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Mathematical model for consensus in group decision making under linguistic assessment using triangular in tuitionistic fuzzy numbers

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ABSTRACT

The aim of this paper is to provide the model for group decision making under intuitionistic fuzzy number. Owing to equivocal concept of frequently represented in decision data, the crisp value are insufficient to real life problems. In this paper, the assessment of each alternative and the encumbrance of each criterion are described by phonological terms which can be articulated in intuitionistic fuzzy numbers, then Hungarian triangular intuitionistic fuzzy number is used to provide the modelling approach select suitable among the all alternatives. This paper provides the alternative method for decision maker in ambiguous concept.

Keywords: Linguistic Variable, Triangular Intuitionistic Fuzzy Number, MCDM.

I INTRODUCTION

Under many conditions, crisp data are inadequate to model real-life Situations. Since human judgements including preferences reoften vague and cannot estimate his preference with an exact numerical value. A more realistic approach maybe to use linguistic assessments instead of numerical values, that is, to suppose that the ratings and weights of the criteria in the problem are assessed by means of linguistic variables[1,3,4,6,9,13].The concept to linguistic variable is very useful in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions [13].For example, "weight" is a linguistic variable; its values are very low, low, medium, high, very high, etc. These linguistic variable can also be represented by in tuitionistic fuzzy numbers.

In this paper, we further extended for solving multi-person multi-criteria decision making (MCDM) problems in in tuitionistic fuzzy environment using optimization techniques. considering the fuzziness in the decision data and group decision-making process, linguistic variables are used to assess the weights of all criteria and the ratings of each alternative with respect to each criterion. we can convert the decision matrix into a in tuitionistic decision matrix and construct in tuitionistic assignment model for decision matrix

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Once the decision makers' in tuitionistic ratings have been pooled. Introduces the basic definitions and notations of the in tuitionistic fuzzy number and linguistic variable, and present a numerical example for linguistic in tuitionistic fuzzy assignment method in group decision making.

II MATHEMATICAL FORMULATION OF ASSIGNMENT PROBLEM

The objective is to minimize the total cost of assignment. If job I is assigned to operator 1, the cost is ($C_{11}X_{11}$). Similarly, for job 1, operator 2 the cost is ($C_{12}X_{12}$). The objective function is:

$$\text{Minimize} = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} X_{ij}$$

Since one job (i) can be assigned to any one of the operators, we have following constraint set:

$$\sum_{i=1}^{n} X_{ij} = 1; \text{ for all } j; j = 1, 2, \ldots n$$

Similarly for each operator, there may be only one assignment of job. For this, the constraint set is:

$$\sum_{i=1}^{n} X_{ij} = 1; \text{ for all } i; i = 1, 2, \ldots n$$

The non-negativity constraint is:

$$X_{ij} \geq 0$$

$$\text{Minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} X_{ij}$$

Subject to $\sum_{i=}^{n} X_{ij} = 1$; for all $j; j = 1, 2, \ldots n$

$\sum_{j=1\ 1}^{n} X_{ij} = 1$; for all $i; i = 1, 2, \ldots n$

$X_{ij} \geq 0$ for all i and all j.

III UNBALANCED ASSIGNMENT PROBLEM TO CHANGE INTO BALANCED ASSIGNMENT PROBLEM

The number of rows is not equal to the number of columns, then the problem is termed as unbalanced assignment problem then this problem changed into balanced assignment problem as follows necessary number of dummy rows / columns are added such that the cost matrix is a square matrix, the values for the entries in the dummy rows / columns are assumed to be zero.

Fuzzy set

A Fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{x, \mu_A(x)\}$; $x€A$, $\mu_A(x)€[0,1]$. In the pair $(x, \mu_A(x))$,the first element x belong to the classical set A,the second element $\mu_A(x)$,belong to the interval [0,1] called membership function.

In tuitionistic Fuzzy Set

An Intuitionstic fuzzy set a$\tilde{A}^I$ assign the each element x of the universe X a membership degree $\mu_{a'}(x) \in [0,1]$ and non membership degree $\upsilon_{a'}(x) \in [0,1]$ such that $\mu_{a'}(x)+ \upsilon_{a'}(x) \leq 1$. An IFS a' is mathematically represented as { $\mu_{a'}(x)$<x, $\upsilon_{a'}(x)$> x €X}

In tuitionistic Triangular Fuzzy Number

A triangular intuitionistic fuzzy number (TIFN) $\tilde{A}^I$ is an intuitionistic fuzzy set in R with the following membership function $\mu_{\tilde{A}^I}(x)$ and non-membership function $\vartheta_{\tilde{A}^I}(x)$

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \frac{x-a_3}{a_2-a_3}, & a_2 \le x \le a_3 \\ 0, & otherwise \end{cases} \text{and } \vartheta_{\tilde{A}^I}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1'}, & a_1' \le x \le a_2 \\ \frac{x-a_2}{a_3'-a_2}, & a_2 \le x \le a_3' \\ 1, & otherwise \end{cases}$$

Where $a_1' \le a_1 \le a_2 \le a_3 \le a_3'$ and $\mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) \le 1$ or $\mu_{\tilde{A}^I}(x) = \vartheta_{\tilde{A}^I}(x)$, for all $x \in R$. This TIFN is denoted by $\tilde{A}^I = (a_1, a_2, a_3;\ a_1', a_2, a_3') = \{(a_1, a_2, a_3);\ (a_1', a_2, a_3')\}$

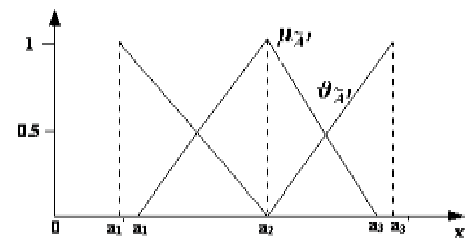

Fig 1: Membership and non-membership functions of TIFN

Positive triangular intuitionistic fuzzy number:

A positive triangular intuitionistic fuzzy number is denoted as $\{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ where all a_i's and a_i''s >0 for all i=1, 2, 3.

Negative triangular intuitionistic fuzzy number:

A negative triangular intuitionistic fuzzy number is denoted as $\{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ where all a_i's and a_i''s <0 for all i=1, 2, 3.

Modified operations of triangular intuitionistic fuzzy numbers using function principle:

The following are the modified operations that can be performed on triangular intuitionistic fuzzy numbers:

Let $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ and $\tilde{B}^I = \{(b_1, b_2, b_3); (b_1', b_2, b_3')\}$.

Then

(i)

Addition: $\tilde{A}^I + \tilde{B}^I = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3); (a_1' + b_1', a_2 + b_2, a_3' + b_3')\}$

(ii)

Subtraction: $\tilde{A}^I - \tilde{B}^I = \{(a_1 - b_3, a_2 - b_2, a_3 - b_1); (a_1' - b_3', a_2 - b_2, a_3' - b_1')\}$

(iii) Multiplication:

$\tilde{A}^I \text{ X } \tilde{B}^I = \{(\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3)$

$(\min(a_1'b_1', a_1'b_3', a_3'b_1', a_3'b_3'), a_2b_2, \max(a_1'b_1', a_1'b_3', a_3'b_1', a_3'b_3')\}$

(iv) Division: $\tilde{A}^I / \tilde{B}^I = \{(\min(\frac{a_1}{b_1}, \frac{a_1}{b_3}, \frac{a_3}{b_1}, \frac{a_3}{b_3}), \frac{a_2}{b_2}, \max(\frac{a_1}{b_1}, \frac{a_1}{b_3}, \frac{a_3}{b_1}, \frac{a_3}{b_3}));$

$(\min(\frac{a_1'}{b_1'}, \frac{a_1'}{b_3'}, \frac{a_3'}{b_1'}, \frac{a_3'}{b_3'}), \frac{a_2}{b_2}, \max(\frac{a_1'}{b_1'}, \frac{a_1'}{b_3'}, \frac{a_3'}{b_1'}, \frac{a_3'}{b_3'}))\}$

Accuracy Function

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Let $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$ be a TIFN then we define $\tilde{A}^I = \frac{[(a_1 + 2a_2 + a_3) + (a_1' + 2a_2 + a_3')}{8}$ an accuracy function of $\widetilde{A}^I$, to defuzzify the given number.

New operation on in tuitionistic triangular fuzzy number Subtraction:

Let $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$and $\tilde{B}^I = \{(b_1, b_2, b_3); (b_1', b_2, b_3')\}$

Then $\tilde{A}^I - \tilde{B}^I = \{(a_1 - b_1, a_2 - b_2, a_3 - b_3); (a_1' - b_1', a_2 - b_2, a_3' - b_3')\}$.

The new subtraction operation exists only if the following conditions are satisfied

$D(\tilde{A}^I) \geq D(\tilde{B}^I)$ and $D(\tilde{A}^{I'}) \geq D(\tilde{B}^{I'})$, where $D(\tilde{A}^I) = \frac{a_3 - a_1}{2}$,$D(\tilde{B}^I) = \frac{b_3 - b_1}{2}$, $D(\tilde{A}^{I'}) = \frac{a_3' - a_1'}{2}$ and $D(\tilde{B}^{I'}) = \frac{b_3' - b_1'}{2}$.

Here D denotes difference point of a intuitionistic triangular fuzzy number.

Division:

Let $\tilde{A}^I = \{(a_1, a_2, a_3); (a_1', a_2, a_3')\}$and $\tilde{B}^I = \{(b_1, b_2, b_3); (b_1', b_2, b_3')\}$.Then

$\tilde{A}^I / \tilde{B}^I = \{(\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}); (\frac{a_1'}{b_1'}, \frac{a_2}{b_2}, \frac{a_3'}{b_3'})\}$.The new division operator exists only if the following conditions are satisfied $\left|\frac{D(\tilde{A}^I)}{M(\tilde{A}^I)}\right| \geq \left|\frac{D(\tilde{B}^I)}{M(\tilde{B}^I)}\right|$; $\left|\frac{D(\tilde{A}^{I'})}{M(\tilde{A}^{I'})}\right| \geq \left|\frac{D(\tilde{B}^{I'})}{M(\tilde{B}^{I'})}\right|$ and the negative triangular in tuitionistic fuzzy number should be changed into negative multiplication of positive triangular in tuitionistic fuzzy number.

Linguistic variable

Linguistic variable is a variable whose values are linguistic terms. The concept of linguistic variable is very use full in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions [13].For example, “weight” is a linguistic variable; its values are Very low, low, medium, high, very high, etc. These linguistic values can also be represented by in tuitionistic fuzzy numbers.

Algorithm

An algorithm of multi criteria decision making with intuitionistic fuzzy approach is

Step1:Form a committee of decision makers,then identify the evaluation criteria.

Step2: choose the appropriate linguistic variables for the importance weight of the criteria and the linguistic rating for alternatives with respect to criteria.

Step3: Assign Suitable weight for each linguistic variable, that would helpful for decision maker to assess the candidate.

Step4: Based on the decision makers assessment form linguistic variable matrix

Step5:Construct the in tuitionistic fuzzy decision matrix corresponding linguistic variable matrix.

Step6: Formulate In tuitionistic triangular fuzzy assignment problem for decision matrix.

Step7:Using Hungarian in tuitionistic fuzzy assignment algorithm to identify suitable decision made from alternative decision variable.

Numerical example

Suppose that a company decide to appoint system administrative engineers, after initial selection process three candidate C_1,C_2,C_3remain for further evaluation. A committee of three decision-makers,D_1,D_2 and D_3 has been formed to conduct the interview and select the most suitable candidate for four quality criteriaareconsidered:

1. Emotional steadiness (Q_1),
2. Oral communication skill (Q_2),
3. Personality (Q_3),
4. Self-confidence (Q_4).

The proposed method is currently applied to solve this problem and computational procedure is expressed as follows.

Step1: the decision-makers use the linguistic weighting variable to assess the importance of the criteria and present in table 1.

Step2: the decision makers use the linguistic rating variable to evaluate the rating of alternatives with respect to each criterion and present it in table2 .

Step3: converting the linguistic evolution into intuitionistic fuzzy numbers to construct the fuzzy decision matrix table 3.

Step4: formulate Intuitionistic triangular fuzzy assignment problem for decision matrix

Step5: converting the minimize intuitionistic fuzzy decision matrix to minimize intuitionistic fuzzy decision matrix table 4.

Step6.using assignment algorithm to assign suitable candidate to corresponding criteria table 5,

Step7: according to intuitionistic fuzzy assignment, out of the candidates A2 Selected for emotional steadiness,A1 for oral communication skill and A3 for personality.

Table 1

The importance weights of each criterion can be made by the decision makers use the Linguistic variables to evaluate the importanceof each criterion and the ratings of alternatives with respect to various criteria.

Linguistic variable	Weight
Very Good(VG)	(8,10,12;7.5,10,12.5)
Good(G)	(7,9,11;6.5,9,11.5)
Medium Good(MG)	(6,8,10;5.5,8,10.5)
Fair(F)	(5,7,9;.4.5,7,9.5)
Poor(P)	(4,6,8;3.5,6,8.5)
Medium Poor(P)	(3,5,7;2.5,5,7.5)
Very Poor(P)	(2,4,6;1.5,4,6.5)

Table 2 linguistic decision metrics

	Q_1	Q_2	Q_3	Q_4
C_1	VG	MG	G	VG
C_2	P	G	MG	G
C_3	MG	F	F	VG

Table 3

	Q_1	Q_2	Q_3	Q_4
C_1	(8,10,12;7.5,10,12.5)	(6,8,10;5.5,8,10.5)	(7,9,11;6.5,9,11.5)	(8,10,12;7.5,10,12.5)
C_2	(4,6,8;3.5,6,8.5)	(7,9,11;6.5,9,11.5)	(6,8,10;5.5,8,10.5)	(7,9,11;6.5,9,11.5)
C_3	(6,8,10;5.5,8,10.5)	(5,7,9;4.5,7,9.5)	(5,7,9;4.5,7,9.5)	(8,10,12;7.5,10,12.5)

The Intuitionistic triangular fuzzy assignment problem can be formulated as following:

Maximize Z = R(8,10,12;7.5,10,12.5) x_{11}+ R(6,8,10;5.5,8,10.5) x_{12}+ R(7,9,11;6.5,9,11.5) x_{13}+ R(8,10,12;7.5,10,12.5) x_{14}+R(4,6,8;3.5,6,8.5)X_{21}+ R(7,9,11;6.5,9,11.5) x_{22}+ R(6,8,10;5.5,8,10.5) x_{23}+ R(7,9,11;6.5,9,11.5) x_{24}+ R(6,8,10;5.5,8,10.5) x_{31}+ R(5,7,9;4.5,7,9.5) x_{32}+ R (5,7,9;4.5,7,9.5) x_{33}+ R(8,10,12;7.5,10,12.5) x_{34}

Such that

$x_{11}+ x_{12}+ x_{13}+ x_{14} = 1$, $x_{11}+ x_{21}+ x_{31} = 1$,

$x_{21}+ x_{22}+ x_{23}+ x_{24} = 1$, $x_{12}+ x_{22}+ x_{32} = 1$,

$x_{31}+ x_{32}+ x_{33}+ x_{34} = 1$, $x_{13}+ x_{23}+ x_{33} = 1$,

$x_{41}+ x_{42}+ x_{43}+ x_{44} = 1$, $x_{14}+ x_{24}+ x_{34} = 1$,

Table 4

Minimization metrics

	Q_1	Q_2	Q_3	Q_4
C_1	(-4,0,4;-5,0,5)	(-2,2,6;-3,2,5)	(-3,1,5;-4,1,6)	(-4,0,4;-5,0,5)
C_2	(0,4,8;-1,4,9)	(-3,1,5;-4,1,6)	(-2,2,2;-3,2,7)	(-3,1,5;-4,1,6)
C_3	(-2,2,2;-3,2,7)	(-1,3,1;-2,3,6)	(-1,3,1;-2,3,6)	(-4,0,4;-5,0,5)
C_4	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(0,0,0;0,0,0)

Table 5

After row and column reduction, we done assignment as follows

	Q_1	Q_2	Q_3	Q_4
C_1	(-2,2,6;-3,2,7)	**[(-4,0,4;-5,0,5)]**	(-3,1,5;-4,1,6)	(-2,2,6;-3,2,7)
C_2	**[(-4,0,4;-5,0,5)]**	(7,3,-1;8,3-2)	(-2,2,6;-3,2,7)	(-1,3,7;-2,3,7)
C_3	(-3,1,5;-4,1,6)	(-4,0,4;-5,0,5)	**[(-4,0,4;-5,0,5)]**	(-2,2,6;-3,2,7)
C_4	(0,0,0;0,0,0)	(0,0,0;0,0,0)	(0,0,0;0,0,0)	**[(0,0,0;0,0,0)]**

Conclusion

In multi-criteria decision making problems follow to uncertain and vaguedata ,and in tuitionistic fuzzy set theory is suitable to deal with it.In this paper, a linguistic decision process is offered to solve the multiple criteria decision-making problem under in tuitionistic fuzzy environment.

Indecision-making process very often, the assessment of alternatives with respect to criteria and the importance weight are suitable to use the linguistic variable sinstead of numerical values. Here ,undergroup decision-making process, it is not difficult to use other aggregation function to pool the in tuitionistic fuzzy assignment of decision makers in the proposed method. Although the method presented in this section is illustrated by a personal selection problem, however, it can also be applied to problems such as material section, project selection, area selection and many other areas of decision making problems.

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