

Effect of an exponentially accelerated ramped vertical plate on an unsteady MHD free convection flow in the presence of magnetic field

B.P. Garg¹, Deepthi²

¹ *Research Supervisor, Department of Mathematics,
Punjab Technical University, Jalandhar, Punjab, India*

² *Research Scholar, Department of Mathematics,
Punjab Technical University, Jalandhar, Punjab, India*

ABSTRACT

An analysis of unsteady MHD free convection flow of a viscous, incompressible electrically conducting and heat absorbing fluid flow past an exponentially accelerated flat plate with non-uniform temperature (ramped) in the presence of magnetic field is carried out. The non-linear partial differential equations governing the flow problem along with its boundary conditions are solved by converting into dimensionless form using non-similar transformations and analytic solutions are obtained. The velocity fields, temperature fields and concentration fields for both non-uniform (ramped) and uniform (isothermal) temperature are shown graphically. The expressions derived for skin friction, Nusselt number and Sherwood number are numerically evaluated for different values of physical parameters and presented in tabular form.

Keywords: Free Convection, Heat and Mass Transfer Flow, Heat Absorption, MHD

1. Introduction

The study of effects of magnetic field on the flow of a viscous, incompressible and electrically conducting fluids has been in the interest of many researchers since a decade due to its importance in many practical applications such as in MHD power generators ,MHD pumps, Nuclear reactors, Geothermal extraction etc. Due to this fact many researchers[1-18] has contributed toward the study of heat and mass transfer of MHD flows under the effects of ohmic heating, viscous dissipation, heat absorption, heat generation, variable suction/injection along a surface.

The above studies have always assumed the velocity and temperature at the interface to be continuous and well defined. But there are several problems of physical interest that may require non-uniform or arbitrary wall conditions. Several researchers [19-22] have investigated problems of free convection from vertical plate with discontinuities in the surface temperature. Unsteady natural convection flow in viscous incompressible fluid near vertical plate with ramped wall temperature was studied by Chandran et al [23]. Seth and Ansari [24]

analysed the MHD natural convection flow past an impulsively moving vertical plate with ramped wall temperature in the presence of thermal diffusion with heat absorption. Heat and mass transfer in MHD flow by natural convection from a permeable inclined surface with variable wall temperature and concentration is discussed by Chen [25]. Seth et al [26] discussed the unsteady natural convection flow of a viscous incompressible electrically conducting fluid past an impulsively moving vertical plate in porous medium with ramped wall temperature along with thermal radiation effect. Recently Raj et al [27] investigated the exact solutions of unsteady MHD free convection in heat absorbing fluid flow past a flat plate with ramped wall temperature.

The aim of the present paper is to study the effect of exponentially accelerating flat plate on hydro magnetic free convective flow of a viscous, incompressible, electrically conducting and heat absorbing fluid under the non-uniformity(ramped) of wall temperature.

2. Mathematical analysis

Consider the unsteady MHD free convective flow of a viscous, incompressible, electrically conducting and heat absorbing fluid passing an infinite non-conducting vertical flat plate. The x' -axis is oriented vertically upward along the plate. The y' -axis is taken perpendicular to plate and a transverse magnetic field of strength B_0 is applied along this axis. For time $t' < 0$ the plate and the fluid is at constant temperature T_∞' and concentration C_∞' in a stationary conditions. At time $t' = 0$, the plate is assumed to be accelerating with velocity $U_0 e^{\alpha t'}$ in its own plane along the x' -axis and non-uniform temperature (which is also known as ramped temperature) is assumed to be varying as $T' = T_\infty' + (T_w' - T_\infty') t'/t_0$ when $t' < t_0$ which are hereafter maintained at a uniform temperature T_w' for $t' > 0$. Also for $t' > 0$, species concentration is raised to C_w' . For free convective flows, here we also assume that all the physical variables except pressure are functions of the space co-ordinate y' and t' only.

The fluid under consideration is a metallic liquid whose magnetic Reynolds number is small and hence induced magnetic field produced by fluid motion is negligible in comparison to applied magnetic field of strength B_0 . In the absence of any input electric field and chemical reaction, the equations governing the unsteady MHD free convection heat and mass transfer flow of a viscous, incompressible, electrically conducting and heat absorbing fluid under above stated configuration are

$$\frac{\partial u'}{\partial t'} = v \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' + g \beta' (T' - T_\infty') + g \beta'' (C' - C_\infty') \quad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{Q_0}{\rho c_p} (T' - T_\infty') \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} \quad (3)$$

Where u' , ρ , g , β' , β^* , T , C , c_p , k , v , σ , D and Q_0 respectively are fluid velocity in the x' - direction, the fluid density, acceleration due to gravity, volumetric coefficient of thermal expansion, volumetric coefficient of expansion for concentration, the temperature of the fluid, species concentration, specific heat at constant pressure, thermal conductivity, the kinematic coefficient of viscosity, electrical conductivity, chemical molecular diffusivity, the heat absorption coefficient.

In the view of the physics of the problem, the initial and boundary conditions are

For $t' \leq 0$: $u' = 0$, $T' = T_\infty'$, $C' = C_\infty'$ for $\forall y' \geq 0$,

For $t' > 0$: $u' = U_0 e^{at}$, $C' = C_w'$ at $y' = 0$ (4)

For $0 < t' \leq t_0$: $T' = T_\infty' + (T_w' - T_\infty') t'/t_0$ at $y' = 0$

For $t' > t_0$: $T' = T_w'$ at $y' = 0$

For $t' > 0$: $u' \rightarrow 0$, $T' \rightarrow T_\infty'$, $C' \rightarrow C_\infty'$ as $y' \rightarrow \infty$

Introducing the following dimensionless variables and parameters as

$$\begin{aligned} y &= \frac{y'}{U_0 t_0}, \quad u = \frac{u'}{U_0}, \quad t = \frac{t'}{t_0}, \quad T = \frac{T' - T_\infty'}{T_w' - T_\infty'}, \quad C = \frac{C' - C_\infty'}{C_w' - C_\infty'}, \quad M = \frac{\sigma B_0^2 \theta}{\rho U_0^2}, \\ Gr &= \frac{g \beta' \theta (T_w' - T_\infty')}{U_0^3}, \quad Gm = \frac{g \beta^* \theta (C_w' - C_\infty')}{U_0^3}, \quad Pr = \frac{\rho c_p \theta}{k}, \quad Sc = \frac{\theta}{D} \text{ and } \phi = \frac{\theta Q_0}{\rho c_p U_0^2} \end{aligned} \quad (5)$$

Equations (1) – (3) becomes

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - Mu + GrT + GmC \quad (6)$$

$$\frac{\partial T}{\partial t} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} - \phi T \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} \quad (8)$$

Where M is the Magnetic number, Gr is the thermal Grashof number, Gm is the mass Grashof number, Pr is the Prandtl number, Sc is the Schmidt number, ϕ the heat absorption parameter and the characteristic time t_0 is defined as $t_0 = \frac{\theta}{U_0^2}$ (9)

The corresponding initial and boundary conditions in non-dimensional form are:

$u = 0$, $T = 0$, $C = 0$ for $y > 0$ and $t \leq 0$

$u = e^{at}$, $C = 1$ for $y = 0$ and $t > 0$

$$T = t \text{ at } y = 0 \text{ and } 0 < t \leq 1 \quad (10)$$

$T = 1$ at $y = 0$ and $t > 1$,

$$u \rightarrow 0, \quad T \rightarrow 0, \quad C \rightarrow 0 \text{ as } y \rightarrow \infty \text{ for } t > 0$$

The system of equations (6) - (8) subject to initial and boundary conditions (10) includes the effect of MHD free convective heat and mass transfer flow of a viscous, incompressible, electrically conducting and heat absorbing fluid passing an exponentially accelerating infinite non-conducting vertical flat plate with non-uniform temperature distribution.

3. Solution of the Problem

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, we assume the trial solution for velocity, temperature and concentration as

$$u(y, t) = u_0 e^{i\omega t}, \quad T(y, t) = T_0 e^{i\omega t}, \quad C(y, t) = C_0 e^{i\omega t} \quad (11)$$

Using (11) in (6) - (8) we get

$$u_0'' - (M + i\omega) u_0 = -Gr T_0 - Gm C_0 \quad (12)$$

$$T_0'' - (\phi + i\omega) Pr T_0 = 0 \quad (13)$$

$$C'' - i\omega Sc C_0 = 0 \quad (14)$$

The corresponding initial and boundary conditions are

$$u_0 = 0, \quad T_0 = 0, \quad C_0 = 0 \quad \forall y \geq 0 \text{ and } t \leq 0$$

$$u_0 = e^{at} e^{-i\omega t}, \quad C_0 = e^{-i\omega t} \text{ at } y = 0 \text{ & } t > 0 \quad (15)$$

$$T_0 = t e^{-i\omega t} \text{ at } y = 0 \text{ and } 0 < t \leq 1$$

$$T_0 = e^{-i\omega t} \text{ at } y = 0 \text{ and } t > 1,$$

$$u_0 = 0, \quad T_0 = 0, \quad C_0 = 0 \text{ as } y \rightarrow \infty \quad \forall t > 0$$

Solutions of equations (12) - (14) with the help of initial and boundary conditions (15), we get

$$u(y, t) = \begin{cases} e^{at} e^{-\zeta y} + \frac{Gr t}{\lambda_1^2 - \zeta^2} (e^{-\zeta y} - e^{-\lambda_1 y}) + \frac{Gm}{\lambda_2^2 - \zeta^2} (e^{-\zeta y} - e^{-\lambda_2 y}), & \text{for } 0 < t \leq 1 \\ e^{at} e^{-\zeta y} + \frac{Gr}{\lambda_1^2 - \zeta^2} (e^{-\zeta y} - e^{-\lambda_1 y}) + \frac{Gm}{\lambda_2^2 - \zeta^2} (e^{-\zeta y} - e^{-\lambda_2 y}), & \text{for } t > 1 \end{cases} \quad (16)$$

$$T(y, t) = \begin{cases} t e^{-\lambda_1 y}, & \text{for } 0 < t \leq 1 \\ e^{-\lambda_1 y}, & \text{for } t > 1 \end{cases} \quad (17)$$

$$C(y, t) = e^{-\lambda_2 y} \quad (18)$$

Where $\zeta = \sqrt{M + i\omega}$, $\lambda_1 = \sqrt{(\varphi + i\omega)Pr}$, $\lambda_2 = \sqrt{i\omega Sc}$

Equations (16) - (18) represent the analytic solution for fluid velocity, fluid temperature and fluid concentration for the flow of a viscous, incompressible, electrically conducting and heat absorbing fluid passing an exponentially accelerating infinite non-conducting vertical flat plate with non-uniform temperature distribution. To study the influence of non-uniform temperature distribution, such a flow is compared with uniform temperature. Solution for the fluid velocity, fluid temperature and fluid concentration for uniform temperature (Isothermal) plate is given by

$$u(y, t) = e^{at} e^{-\zeta y} + \frac{Gr}{\lambda_1^2 - \zeta^2} (e^{-\zeta y} - e^{-\lambda_1 y}) + \frac{Gm}{\lambda_2^2 - \zeta^2} (e^{-\zeta y} - e^{-\lambda_2 y}) \quad (19)$$

$$T(y, t) = e^{-\lambda_1 y} \quad (20)$$

$$C(y, t) = e^{-\lambda_2 y} \quad (21)$$

4. Skin Friction, Nusselt Number and Sherwood Number

Important physical parameters viz. skin friction, Nusselt number and Sherwood number are evaluated for such type of flows as

Knowing the velocity field, the skin friction at the plate can be obtained as it measures the rate of friction at the plate and is given by

$$\tau = \left(\frac{\partial u}{\partial y} \right)_{y=0} = \begin{cases} -\zeta e^{at} - \frac{Gr t}{\lambda_1^2 - \zeta^2} (\zeta - \lambda_1) - \frac{Gm}{\lambda_2^2 - \zeta^2} (\zeta - \lambda_2) & \text{for Ramped plate} \\ -\zeta e^{at} - \frac{Gr}{\lambda_1^2 - \zeta^2} (\zeta - \lambda_1) - \frac{Gm}{\lambda_2^2 - \zeta^2} (\zeta - \lambda_2) & \text{for Isothermal plate} \end{cases} \quad (22)$$

Knowing the Temperature field, the Nusselt number can be obtained as it measures the rate of heat transfer at the plate and is given by

$$Nu = - \left(\frac{\partial T}{\partial y} \right)_{y=0} = \begin{cases} t\lambda_1 & \text{for Ramped plate} \\ \lambda_1 & \text{for Isothermal plate} \end{cases} \quad (23)$$

Knowing the Concentration field, the Sherwood number (Sh) can be obtained which measures the rate of mass transfer at the plate and is given by

$$Sh = - \left(\frac{\partial C}{\partial y} \right)_{y=0} = \lambda_2 \quad (24)$$

From the expression (21) it is clear that Sherwood number (Sh) is proportional to \sqrt{Sc} ($= \sqrt{i\omega Sc}$) i.e. Sherwood number increases on increasing the Schmidt's number. As Sc is the relative strength of viscosity to mass diffusivity of the fluid, so Sc decreases on increasing mass diffusivity. This implies that mass diffusivity tends to reduce the rate of mass transfer at the plate.

5. Results and Discussions

The effects of various governing parameters on physical quantities for both isothermal and Ramped temperature plates are displayed graphically in the figures (1) - (12) and discussed in detail.

Fig.1 and Fig. 2 depicts the effects of plate acceleration parameter ‘ a ’ and time respectively on the velocity profiles for both ramped and isothermal plate which indicates that the fluid velocity increases with the increase in values of ‘ a ’ and the progression in time. However Fig 3 and Fig. 4 indicates that the increase in Prandtl number and Schmidt’s number decreases the fluid velocity . Fig. 5 and Fig 6 shows the effect of Thermal Grashof number and mass Grashoff number respectively on velocity field where it is noticed that the increase in Thermal Grashof number and mass Grashoff number ,increases velocity field. Fig. 7 shows the effect of heat absorption parameter on velocity field. It is observed that as the heat absorption parameter increases there is increase in velocity initially but after certain distance it gradually decreases. Fig. 8 indicated the effect of Magnetic number on velocity field where it is observed that velocity decreases with increase of Magnetic number. So in general we can say that magnetic field, heat absorption and viscosity of fluid tend to decrease the fluid flow, however thermal diffusion, mass diffusion, thermal buoyancy force and mass buoyancy force shows the opposite effects.

Fig. 9 illustrates that the fluid temperature increases with t . However Fig. 10 and 11 shows that fluid temperature decreases with increase in heat absorption parameter and Prandtl number. So we can conclude that the heat absorption and viscosity tends to reduce the fluid temperature, whereas thermal diffusion and time have adverse effects. In Fig. 12, it is observed that fluid concentration increases with increase in Schmidt’s number. Further from figures 1-11, it is observed that fluid flow and fluid temperature for an isothermal plate are more dominant as compared with ramped temperature plate.

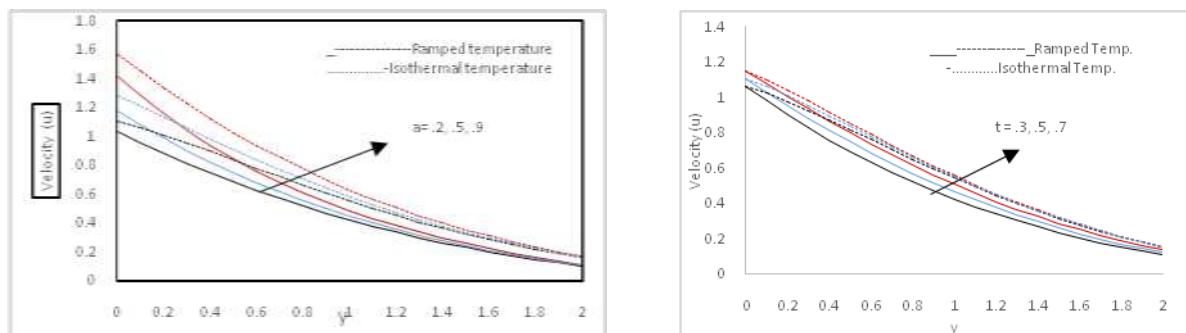


Fig. 1

Fig.1. Velocity profiles when $t = .5$, $Sc = .6$, $Pr = .71$, $\phi = 1$, $\omega = 1$, $M = 1$ $Gm = Gr = 2$,

Fig.2

Fig.2. Velocity profiles when $a = .2$, $Sc = .6$, $Pr = .71$, $\phi = 1$, $\omega = 1$, $M = 1$ $Gm = Gr = 2$

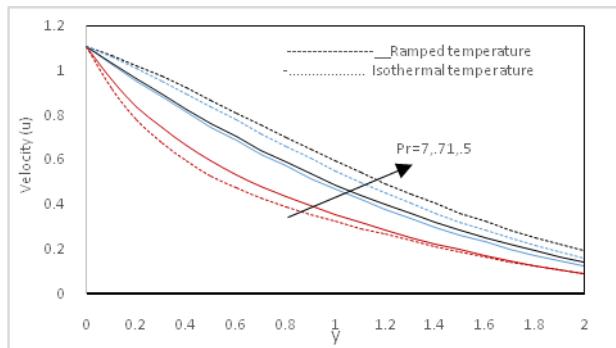


Fig. 3

Fig.3. Velocity profiles when $a = .2$, $t = .5$, $Sc = .6$, $\phi = 1$, $\omega = 1$, $M = 1$ $Gm = Gr = 2$

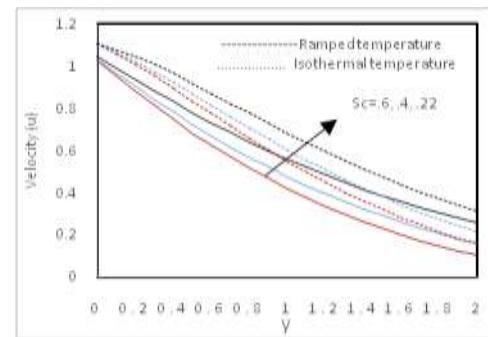


Fig.4

Fig.4. Velocity profiles when $a = .2$, $t = .5$, $Pr = .71$, $\phi = 1$, $\omega = 1$, $M = 1$ $Gm = Gr = 2$

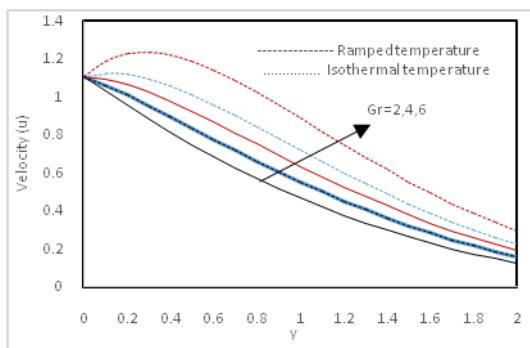


Fig. 5

Fig.5. Velocity profiles when $a = .2$, $t = .5$, $Sc = .6$, $Pr = .71$, $\phi = 1$, $\omega = 1$, $M = 1$ $Gm = 2$

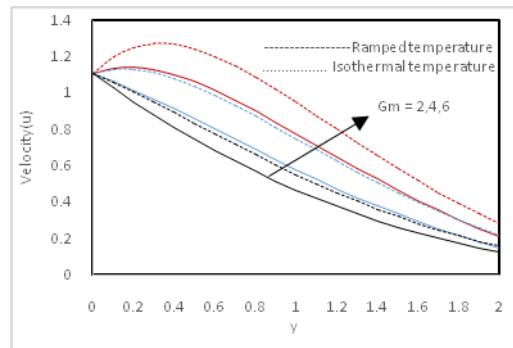


Fig.6

Fig.6. Velocity profiles when $a = .2$, $t = .5$, $Sc = .6$, $Pr = .71$, $\phi = 1$, $\omega = 1$, $M = 1$, $Gr = 2$

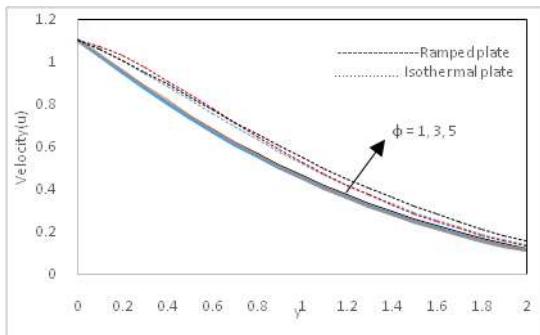


Fig. 7

Fig.7. Velocity profiles when $a = .2$, $t = .5$, $Sc = .6$, $Pr = .71$, $\omega = 1$, $M = 1$ $Gm = Gr = 2$

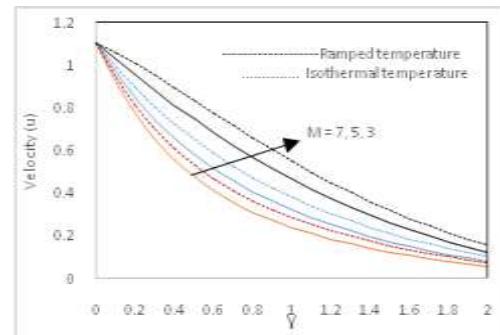


Fig.8

Fig.8. Velocity profiles when $a = .2$, $t = .5$, $Sc = .6$, $Pr = .71$, $\phi = 1$, $\omega = 1$, $Gm = Gr = 2$

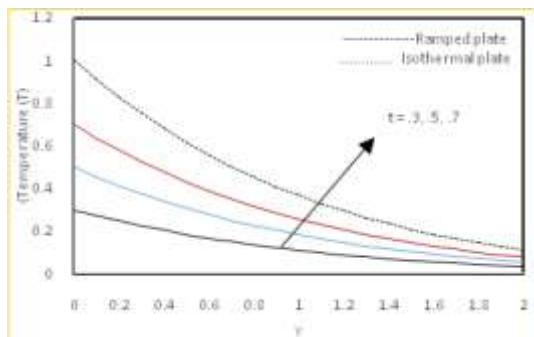


Fig. 9

Fig.9. Temperature profiles when, $Pr = .71$, $\phi = 1$, $\omega = 1$

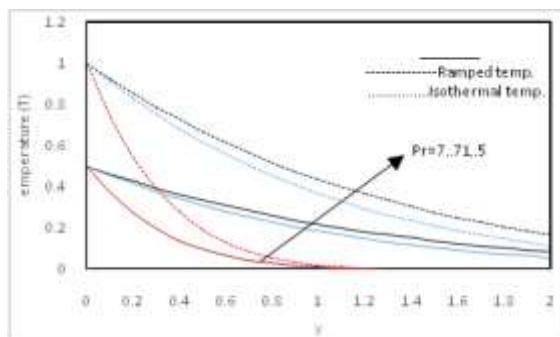


Fig.10

Fig.10. Temperature profiles when, $t = .5$, $\phi = 1$, $\omega = 1$

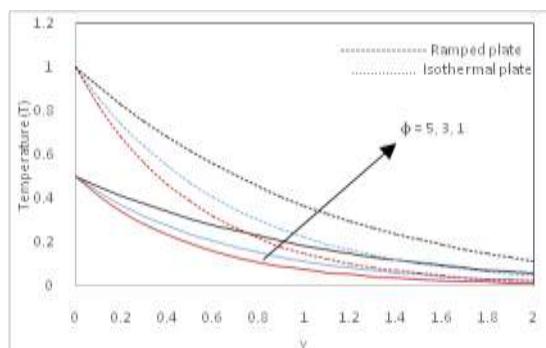


Fig. 11

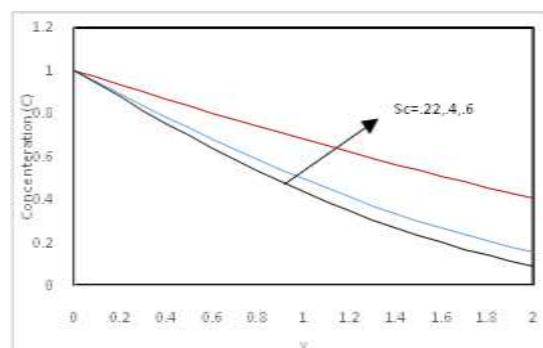


Fig.12

Fig.11. Temperature profiles when $t = .5$, $Pr = .71$, $\omega = 1$,

Fig.12. Concentration profiles when $t = .5$, $\varphi = 1$, $\omega = 1$

The numerical values of skin friction and Nusselt number for both uniform(isothermal) and non uniform temperature(ramped) plates are computed from the equations (22) - (24) and are represented in tabular form in tables 1 to 6 for various values of t, Pr, Sc, Gr, Gm, φ , and M.

Table 1. The Nusselt number at ramped temperature and isothermal plate when $w=1$, $Pr=.71$

	$t \downarrow / \varphi \rightarrow$	1	3	5
Nu_R	.3	.2777302970	.4437169950	.5680158994
	.5	.4628838283	.7395283251	.9466931657
	.7	.6480373596	1.0353396551	1.3253704321
Nu_I		.9257676567	1.47905665019	1.8933863315

Table 2. The Nusselt number at ramped temperature and isothermal plate when $w=1$, $\varphi=1$

	$t \downarrow / Pr \rightarrow$.3	.5	.71
Nu_R	.3	.1805322278	.2330660878	.2777302970
	.5	.3008870464	.3884434798	.4628838283
	.7	.4212418649	.5438208717	.6480373596
Nu_I		.6017740928	.7768869596	.9257676567

Table 3. Skin friction at ramped temperature and isothermal plate when, $\varphi = 1, \omega = 1$, $Sc = .6, M = 1$, $Gm = 2$, $t = .5$

	$Gr \downarrow / Pr \rightarrow$	0.3	0.5	0.71
τ_R	2	-0.676873741	-0.739806629	-0.794145715
	4	-0.335532583	-0.335532583	-0.444210755
	6	0.257540127	0.068741463	-0.094275795
τ_I	2	-0.209666807	-0.335532583	-0.444210755
	4	0.724747061	0.47301551	0.255659166
	6	1.659160929	1.281563602	0.955529086

Table 4. Skin friction at ramped temperature and isothermal plate when, $\varphi = 1$, $\omega = 1$, $Pr = .71$, $M = 1$, $Gr = 2$, $t = .5$

	Gm↓/Sc→	0.22	0.4	0.6
τ_R	2	-0.580779132	-0.704135991	-0.794145715
	4	0.428432409	0.181718693	0.001699243
	6	1.437643951	1.067573376	0.797544202
τ_I	2	-0.230844172	-0.35420103	-0.444210755
	4	0.77836737	0.531653653	0.351634204
	6	1.787578911	1.417508336	1.147479162

Table 5. Skin friction at ramped temperature and isothermal plate when, $\omega = 1$, $Pr = .71$, $Sc = .6$, $M = 1$, $Gr = 2$, $Gm = 2$

	t↓/φ→	1	3	5
τ_R	0.3	-0.858054127	-0.842080735	-0.781731222
	0.5	-0.794145715	-0.767523394	-0.666940873
	0.7	-0.733341598	-0.696070349	-0.555254819
τ_I	0.3	-0.368145183	-0.314900541	-0.113735498
	0.5	-0.444210755	-0.390966113	-0.18980107
	0.7	-0.523380622	-0.47013598	-0.268970937

Table 6. Skin friction at ramped temperature and isothermal plate when, $\varphi = 1$, $\omega = 1$, $Pr = .71$, $Sc = .6$, $Gr = 2$, $Gm = 2$

	t↓/M→	3	5	7
τ_R	0.3	-0.858054127	-1.518420799	-2.053530923
	0.5	-0.794145715	-1.490664767	-2.058135547
	0.7	-0.733341598	-1.466882773	-2.067431068
τ_I	0.3	-0.368145183	-1.080454624	-1.667347993
	0.5	-0.444210755	-1.177831785	-1.782290597
	0.7	-0.523380622	-1.279182984	-1.901924098

In Tabs 1&2, we observe that the Nusselt number for both ramped and isothermal plate's increases with increase in Pr , φ and t which shows that thermal diffusion tends to retard the rate of heat transfer whereas time and heat absorption increases the rate of heat transfer at plate. In Tables 3-6, it is evident that skin friction increases with

Gr, Gm or ϕ but it decreases with Pr, Sc or M. This implies that mass buoyancy force, thermal buoyancy force and heat absorption tend to increase the shear stress at the plate (both Ramped and Isothermal) whereas thermal diffusion, mass diffusion and magnetic field decreases the shear stress at the plate. Also it is noticed from tabs 5&6 that skin friction increases for the ramped plate whereas it decreases for isothermal plate on increasing t. This implies that with the time progression, the shear stress at ramped temperature plate is increased whereas it is reduced at isothermal plate.

6. Conclusion

The important findings of the theoretical study of effect of exponentially accelerating vertical ramped flat plate on an unsteady MHD free convection flow of a viscous, incompressible, electrically conducting and heat absorbing fluid in the presence of magnetic field are concluded as:

- The plate acceleration parameter accelerates the fluid velocities in boundary layer region
- Thermal diffusion and time enhance the both fluid velocity and fluid temperature whereas the heat absorption has adverse effects on them.
- Magnetic field tends to retard the fluid flow whereas thermal buoyancy force and mass buoyancy force accelerates the fluid flow in boundary layer region.
- Fluid velocities and fluid temperature are higher in case of isothermal plate as compared to ramped temperature plate.
- Mass diffusivity tends to increase both the species concentration and fluid velocity.
- Heat absorption and time increases the rate of heat transfer at the plate whereas thermal diffusion has reverse effect.
- Mass diffusion and magnetic field tend to decrease the shear stress at the plate while mass buoyancy force and thermal buoyancy force increases the same. As time progresses, there is a reduction in shear stress at isothermal plate whereas there is an enhancement in shear stress at ramped temperature plate.
- Mass diffusivity tends to reduce the rate of mass transfer at the plate.

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