

INVESTIGATION OF DIVIDED HELMHOLTZ RESONATOR BY PERFORATED PLATE

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ABSTRACT

Now-a-days we have witnessed a great growth in noise in our surrounding .In this project we are going to investigate the performance of a Helmholtz resonator using a perforated panel. The current resonator has a drawback of low frequency bandwidth, it has simple geometry. In the proposed resonator we have attached a perforated panel in its geometry. In our new modified resonator it is expected an increase in frequency bandwidth and increase in transmission loss. In this project we have first analytically transformed the resonator into mathematical model in single degree and two degree of freedom and obtained the dimensions. We have made a CAD model from the theoretical dimensions and also programmed it in matlab for easy calculations. Theoretically the sound reduces by 2dB and experimental analysis will we done after construction of the physical model in the next semester.

1.INTRODUCTION

Sound is all around. Sometimes it is experienced as pleasant, sometimes as unpleasant. Unwanted sound is generally referred to as noise. Noise is a frequently encountered problem in modern society. One of the environments where the presence of noise causes a deterioration in people's comfort is in aircraft cabins. Noise encountered in daily life can, for example, be caused by domestic appliances such as vacuum cleaners and washing machines, vehicles such as cars and aeroplanes. So it has to be reduce to some extent level for reducing the undesirable effects on environment and social area.

Common methods of passive noise reduction are the use of porous materials such as glass wool or foam, double wall panels with thin air layers, or the application of acoustic resonators. Knowledge of Helmholtz resonance and its theoretical description have been used over the years to design and analyse various systems. An advantage of the Helmholtz resonator is that it has the characteristic of strong sound attenuation, even though its geometry is relatively simple. When it, is appropriately tuned, it can substantially reduce noise over the low frequency domain. Many researchers and engineers have been interested in, and employed, the Helmholtz resonator for a variety of applications, including a range of geometries, some of them as part of complex configurations. Helmholtz resonators are in common use in applications such as acoustic elements in rooms

and in duct silencers. They can take two principal forms: single resonators or distributed resonators. The perforated panels often used in rooms is an example of distributed Helmholtz resonators. The geometries of Helmholtz resonators are very diverse, but they all have two characteristic features in common: A cavity and a relatively small opening through which the sound energy enters the cavity. In the case of the distributed Helmholtz resonator, the cavity is shared by the resonator openings. For a limited frequency range, a Helmholtz resonator is analogous to a simple mechanical damped resonator system. The resilience of the air in the cavity makes it similar to a spring. The mass of the air in and around the orifice is equivalent to a mechanical mass. The value of the “spring constant” and the mass is what mainly determines the resonance frequency.

II. CONVENTIONAL HELMHOLTZ RESONATOR MODEL

Hermann Ludwig Ferdinand von Helmholtz (August 31, 1821 – September 8, 1894) created the device known as the Helmholtz resonator in the 1860s. Helmholtz resonance is widely known as the phenomenon of air resonance in the cavity or chamber that contains a gas. The highest amplitude sound is generated near and at the resonant frequency, which is determined by the volume and the neck dimensions of the Helmholtz resonator. A well-known configuration of such a resonator is the musical tone generated when air is blown across the top of an empty soda bottle.

A Helmholtz resonator is an acoustic band stop filter comprises of a rigid cavity with a protruding neck that connects the cavity to the system of interest. The behaviour of HR is analogues to that of spring-mass-damper system as shown in fig.1.4. The excitation is provided by tonal pressure fluctuations acting over the opening of the neck, resulting in the oscillations of the volume of air in the neck. The pressure increase in the cavity provides a reacting force analogous to that of spring. Damping appears in the form of radiation losses at the neck end and viscous losses due to friction of the oscillating air in the neck.

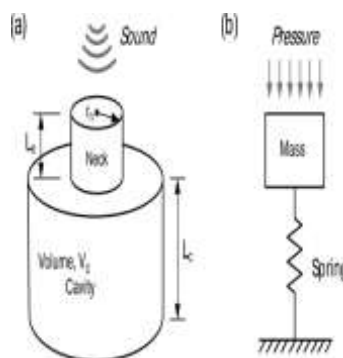


Fig1.2: Conventional HR with spring mass system

In this work, we will study the effect of perforation of plate inside the resonator on transmission loss & bandwidth of the single Helmholtz resonator as shown in fig.1.5 below

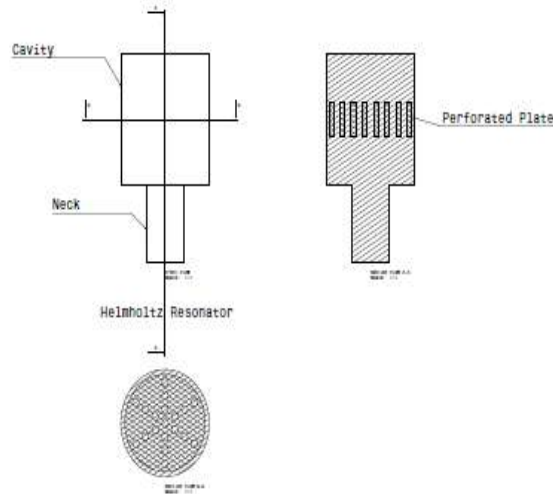


Fig1.3: Drafting of HR with plate

III.ANALYSIS

3.1 RESONANT FREQUENCY OF THE HELMHOLTZ RESONATOR

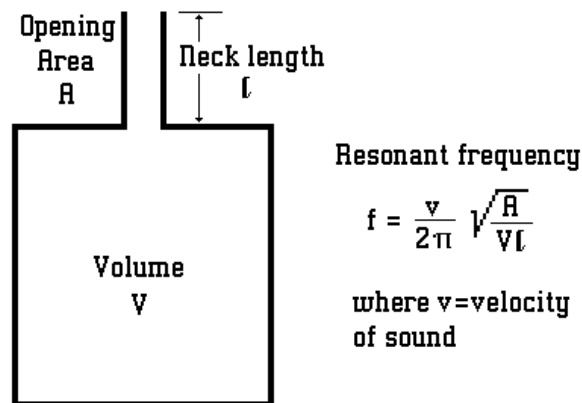


Fig3.1: schematic of HR

The effective mass of the Helmholtz resonator is given by

$$m = \rho S L_{\text{eff}}$$

The effective length is given by

$$L_{\text{eff}} = L + 1.7a$$

Where,

a=radius of neck &

L =actual neck length

The stiffness of the resonator is define as the reciprocal of the compliance.

The force is written as $F=PS$

Where,

P=pressure of the neck entrance

S=surface area of the neck

To develop the resonator stiffness as a function of resonator dimensions, consider the thermodynamics analysis of the resonator cavity. Assuming the system is adiabatic and the air is an ideal gas with constant specific heat constant. The polytrophic process equation for the resonator is

$$PV^{\gamma}=\text{constant}$$

Where,

γ =ratio of specific heat

V=cavity volume of the Helmholtz resonator

$$\therefore \frac{P}{Pa} = \frac{-\Delta V}{V} = \frac{\Delta Sx}{V}$$

Where,

A=acceleration

Or

$$\frac{d^2x}{dt^2} = \frac{F}{m}$$

$$\therefore \frac{d^2x}{dt^2} = \frac{PS}{\rho SL} = \frac{-\Delta SPa}{\rho VL} * x$$

As process is simple harmonic motion,

Hence,

$$F_{net} = m \frac{d^2x}{dt^2} = -Kx \quad \dots \text{as } F=-Kx$$

$$\therefore \frac{d^2x}{dt^2} = \frac{-k}{m} * x$$

And

$$\therefore \omega = \sqrt{\frac{K}{M}}$$

Where, K is the spring constant (N/m) of the spring and M is the mass (kg). In a Helmholtz resonator an arbitrarily-shaped container of volume V (m³) with an circular opening of cross-sectional area S (m²), the mass of air M contained within the opening “neck” of the container oscillates back and forth due to the effective spring constant associated with the air contained within the volume V of the whole container alternatively being compressed/rarified as above/below atmospheric pressure once each cycle of oscillation. Since the mass of air oscillating back and forth in the neck of the Helmholtz resonator at a frequency shown below

Equation of the frequency is given by

$$F = \frac{\omega}{2\pi}$$

$$\therefore F = \frac{1}{2\pi} * \frac{\square SP\alpha}{\rho VL}$$

Where C is determined by density, pressure and \square ,

$$F = \frac{C}{2\pi} * \sqrt{\frac{S}{VL}}$$

3.2 HELMHOLTZ RESONATOR WITH PERFORATED PLATE FOR n NUMBER OF HOLES

3.2.1 For plate with one hole i.e. orifice:

From FBD of two masses m₁ and m₂ the two differential equations of motion are:

$$m_1 \ddot{x}_1 - k_1(x_2 - x_1) = 0 \quad \dots 1$$

$$m_2 \ddot{x}_2 + k_2 x_2 + k_1(x_2 - x_1) = 0 \quad \dots 2$$

By rearranging the terms of equations, we get,

$$m_1 \ddot{x}_1 - k_1 x_2 - k_1 x_1 = 0 \quad \dots 3$$

$$m_2 \ddot{x}_2 + (k_2 + k_1)x_2 - k_1 x_1 = 0 \quad \dots 4$$

Assuming masses m_1 and m_2 executes harmonic vibration at frequency ω , the solutions for x_1 and x_2 under steady state condition are:

$$x_1 = A \sin(\omega t) \quad \dots 5$$

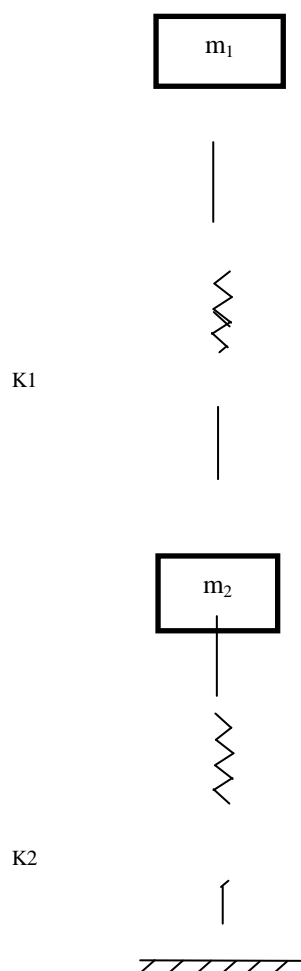


Fig3.2: Two degree of freedom system of perforated plate with one hole

$$x_2 = B \sin(\omega t) \quad \dots 6$$

where A and B are the amplitudes of the two vibrations of the two masses m_1 and m_2 under steady condition.

Therefore,

$$\therefore \ddot{x}_1 = -A\omega^2 \sin(\omega t) \quad \dots 7$$

$$\therefore \ddot{x}_2 = -B\omega^2 \sin(\omega t) \quad \dots 8$$

Substituting equ. 5, 6 & 7 in equ. 3, we get,

$$-m_1 A \omega^2 \sin(\omega t) - k_1 B \sin(\omega t) + k_1 A \sin(\omega t) = 0$$

$$A(-m_1 \omega^2 \sin(\omega t) + k_1 \sin(\omega t)) = k_1 B \sin(\omega t)$$

$$A(-\omega^2 m_2 + k_2 + k_1) = k_1 B$$

$$\therefore \frac{A}{B} = \frac{k_1}{k_1 - m_1 \omega^2} \quad \dots 9$$

Substituting equ. 5, 6 & 8 in equ. 4, we get,

$$-\omega^2 \sin(\omega t) B m_2 + k_2 B \sin(\omega t) + k_1 B \sin(\omega t) - k_1 A \sin(\omega t) = 0$$

$$B(-\omega^2 m_2 + k_2 + k_1) = k_1 A$$

$$\therefore \frac{A}{B} = \frac{k_1 + k_2 - \omega^2 m_2}{k_1} \quad \dots 10$$

From equ. 9 & 10 we get,

$$\therefore \frac{k_1}{k_1 - m_1 \omega^2} = \frac{k_1 + k_2 - \omega^2 m_2}{k_1}$$

$$\therefore k_1^2 = (-\omega^2 m_2 + k_2 + k_1)(-\omega^2 m_2 + k_2 + k_1)$$

$$k_1^2 = k_1^2 - m_1 \omega^2 k_1 + k_1 k_2 - m_1 \omega^2 k_2 - k_1 \omega^2 m_2 + m_1 \omega^4 m_2$$

$$\therefore m_1 m_2 \omega^4 + (-m_1 k_1 - m_1 k_2 - k_1 m_2) \omega^2 + k_1 k_2 = 0 \quad \dots 11$$

the above equ. is quadratic in ω^2 and gives two values of ω^2 , (two positive values of ω and two negative values of ω). The two positive values of ω give natural frequencies ω_{n1} and ω_{n2} of the system.

Therefore above equ is called as **frequency equation**.

Consider following special cases to study the behaviour of the system.

Hence,

$$\therefore m_1 m_2 \omega^4 + (-m_1 k_1 - m_1 k_2 - k_1 m_2) \omega^2 + k_1 k_2 = 0$$

by comparing above equ with quadratic equ as shown below

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Hence,

$$a = m_1 m_2$$

$$b = -(k_1 m_1 + m_1 (k_1 + k_2))$$

$$c = k_1 k_2$$

3.2.2 For perforated plate with two hole:

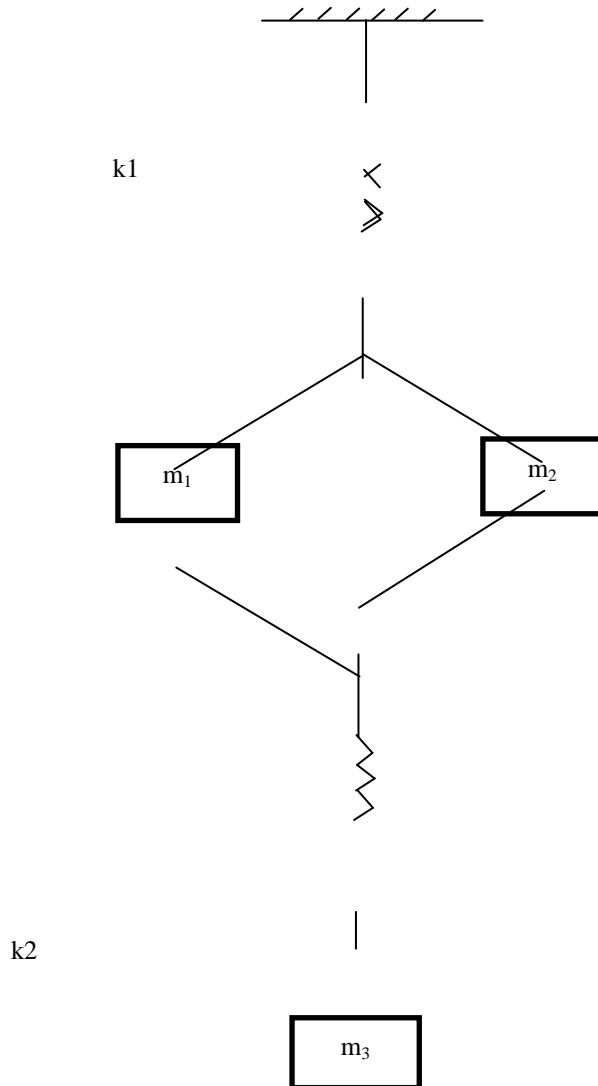


Fig3.3: Two degree of freedom system of perforated plate with two hole

From FBD of two masses m_1 and m_2 the two differential equations of motion are:

$$m_1 \ddot{x}_1 + m_2 \ddot{x}_1 + k_1 x_1 - k_2 (x_2 - x_1) = 0$$

$$m_3 \ddot{x}_2 + k_2 (x_2 - x_1) = 0$$

Assuming masses m_1 and m_2 executes harmonic vibration at frequency ω , the solutions for x_1 and x_2 under steady state condition are:

$$\ddot{x}_1 = -A\omega^2 \sin(\omega t)$$

$$\ddot{x}_2 = -B\omega^2 \sin(\omega t)$$

$$-m_1 A \omega^2 \sin(\omega t) - m_2 A \omega^2 \sin(\omega t) + k_1 A \sin(\omega t) + k_2 (B \sin(\omega t) - A \sin(\omega t)) = 0$$

$$(-m_1 \omega^2 - m_2 \omega^2 + k_1 + k_2) A = k_2 B$$

$$\therefore \frac{A}{B} = \frac{k_2}{k_1 + k_2 - \omega^2 (m_1 + m_2)}$$

$$-m_3 A \omega^2 \sin(\omega t) + k_2 (B \sin(\omega t) - A \sin(\omega t)) = 0$$

$$-m_3 B \omega^2 + k_2 B - k_2 A = 0$$

$$\therefore \frac{A}{B} = \frac{k_2 - \omega^2 m_3}{k_2}$$

$$\therefore \frac{k_2}{k_1 + k_2 - \omega^2 (m_1 + m_2)} = \frac{k_2 - \omega^2 m_3}{k_2}$$

$$m_1 = m_2 = m$$

$$(k_2 - m_3 \omega^2)(k_1 + k_2 - 2m \omega^2) = k_2^2$$

$$k_1 k_2 + k_2^2 - 2m \omega^2 k_2 - m_3 k_1 \omega^2 - m_3 \omega^2 k_2 + 2m m_3 \omega^4 = k_2^2$$

$$2m m_3 \omega^4 + (-2m k_2 - m_3 k_1 - m_3 k_2) \omega^2 + k_1 k_2 = 0$$

by comparing above equ with quadratic equ as shown below

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Hence,

$$a = 2m_1m_2$$

$$b = -(2mk_2 + m_3(k_1 + k_2))$$

$$c = k_1k_2$$

Similarly,

For perforated plate with three holes, Quadratic parameters are as follows,

$$a = 3m_1m_2$$

$$b = -(3mk_2 + m_3(k_1 + k_2))$$

$$c = k_1k_2$$

Therefore, perforated plate with n number of holes, Quadratic parameters are as follows,

$$a = nm_1m_2$$

$$b = -(nmk_2 + m_3(k_1 + k_2))$$

$$c = k_1k_2$$

By solving quadratic equ.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \left(\frac{k_2}{2m_3} + \frac{k_1 + k_2}{2nm} \right) \pm \sqrt{\frac{k_2^2}{4m_3^2} + \frac{(k_1 + k_2)^2}{4n^2m^2} + \frac{k_2^2}{2nmm_3} - \frac{k_1k_2}{2nm m_3}}$$

$$= \omega^2$$

Hence,

$$\omega_{n_1, n_2} = \sqrt{\left(\frac{k_2}{2m_3} + \frac{k_1 + k_2}{2nm} \right) \pm \sqrt{\frac{k_2^2}{4m_3^2} + \frac{(k_1 + k_2)^2}{4n^2m^2} + \frac{k_2^2}{2nmm_3} - \frac{k_1k_2}{2nm m_3}}}$$

Where,

N= total number of holes

K1= spring (air) stiffness for hole mass in N/mm

K2= spring (air) stiffness for neck mass in N/mm

m= Total mass of air in holes of perforated plate

m₃= Mass of air inside the neck

Therefore by analytically we get the equation of resonance frequency of Helmholtz resonator with perforated plate inside it.

3.3 DESIGN MODEL OF HELMHOLTZ RESONATOR WITH PLATE

3.3.1 Design parameters selected for Helmholtz resonator with perforated plate

We calculate resonator frequency value which further required for determining its application on the basis of its frequency range. The neck hole area should be equal to total number of holes in perforated plate in order to reduce the back pressure. We have done the calculation on basis of getting lesser resonant frequency for noise attenuation criteria due to sound energy loss. Analytical calculation was done on basis of neck length and cavity length variation represented in table

1. There are two constrains in the design of HR are as follows

Diameter of cavity (d_c) 100mm

Diameter of neck (d_n) 37mm

2. From using constrain values, for the frequency of 200HZ, other various parameters calculated by using HR resonance formula are as follows:

$$F = \frac{C}{2\pi} * \sqrt{\frac{S}{VL}}$$

Table-1: Resonator frequency calculation with ±10% variation in neck & cavity length

SR.NO.	Speed of sound(mm/s)	Cavity diameter(mm)	Neck diameter(mm)	Cavity length(mm)	Neck length(mm ²)	Frequency(HZ)
1.	3.4902E+05	100	37	188.8	72	210.823
2.	3.4902E+05	100	37	132	80	200
3.	3.4902E+05	100	37	145.2	88	190.697

Also following graphs show frequency vs parameters with ±10% variation in neck & cavity length

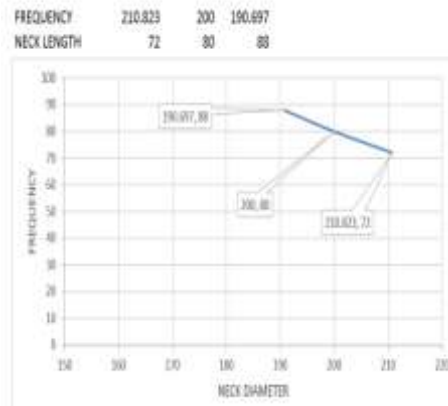


Fig3.4: Graph between frequency vs $\pm 10\%$ variation in neck length

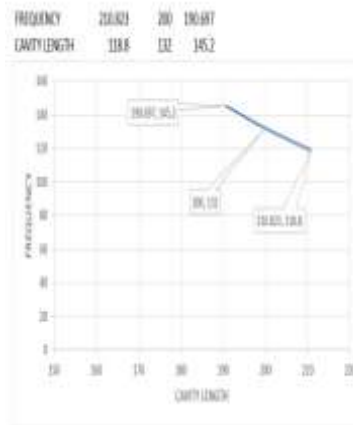


Fig3.5: Graph between frequency vs $\pm 10\%$ variation in cavity length



Fig3.6: Graph between frequency vs $\pm 10\%$ variation in sound velocity

The above calculation is done on a basis of given resonance frequency of resonator considering with variation in neck & cavity length. The layout of HR represented with dimensional nomenclature shown in fig

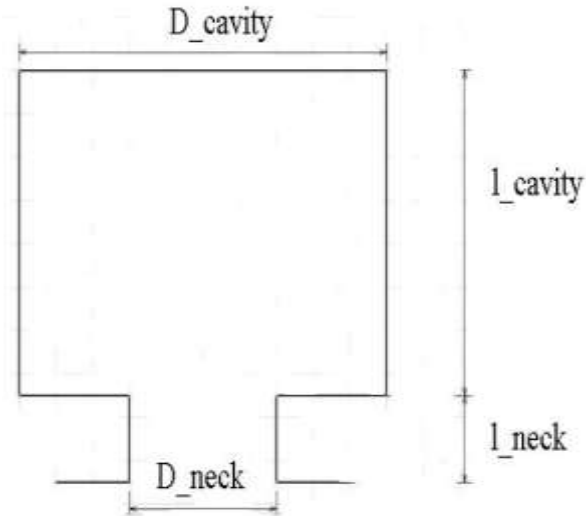


Fig3.7: Layout of Helmholtz resonator

From above table, we got resonator dimension as:

Length of cavity of HR (L_c)	132mm
Length of neck of HR (L_n)	80mm
Velocity of sound (at 28 ⁰) (C)	349.02m/s ²

Design model of Helmholtz resonator was done in CATIA software include above dimensions shown in below fig.

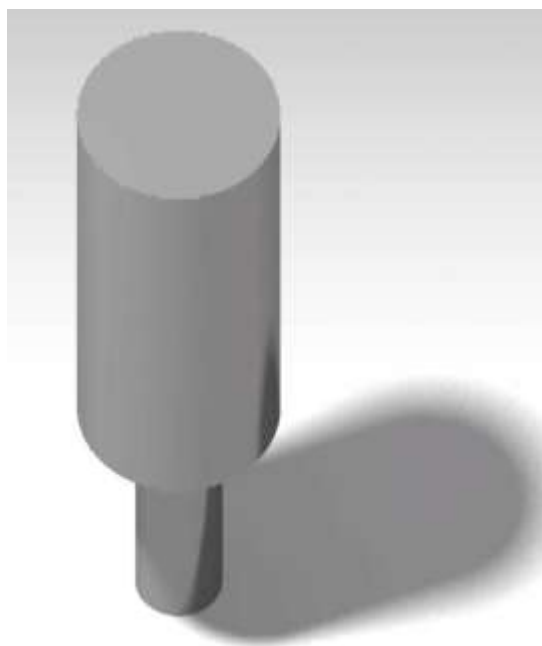


Fig3.8: CAD model of Helmholtz resonator

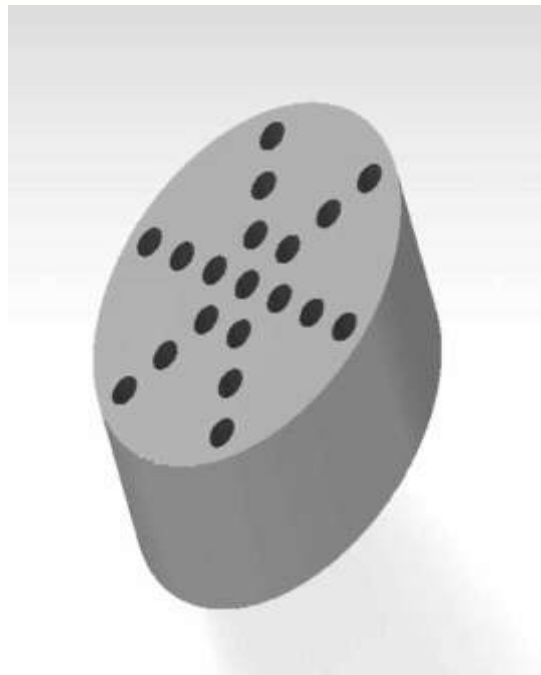


Fig3.9: CAD model of perforated plate

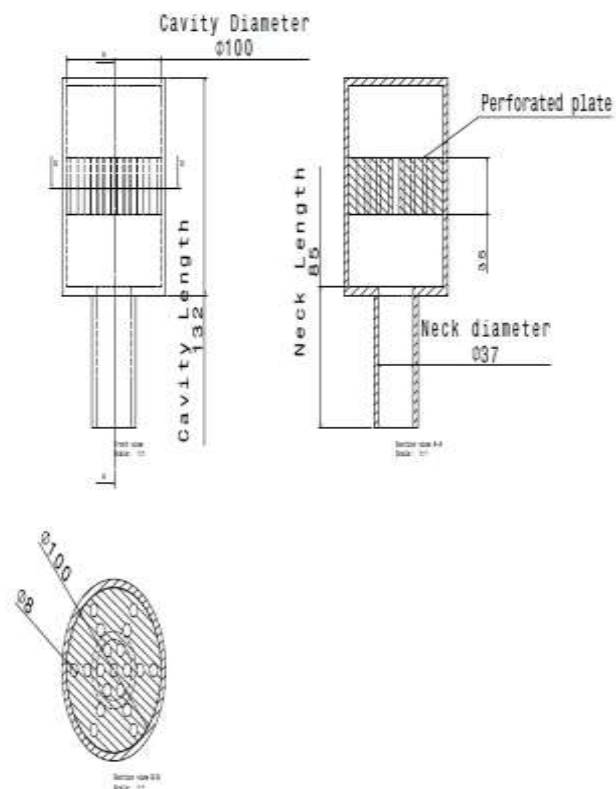


Fig3.10: Draft sheet of HR with perforated plate

3.3.2 THEORETICAL RESONANT FREQUENCY OF HELMHOLTZ RESONATOR WITH PERFORATED PLATE

The derived equation for resonance frequency of Helmholtz resonator with plate is as follow:

$$\therefore \omega_{n_1, n_2} = \sqrt{\left(\frac{k_2}{2m_3} + \frac{k_1 + k_2}{2nm}\right) \pm \sqrt{\frac{k_2^2}{4m_3^2} + \frac{(k_1 + k_2)^2}{4n^2m^2} + \frac{k_2^2}{2nmm_3} - \frac{k_1k_2}{2nmm_3}}}$$

By putting all calculated design parameters values in the following equations,

$$k_1 = \frac{\rho C_1^2 S_1^2}{V_1}, \quad k_2 = \frac{\rho C_2^2 S_2^2}{V_2}, \quad m_3 = \rho S_2 L, \quad m = n \rho S_1 t$$

Hence,

$$\therefore k_1 = 2153.5252 \text{ N/m}$$

$$\therefore k_2 = 985.3644 * 10^3 \text{ N/m}$$

$$\therefore m = 4.01118 * 10^{-5} \text{ kg}$$

$$\therefore m_3 = 1.0322 * 10^{-4} \text{ kg}$$

Assume the perforated plate placed in such a way that the cavity is divided into two equal parts. Hence, the volume is divided into two parts

$$\therefore V_1 = \frac{\pi}{4} * d_c^2 * L_1$$

$$\therefore V_2 = \frac{\pi}{4} * d_c^2 * L_2$$

Where,

$$L_1 = L_2 = 48.5 \text{ mm.}$$

By putting all values in equation of frequency

$$\therefore F_1 = 176.6548 \text{ HZ}$$

$$\therefore F_2 = 9505.629 \text{ HZ}$$

3.4 TRANSMISSION LOSS

The conventional measure for sound insulation is the sound transmission loss, which is the ratio of the incident and transmitted sound powers in logarithmic form.

$$TL = 20 \log_{10} \left(\frac{P_i}{P_o} \right)$$

Sound transmission is significantly reduced when the dimensions of the partition or boundary are larger the largest wavelength of incident sound wave.

3.4.1 The equation for transmission loss for HR is as follow:

$$TL = 10 \log_{10} \left(1 + \left(\frac{a_n}{2a_d} * \frac{(1/A) \tan(kL_c) + \tan(kL_n)}{(1/A) \tan(kL_n) + \tan(kL_c)} \right)^2 \right)$$

This is equation for TL of the single degree of freedom or conventional HR and it difficult to derive the TL equation for two degree of freedom. So we will do it experimentally. [3]

IV.SUMMARY

The conventional Helmholtz resonator has a limitation of low band frequency width of transmission loss. So, perforated plate is main module of this project which was implemented for noise attenuation criteria. Other authors had done it with duct and muffler application and in this project we are going to place in the inside of HR and trying to find out how much there is noise reduction when using perforated plate.

The present study involves the acoustic performance of Helmholtz resonator with perforated plate application theoretically and analytically. Since the approach of this project work involve only to find out the resonance frequency and since given frequency range is low range frequency hence, it is only useful for noise attenuation application. We have derived the resonance frequency of HR for two degree of freedom since conventional HR is single degree of freedom problem, but using plate it becomes double degree of freedom problem. Analytical analysis was done on perforated plate by first of all assuming with single hole plate and then with n number of hole plate with the variation in neck length of Helmholtz resonator to get resonator frequency and the same will be investigated experimentally after manufacturing the HR.

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