The Split Domination and Irredundant Number of a Graph

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ABSTRACT

For a simple connected graph G and D $\subseteq$ V = V (G). Here we have continue some relation between split domination number and irredundant number. We give a characterization of graph G for which $\gamma_s(G) \le ir(G)$ for every graph G. Moreover some special graphs, sufficient condition of the domination number $\gamma_s(G) \ge 2$. Further, we have developed a method of construction of a graph with a given number as the split domination number.

***Key Words**: Irredundant set, Irredundant number, P-N set, Split Domination set, Split Domination number.*

I. INTRODUCTION

Graph theory is one of the most important branches of modern mathematics and computer applications. Here we discuss the relation of split domination and irredundant number of a graph. For a graph G = (V, E) and a vertex v $\in$ V. The open neighbourhood of v is the set N(v) = {u$\in$ V |uv $\in$ E} and the closed neighbourhood of v is N[v] = N(v) $\cup$ {v}. A dominating set is a set w $\subseteq$ V for which N[w] = V , or equivalently, for every vertex u $\in$ V - D, we have N (u) $\cap$ D $\neq \emptyset$. A set D of vertices in a graph G is a dominating set of G if every vertex in V - D is adjacent to some vertex in D.

A vertex v in a graph G is said to dominate itself and each of its neighbours. We say with other words, v dominates the vertices of its closed neighbourhood N[v]. The domination number of G equals the minimum cardinality over all dominating set in G and it is denoted by γ(G). A dominating set of cardinality γ(G) is referred to as a minimum dominating set. A dominating set of cardinality γ(G) is called a γ - set. A minimal dominating set in a graph G is a dominating set that contains no dominating set as a proper subset. A minimal dominating set of minimum cardinality is a minimum dominating set and consists of γ(G) vertices. It was in [1].

A set D is irredundant if for every vertex u $\in$ D, we have N [u]– N [D – {u}] $\neq \emptyset$. A subset D $\subseteq$ V is irredundant if for each x$\in$ D, D – {x} does not dominate N [x]. Equivalently, D is an irredundant set of vertices if N [D – {v}] $\neq$ N [D] for every vertex $v \in D$. The irredundant number of G, denoted by ir(G), is the minimum cardinality taken over all maximal irredundant set of vertices of G. An irredundant set of cardinality ir(G) is called an ir-set. It was specified in [2, 3].

A set $D \subseteq V$ is a dominating set of G if $N[D] = v$, while $D \subseteq V$ is an irredundant set of G if $PN(x, D) = N[x] - N[D - \{x\}] \neq \emptyset$ for every $x \in D$. D is a minimal dominating if and only if it is dominating and irredundant. An irredundant set D is maximal irredundant if no proper superset of D is irredundant. Which are indicated in [4]. Every vertex v with the property $N[D - \{v\}] \neq N[D]$ is an irredundant vertex. Consequently, every vertex in an irredundant set is an irredundant vertex. we refer to [5]. For a simple graph G and we observe that D is a P-N set if and only if the set of D - perfect vertices is a dominating set of G. It was specified in [6].

Theorem 1.1 [1] If G is a graph and $D \subseteq V(G)$ is a dominating set then D is an irredundant set.

The above observation motivate us to study of relation between split domination and irredundant number. The split domination and irredundant number of certain standard classes of graphs are determined. Various characterization results are proved.

II. BASIC DEFINITIONS

Definition 2.1 (Kulli and Janakiram[7]) A dominating set $D \subseteq V(G)$ is a split dominating set if the induced subgraph $< V \setminus D >$ is either disconnected or a K_1.

Definition 2.2 [7] A split domination set D is a minimal split dominating set if

i) every vertex $v \in D$ has a private neighbour with respect to D or

ii) for every vertex $v \in D$, the induced graph $< (V \setminus D) \cup \{v\} >$ is connected.

Definition 2.3 [7] Let G = (V,E) be a graph. Then $\gamma_s(G)$ = min{ |D| : D is a split domination set} is the split domination number and $\gamma_s(G)$ = max{ |D| : D is a minimal split dominating set} is the upper split domination number.

Definition 2.4 [8] A split irredundant set is a set D such that

i) for $u \in D$, u has a private neighbour with respect to V (S) and

ii) the induced graph $< V \setminus D >$ is either disconnected or K_1.

Definition 2.5 [8] A maximal split irredundant set D is a split irredundant set such that for every $v \in V \setminus D$one of the following holds true :

i) v does not have a private neighbour with respect to V $(D \cup \{v\})$ or

ii) the induced graph $< V \setminus \{D \cup v\} >$ is connected

Definition 2.6 [8] Let G = (V, E) be a graph. Then $ir_s(G)$ = min{ |D| : D is a maximal split irredundant set} is the lower split irredundant number and IRs(G) = max{ |D| : D is a maximal split irredundant set} is the upper split irredundant number.

Definition 2.7 [6] A set $D \subseteq V = V(G)$, vertex u of G is known as D - perfect if $|N[u] \cap D| = 1$.

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Definition 2.8 [6] The set D is called a perfect neighbourhood set if for all v $\in$ V, v or some neighbour of v is D - perfect.

III. IMPORTANT RESULTS

Some known inequalities and bounds that are of interest are the following:

- [7] $\gamma(G) \leq \gamma_s(G)$
- [7] $\gamma_s(G) \leq n.\Delta(G) / (\Delta(G) + 1)$.
- [9] $\gamma(G) + \gamma_s(G) \leq n$.
- [8] For the bipartite graph $K_{m,n}$

  $ir_s(G) = \gamma_s(G) = \min\{m, n\}$.

  $\Gamma s(G) = IRs(G) = \max\{m, n\}$.
- [8] For a path P_n,

  $ir_s(G) = \gamma_s(G) = \lceil n/3 \rceil$.

  $\Gamma s(G) = IRs(G) = \lceil n/2 \rceil$.
- [8] For a cycle C_n,

  $ir_s(G) = \gamma_s(G) = \lceil n/3 \rceil$.

  $\Gamma s(G) = IRs(G) = \lceil n/2 \rceil$.
- [8] $\gamma_s(C_p) = [p / 3]$, where [x] is the least + ve integer not less than x and C_p is a cycle with $p \geq 4$ vertices.
- [8] $\gamma_s(W_p) = 3$, where W_p is a wheel with $p \geq 5$ vertices.
- [8] $\gamma_s(K_{m,n}) = m$, where $2 \geq m \geq n$ and $K_{m,n}$ is a complete bipartite graph.

IV. SPLIT DOMINATING NUMBER FOR SOME SPECIAL CLASS GRAPHS

4.1 K_4 Graph

K_4 graph contains a vertex set V = {v_1, v_2, v_3, v_4}. It has the subset namely D = {v_2, v_4}. Every vertex of D is incident with < V - D >. If we remove D from V, then < V - D > will be disconnected. Therefore the split domination number for K_4 graph is 2. i.e., $\gamma_s(G) \geq 2$.

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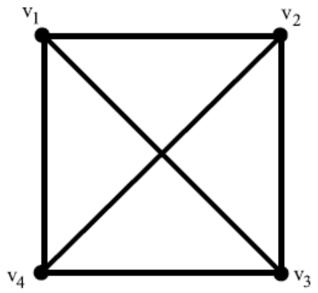

Figure 1: K_4 Graph.

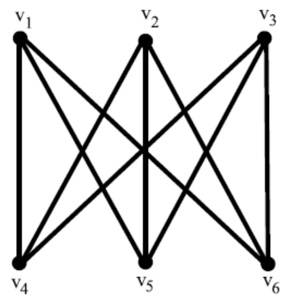

Figure 2: Thomsen Graph.

4.2 Thomsen Graph

In Thomsen graph the vertex set V = {v_1, v_2, v_3, v_4, v_5, v_6} and it has the subset D = {v_4, v_5, v_6} of vertex set. Which is incident with < V - D >. If we remove D from V, then D is disconnected. Therefore the split domination number is greater than 2. i.e., $\gamma_s(G) \geq 2$.

4.3 Cycle Graph

Cycle graph vertices V = {v_1, v_2, v_3, v_4, v_5} and it has a subset D whose vertices are {v_2, v_5} and every vertex of D is incident with < V - D >. When D is removed from V, < V - D > is disconnected. Hence the split domination number is greater than 2. i.e., $\gamma_s(G) \geq 2$.

4.4 Cube Graph

Cube graph contains a vertex set V = {v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8}. It has the subset namely D = {v_2, v_6, v_8}. Every vertex of D is incident with < V - D >. If we remove D from V, then < V - D > will be disconnected. Therefore the split domination number for cube graph is 3. i.e., $\gamma_s(G) \geq 3$.

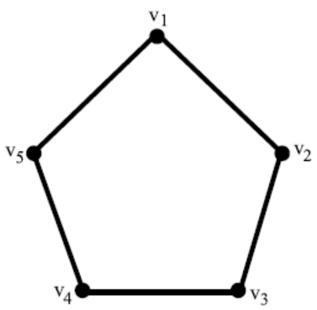

Figure 3: Cycle Graph.

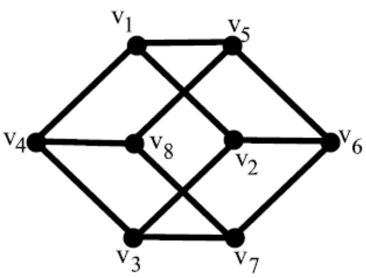

Figure 4: Cube Graph.

Theorem 10. A split dominating set D if there exists two vertices w_1, $w_2 \in$ V - D such that every w_1 - w_2 path contains a vertex D.

Proof. Let D be any set split dominating set of G. Then <V - D > has atleast two vertices from different components.

The vertices w_1 and w_2 are not adjacent, but they are connected by a path contains atleast one vertex in D. Hence the result.

Theorem 11. If D is a split dominating set of G then <V - D > is a split dominating set of G.

Proof. Since D is a minimal split dominating set of G. Let us consider a vertex v $\in$ D. There exists u $\in$ V - D such that V does not satisfy N(u) $\cap$ D = {v}. Therefore V - D is a dominating set of G and further it is a split dominating set. Since < D > is disconnected. Hence V - D is also a split dominating set.

Theorem 12. For any graph G then $\gamma_s(G) \leq$ ir(G).

Proof. Let D be a maximum irredundant set of vertices of G. Clearly V - D is a dominating set of G. Then D has atleast two vertices and every vertex in D is adjacent to some vertex in V - D. This implies that V - D is a split dominating set of G. Also V - D is the maximum irredundant set.

Therefore $\gamma_s(G) \leq$ |V – D| = ir(G).

Hence $\gamma_s(G) \leq$ ir(G).

Theorem 13. If D is a P-N set of a 4-regular graph, then D is irredundant.

Proof. Suppose D is not irredundant. Then there exists w $\in$ D with I(w, D) = ∅. Neither w nor any neighbour of w is D - perfect. Therefore D is not a P-N set. Hence, D is irredundant.

V. CONCLUSION

In this paper we discussed the relation between the split domination number and irredundant number. Then here we investigate the sufficient condition of the split domination number for some special graph. Also we develop the construction of graph using the split domination number and irredundant number. Domination and irredundant can stand together to facilitate the network communication process, electrical network, etc.

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