probability mass function (pmf) of two- parameter Poisson- Lindley (TPL) distribution

$$P_r^1(X=x) = \frac{\theta^2}{(\theta+1)^{x+2}}\left[1+\frac{\alpha x+1}{\theta+\alpha}\right],\ \ x=0,1,2,\ \ldots,\ \theta>0, \alpha>-\theta,$$

which had been derived by compounding a Poisson distribution with a two- parameter Lindley distribution with probability density function (pdf)

$$f_1(x;\ \alpha,\theta)=\frac{\theta^2}{\alpha+\theta}(1+\alpha x)e^{-\theta x}, x>0, \theta>0, \alpha>-\theta.$$

Researchers like Adil, Zahoor and Jan obtained several compound distributions for instance, (2013) a compound of zero truncated generalized negative binomial distribution with generalized beta distribution, (2014a) they obtained compound of Geeta distribution with generalized beta distribution and (2014b) compound of Geeta distribution with generalized beta distribution reecently Adil and Jan (2014c) explored a mixture of generalized negative binomial distribution with that of generalized exponential distribution which contains several compound distributions as its sub cases and proved that this particular model is better in comparison to others when it comes to fit observed count data set. Most recently Adil and Jan (2015,) developed a new count data models that can be used as a tool for modeling overdispersion.

In this paper certain properties of the Generalized two parameter Poisson Lindley (GTPL) distribution have been investigated. The methods of estimation of its parameters have also been discussed. The model has been applied to real data set to test its goodness of fit. A comparison has been made between GTPL and TPL distributions based on χ^2 –test.

II Probability Function of Generalized Poisson Lindley Distribution

The p.m.f of the generalized Poisson Lindley distribution due to Shanker and Mishra (2014) is given by:

$$f(x;\ \theta,\alpha)=\frac{\theta^2}{(\theta+1)^{x+1}(\alpha\theta+1)}\left[\alpha+\frac{x+1}{\theta+1}\right]$$

Particular cases:

(i) For $\alpha=0$, it reduces to negative binomial distribution with parameter $r=2$ and $p=\frac{\theta}{\theta+1}$ .For $\alpha=1$, it reduces to one parameter Poisson- Lindley distribution.

(ii) For $\alpha\to\infty$, it reduces to geometric distribution with parameter $\frac{\theta}{\theta+1}$.

We have,

$$\frac{d}{dx}f(x;\ \theta,\alpha)=\frac{-\theta^2}{(\alpha\theta+1)}\{log(\theta+1)(\alpha\theta+\alpha+x+1)-1\}.$$

and,

$$\frac{d^2}{dx^2}f(x;\ \theta,\alpha)=\frac{\theta^2 log(\theta+1)}{(\alpha\theta+1)(\theta+1)^{x+2}}\{log(\theta+1)(\alpha\theta+\alpha+x+1)-2\}.$$

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ISSN: 2319-8354

Now, $\frac{d}{dx} f(x;\, \theta, \alpha) = 0$, the solution is $x = \frac{1-\alpha log(\theta+1)-\alpha\theta log(\theta+1)-log(\theta+1)}{log(\theta+1)}$ and

$$\frac{d^2}{dx^2} f(x;\, \theta, \alpha) = \frac{-\theta^2(\theta+1)}{(\alpha\theta+1)}\left(\alpha+\alpha\theta-1-\frac{1}{log(\theta+1)}\right) < 0$$

For $\theta, \alpha, \hat{x} > 0$, $\hat{x} = \frac{1}{log(\theta+1)} - (\alpha+\alpha\theta+1)$ is the unique critical point which $f(x;\, \theta, \alpha)$ is maximum and $f(x;\, \theta, \alpha)$ is concave.

Therefore, the mode of GTPLD is given by

$$Mode(X) = \frac{1}{log(\theta+1)} - (\alpha+\alpha\theta+1) \text{ for } \theta, \alpha > 0.$$

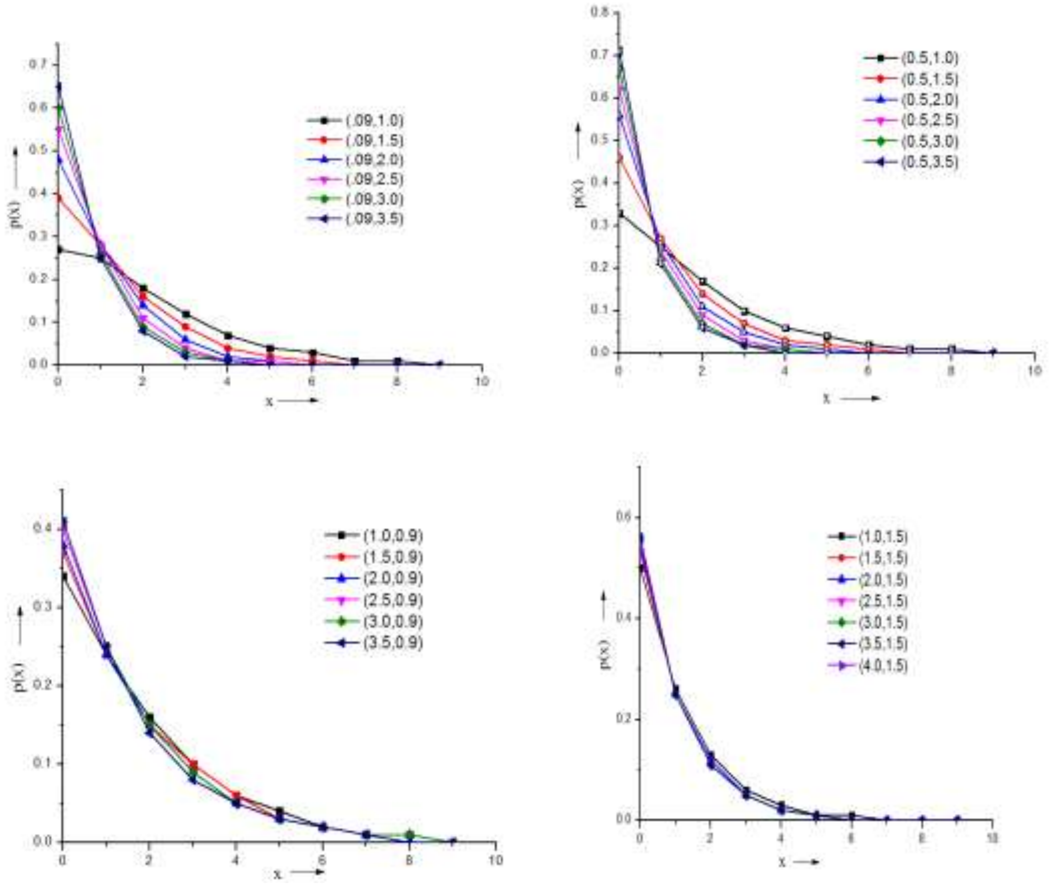

Figure 1: Pmf plots fordifferent values of α and θ

III Properties of the Generalized Poisson Lindley distribution

3.1 Shape of the probability function

It can be seen that

$$p(0) = \frac{\theta^2(\alpha\theta+\alpha+1)}{(1+\theta)^2(1+\alpha\theta)}$$

$$\frac{p(x+1)}{p(x)} = \frac{1}{(1+\theta)}\left\{1+\frac{1}{\alpha(\theta+1)+x+1}\right\},\ x = 1,2,\dots\dots \quad (2)$$

which is a decreasing function in x, $f(x;\theta,\alpha)$ is log-concave. Therefore, it has an increasing hazard rate and is unimodal.

3.2 GeneratingFunctions

The probability generating function (p.g.f) has been obtained as

$$P_X(t) = E(t^x)$$

$$= \sum_{x=0}^{\infty} t^x \frac{\theta^2}{(\theta+1)^{x+1}(\alpha\theta+1)}\left[\alpha+\frac{x+1}{\theta+1}\right]$$

$$= \frac{\theta^2}{(\alpha\theta+1)(\theta+1-t)^2}\{\alpha(\theta+1-t)+1\}. \quad (3)$$

The Moment generating function is

$$M_x(t) = \sum_{x=0}^{\infty} e^{tx}\frac{\theta^2}{(\theta+1)^{x+1}(\alpha\theta+1)}\left[\alpha+\frac{x+1}{\theta+1}\right]$$

$$= \frac{\theta^2}{(\alpha\theta+1)(\theta+1-e^t)^2}\{\alpha(\theta+1-e^t)+1\} \quad (4)$$

The recursive expression has been obtained as

$$p_r = \frac{1}{(\theta+1)^2}\{2(\theta+1)p_{r-1}-p_{r-2}\},\ r \geq 2,$$

where $p_0 = \frac{\theta^2}{(\theta+1)^2(\alpha\theta+1)}(\alpha+\alpha\theta+1)$

$$p_1 = \frac{\theta^2}{(\theta+1)^3(\alpha\theta+1)}(\alpha+\alpha\theta+2)$$

3.3Moments, Skewness and Kurtosis

The first four moments of Xhave been obtained as

$$\mu_1' = \frac{\alpha\theta+2}{\theta(\alpha\theta+1)},$$

$$\mu_2' = \frac{\alpha\theta + 2}{\theta(\alpha\theta + 1)} + \frac{2(\alpha\theta + 3)}{\theta^2(\alpha\theta + 1)},$$

$$\mu_3' = \frac{\alpha\theta+2}{\theta(\alpha\theta+1)} + \frac{6(\alpha\theta+3)}{\theta^2(\alpha\theta+1)} + \frac{6(\alpha\theta+4)}{\theta^3(\alpha\theta+1)},$$

$$\mu_4' = \frac{\alpha\theta+2}{\theta(\alpha\theta+1)} + \frac{14(\alpha\theta+3)}{\theta^2(\alpha\theta+1)} + \frac{36(\alpha\theta+4)}{\theta^3(\alpha\theta+1)} + \frac{24(\alpha\theta+5)}{\theta^4(\alpha\theta+1)}.$$

And the moment about mean are

$$\mu_2 = \frac{(\alpha\theta + 2)(\theta + 1)(\alpha\theta + 1) + \alpha\theta}{\theta^2(\alpha\theta + 1)^2}$$

$$\mu_3 = \frac{\alpha^3\theta^3(\theta + 1)(\theta + 2) + \alpha^2\theta^2((2\theta + 1)^2 + 11\theta + 1) + \alpha\theta\big((5\theta + 6)(\theta + 2) + 2\theta\big) + 2(\theta + 1)(\theta + 2)}{\theta^3(\alpha\theta + 1)^3}$$

$$\mu_4 = \frac{\begin{array}{c}\alpha^4\theta^7 + \theta^6\alpha^3(5 + 10\alpha) + \theta^5\alpha^2(20\alpha^2 + 60\alpha + 9) + \theta^4\alpha(7 + 116\alpha + 120\alpha^2 + 9\alpha^4) + \theta^3(2 + 92\alpha + 240\alpha^2 + 72\alpha^3) \\ + \theta^2(26 + 180\alpha + 132\alpha^2) + \theta(48 + 168\alpha) + 24\end{array}}{\theta^4(\alpha\theta + 1)^4}$$

Skewness:

$$\gamma = \frac{\mu_3}{{\mu_2}^{3/2}}$$

$$= \frac{\alpha^3\theta^5 + \theta^4(4\alpha^2 + 3\alpha^3) + \theta^3(5\alpha + 15\alpha^2 + 2\alpha^3) + \theta^2(2 + 18\alpha + 2\alpha^2) + \theta(6 + 12\alpha) + 4}{(\alpha^2\theta^3 + 3\alpha\theta^2 + \alpha^2\theta^2 + 2\theta + 4\alpha\theta + 2)^{3/2}}$$

Kurtosis:

$$\beta_2 = \frac{\mu_4}{{\mu_2}^2}$$

$$= \frac{\begin{array}{c}\alpha^4\theta^7 + \theta^6\alpha^3(5 + 10\alpha) + \theta^5\alpha^2(20\alpha^2 + 60\alpha + 9) + \theta^4\alpha(7 + 116\alpha + 120\alpha^2 + 9\alpha^4) + \theta^3(2 + 92\alpha + 240\alpha^2 + 72\alpha^3) \\ + \theta^2(26 + 180\alpha + 132\alpha^2) + \theta(48 + 168\alpha) + 24\end{array}}{\big((\alpha\theta + 2)(\theta + 1)(\alpha\theta + 1) + \alpha\theta\big)^2}$$

Index of Dispersion:

Index of dispersion (ID) is defined as the ratio of variance to mean, to indicate whether a distribution is over dispersed or under dispersed. If $ID \geq 1$, the distribution is over dispersed, otherwise it is called under dispersed.

$$ID = \frac{\sigma^2}{\mu}$$

$$= \frac{(\alpha\theta + 2)(\theta + 1)(\alpha\theta + 1) + \alpha\theta}{\theta^2(\alpha\theta + 1)(\alpha\theta + 2)}$$

IV METHOD OF ESTIMATION

4.1 Method of Moments

Suppose, considering a sample of size n, say $x_1, x_2, \dots \dots \dots, x_n$ from equation (1). Then the moment estimates $\tilde{\alpha}$ and $\tilde{\theta}$ of α and θ can be obtained by solving the equations as follows

$$m_1 = \frac{\alpha\theta + 2}{\theta(\alpha\theta + 1)}$$

and $$m_2 = \frac{\theta(\alpha\theta+2)+2(\alpha\theta+3)}{\theta^2(\alpha\theta+1)},$$

where m_1 and m_2 denote the first and second order moments. Solving the above two equations we got,

$$\alpha = \frac{2-m_1\theta}{(m_1\theta-1)\theta}$$

and $$\theta = \frac{2m_1+\sqrt{4m_1{}^2+2m_1-2m_2}}{(m_2-m_1)}.$$

4.2 Maximum Likelihood Estimation

Suppose $x_1, x_2, x_3, \dots \dots, x_n$ are random sample of size n from generalized Poisson Lindley distribution. Then the log likelihood function is given as,

$$\log \mathrm{L} = \prod_{i=1}^{n} f(x_i; \theta, \alpha)$$

$$= 2n\log\theta - \sum_{i=1}^{n}(x_i + 1)\log(1+\theta) - n\log(\alpha\theta + 1) + \sum_{i=1}^{n}\log\left(1 + \frac{x_i+1}{\theta+1}\right) .(4.3)$$

Differentiating $\log \mathrm{L}$ w.r.t θ and α we get,

$$\frac{\partial \log \mathrm{L}}{\partial\theta} = \frac{2n}{\theta} - \frac{\sum_{i=1}^{n} x_i}{1+\theta} - \frac{n\alpha}{\alpha\theta+1} - \sum_{i=1}^{n}\frac{x_i+1}{\{1+\theta+(x_i+1)\}(1+\theta)}$$

$$\frac{\partial \log \mathrm{L}}{\partial\alpha} = -\frac{n\theta}{\alpha\theta + 1}$$

The second derivatives are,

$$\frac{\partial^2 \log \mathrm{L}}{\partial^2\theta} = \frac{n\alpha^2}{(\alpha\theta+1)^2} - \frac{2n}{\theta^2} + \frac{\sum_{i=1}^{n} x_i + n}{(1+\theta)^2} + \sum_{i=1}^{n}\frac{(x_i+1)(3+\theta+x_i)}{(1+\theta)^2(2+\theta+x_i)^2}$$

$$\frac{\partial^2 \log \mathrm{L}}{\partial\theta\partial\alpha} = \frac{-n(2\alpha+1)}{(\alpha\theta+1)^2}$$

$$\frac{\partial^2 \log \mathrm{L}}{\partial^2\alpha} = \frac{n\theta^2}{(\alpha\theta+1)^2}$$

The following equations for $\hat{\theta}$ and $\hat{\alpha}$ can be solved

$$\begin{bmatrix} \frac{\partial^2 \log \mathrm{L}}{\partial^2\theta} & \frac{\partial^2 \log \mathrm{L}}{\partial\theta\partial\alpha} \\ \frac{\partial^2 \log \mathrm{L}}{\partial\theta\partial\alpha} & \frac{\partial^2 \log \mathrm{L}}{\partial^2\alpha} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}} \begin{bmatrix} \hat{\theta} - \theta_0 \\ \hat{\alpha} - \alpha_0 \end{bmatrix} = \begin{bmatrix} \frac{\partial \log \mathrm{L}}{\partial\theta} \\ \frac{\partial \log \mathrm{L}}{\partial\alpha} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}}$$

where θ_0 and α_0 are the initial values of θ and α respectively. These equations are solved iteratively till sufficiently closed values of $\hat{\theta}$ and $\hat{\alpha}$ can be obtained.

Theorem: The estimator $\tilde{\theta}$ of θ is positively biased, for fixed α, i.e $E(\tilde{\theta}) > \theta$.

Proof: Let $\tilde{\theta} = g(\bar{X})$ and $g(t) = \frac{(\alpha-t)+\sqrt{t^2+6\alpha t+\alpha^2}}{2\alpha t}$ for $t > 0$.

Then, $g''(t) = \frac{1}{t^3}\left[1 + \frac{15\alpha t^2+9\alpha^2 t+3t^3+\alpha^3}{(t^2+6\alpha t+\alpha^2)^{3/2}}\right] > 0.$

Therefore, $g(t)$ is strictly concave. Thus, by Jensens inequality, we have

$E\{g(X)\} > g\{E(X)\}$. Thus we get $E(\tilde{\theta}) > \theta$, since

$$g\{E(\bar{X})\} = g\left(\frac{\alpha\theta+2}{\theta(\alpha\theta+1)}\right) = \theta.$$

Theorem: The moment estimate $\tilde{\theta}$ of θ is consistent and asymptotically normal,

for fixed values α, and is distributed as

$$\sqrt{n}(\tilde{\theta}-\theta) \xrightarrow{d} N(0, v^2(\theta)),$$

where, $v^2(\theta) = \frac{\theta^2(\alpha\theta+1)^2\{(\theta+1)(\alpha^2\theta^2+3\alpha\theta+2)+\alpha\theta\}}{(4\alpha\theta+\alpha^2\theta^2+2)^2}$.

V APPLICATION TO REAL DATA SET

The fitted frequencies of two- parameter Poisson- Lindley distribution [Sankaran (1970)] and generalized two-parameter Poisson- Lindley (GTPL)distribution has been shown in the following Tables. The parameters of GTPL distribution have been estimated using a composite method. Goodness of fit has been tested based on χ^2-statistic.

Table 1. Comparison of observed frequencies for mistakes in copying groups of random digits with expected frequencies, estimated parameters and χ^2-statistic obtained by fitting GTPL distribution. [Data from Sankaran (1970)].

IJARSE
ISSN: 2319-8354

No. of errors per group	Observed frequencies	Expected frequencies	
		One- parameter PLD $(\hat{\theta})$	GTPL(θ,α)
0	35	33.1	32.4
1	11	15.3	15.8
2	8	6.8	7.0
3	4	2.9	2.9
4	2	1.2	1.9
Total	60	59.3	60.0
Parameter $\hat{\theta}$		1.743	2.000
estimates $\hat{\alpha}$			0.3824
χ^2		2.20	2.11

Table 2. Fitting of GTPL distribution along with expected frequencies, estimated parameters and χ^2-statistic [Data of Pyraustanublilalis].

No. of insects	Observed frequencies	Expected frequencies	
		One- parameter PLD $(\hat{\theta})$	GTPL (θ,α)
0	33	31.5	31.90
1	12	14.2	13.80
2	6	6.1	6.00
3	3	2.5	2.53
4	1	1.0	1.27
5	1	0.7	0.58
Total	56	56	55.67
Parameter	$\hat{\theta}$	1.8081	1.5255
estimates	$\hat{\alpha}$		3.8919
χ^2		0.53	0.36

IJARSE
ISSN: 2319-8354

Table 3. A comparison of generalized two- parameter Poisson-Lindley (GTPL)with two- parameter Poisson-Lindley (TPL) distribution.

	Two- parameter Poisson-Lindley: TPL$(\boldsymbol{\theta}, \boldsymbol{\alpha})$	**Generalized Two- parameter Poisson-Lindley: GTPL$(\boldsymbol{\theta}, \boldsymbol{\alpha})$**
Pmf	$\frac{\theta^2}{(\theta+1)^{x+2}}\left[1+\frac{\alpha x+1}{\theta+\alpha}\right]$	$\frac{\theta^2}{(\theta+1)^{x+1}(\alpha\theta+1)}\left[\alpha+\frac{x+1}{\theta+1}\right]$
pgf$g(t)$	$\frac{\theta^2(\theta+1-t)+\alpha\theta^2}{(\theta+\alpha)(\theta+1-t)^2}$	$\frac{\theta^2}{(\alpha\theta+1)(\theta+1-t)^2}\{\alpha(\theta+1-t)+1\}$
Recurrence relation	$p_r=\frac{1}{(\theta+1)^2}\{2(\theta+1)p_{r-1}-p_{r-2}\}$	$p_r=\frac{1}{(\theta+1)^2}\{2(\theta+1)p_{r-1}-p_{r-2}\},$ $for\, r\geq 2,$
Meanm_1	$\frac{\theta+2\alpha}{\theta(\theta+\alpha)}$	$\frac{\alpha\theta+2}{\theta(\alpha\theta+1)}$
m_2	$\frac{\theta+2\alpha}{\theta(\theta+\alpha)}+\frac{2(\theta+3\alpha)}{\theta^2(\theta+\alpha)}$	$\frac{\alpha\theta+2}{\theta(\alpha\theta+1)}+\frac{2(\alpha\theta+3)}{\theta^2(\alpha\theta+1)}$
m_3	$\frac{\theta+2\alpha}{\theta(\theta+\alpha)}+\frac{6(\theta+3\alpha)}{\theta^2(\theta+\alpha)}+\frac{6(\theta+4\alpha)}{\theta^3(\theta+\alpha)}$	$\frac{\alpha\theta+2}{\theta(\alpha\theta+1)}+\frac{6(\alpha\theta+3)}{\theta^2(\alpha\theta+1)}+\frac{6(\alpha\theta+4)}{\theta^3(\alpha\theta+1)}$
m_4	$\frac{\theta+2\alpha}{\theta(\theta+\alpha)}+\frac{14(\theta+3\alpha)}{\theta^2(\theta+\alpha)}+\frac{36(\theta+4\alpha)}{\theta^3(\theta+\alpha)}+\frac{2}{\theta}$	$\frac{\alpha\theta+2}{\theta(\alpha\theta+1)}+\frac{14(\alpha\theta+3)}{\theta^2(\alpha\theta+1)}+\frac{36(\alpha\theta+4)}{\theta^3(\alpha\theta+1)}+\frac{2}{\theta}$
Variance μ_2	$\frac{\theta^3+\theta^2+3\alpha\theta^2+2\alpha^2\theta+4\alpha\theta+2\alpha^2}{\theta^2(\theta+\alpha)^2}$	$\frac{\alpha^2\theta^3+3\alpha\theta^2+\alpha^2\theta^2+2\theta+4\alpha\theta+2}{\theta^2(\alpha\theta+1)^2}$
mgf $M_X(t)$	$\frac{\theta^2(\theta+1-e^t)+\alpha\theta^2}{(\theta+\alpha)(\theta+1-e^t)^2}$	$\frac{\theta^2}{(\alpha\theta+1)(\theta+1-e^t)^2}\{\alpha(\theta+1-e^t)+1\}$
Estimate θ	$\left(\frac{b+2}{b+1}\right)\frac{1}{m_1}$ where $m_1$denotes the mean and b = $\frac{\theta}{\alpha}$	$\frac{2m_1+\sqrt{4m_1{}^2+2m_1-2m_2}}{(m_2-m_1)}$, where m_1, m_2 denote the raw moments
Estimate $\boldsymbol{\alpha}$	$\frac{b+2}{b(b+1)}.\frac{1}{m_1}$	$\frac{2-m_1\theta}{(m_1\theta-1)\theta}$

VI RESULTS AND DISCUSSION

In this paper, the first four moment about origin has been obtained. The expression for probability generating function and moment generating function has also been derived. The GTPL distribution has been fitted to two data sets for testing its goodness of fit and it has been found that it provides a closer fit than one parameter Poisson Lindley distribution.

VII ACKNOWLEDGEMENT

The authors would like to sincerely thank the Associate Editor for carefully reading the paper and whose useful and valuable suggestions led to the improvement of the paper.

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