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ALMOST PERIODIC FUNCTIONS ON INTUITIONISTIC FUZZY METRIC SPACE

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ABSTRACT

In this paper we have proved the result regarding almost periodic functions in an intuitionistic fuzzy metric spaces.We conclude with an example to support our main results. Our results unify, generalize and complement the comparable results from the current literature.

I.INTRODUCTION AND PRELIMINARIES

Zadeh [7] Introduced the concept of fuzzy sets in 1965. Following the concept of fuzzy sets, fuzzy metric spaces have been introduced by Kramosil and Michalek [4] in the next decade in 1975 which was the great path for the research in Fuzzy based analysis. As a generalization of fuzzy sets, Atanassov [2] introduced and studied the concept of intuitionistic fuzzy sets in 1984. It has a wide range of applications in the field of population dynamics, Chaoscontrol, computer programming,medicine,etc. Using the idea of intuitionistic fuzzy sets, Park [5] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-co-norm as a generalization of fuzzy metric space. Here we will prove the result by generalizing the results given by Sharma[6]on fuzzy metric space.

To prove the result, we will need following definitions:

Definition 1.1. [3] A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$ is called a continuous t-norm if

$([0,1], *)$ is an abelian topological monoid with the unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$.

Definition 1.2. [2] A binary operation $\lozenge: [0,1] \times [0,1] \rightarrow [0,1]$ is continuous t-co-norm if $\lozenge$ is satisfying the following condition

(1.2.1)$\Diamond$ is commutative and associative

(1.2.2)$\Diamond$ is continuous

(1.2.3)$a \Diamond 0 = a$, for all $a \in [0,1]$

(1.2.4) $a \Diamond b \leq c \Diamond d$, whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$

Definition 1.3. [3] The 3-tuple (X, M, $*$) is called a fuzzy metric space (FM-space) if X is an arbitrary set,$*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0,\infty]$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$:

(1.3.1) $M(x,y,0) > 0$

(1.3.2)$M(x,y,t) = 1, \forall\, t > 0, iff\ x = y$

(1.3.3)$M(x,y,t) = M(y,x,t),$

(1.3.4) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$

(1.3.5)$M(x,y,\ .): [0,\infty] \to [0,1]$ is continuous.

Definition 1.4 [1] A five–tuple (X, M, N, $*$, $\Diamond$) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set,$*$ is a continuous t– norm, $\Diamond$ is a continuous t-co-norm and M, N are fuzzy sets on $X^2 \times (0,\infty)$ satisfying the following conditions for all $x, y, z \in X, s, t > 0$

(2.4.1) $M(x,y,t) + N(x,y,t) \leq 1$

(2.4.2)$M(x,y,t) > 0$

(2.4.3)$M(x,y,t) = M(y,x,t)$

(2.4.4)$M(x,y,t) * M(y,z,s) \leq M(x,z,t+s)$

(2.4.5) $M(x,y,.): (0,\infty) \to (0,1)$ is continuous

(2.4.6)$N(x,y,t) > 0$

(2.4.7)$N(x,y,t) = N(y,x,t)$

(2.4.8) $N(x,y,t) \Diamond N(y,z,s) \geq N(x,z,t+s)$

(2.4.9)$N(x,y,.): (0,\infty) \to (0,1]$ is continuous .

Here (M, N) is called an intuitionistic fuzzy metric on X, the function $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non- nearness between x and y with respect to t respectively

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Example 1.1 [1] Let (X, d) be a metric space, denote $a * b = ab$ and $a \lozenge b = \min\{1, a + b\}$, for all $a, b \in [0,1]$ and let M, and N be fuzzy sets on $X^2 \times (0,1)$ defined as follows

$$M(x,y,t) = \frac{t}{t+d(x,y)}, \quad N(x,y,t) = \frac{d(x,y)}{t+d(x,y)}$$

Then$(X, M, N, *, \lozenge)$ is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric induced by a metric d, the standard intuitionistic fuzzy metric

Definition 1.6 [6]A continuous function on R(real). $T: R \to (X, M, N, *, \lozenge)$ is called almost periodic if for any member $t > 0$, one can find $l(t) > 0$ such that any interval of length$l(t)$ contains atleast one point $\tau_T$with property:

$$M(T(t+\tau), T(t), \varepsilon) = 1, t \in R$$

II.MAIN RESULT

Theorem 2.1. An almost periodic function on intuitionistic fuzzy metric space is uniformly continuous.

Proof. Let $l = l(\varepsilon/3) > 0$ the definitionof almost periodic function. On the interval $[-1, 1 + l]$ the function is continuous and hence it is also uniform continuous. Let $\delta = \delta(\varepsilon/3) > 0, \delta < 1$ such that $t_1, t_2 \in [-1, 1 + l]$ with $|t_1 - t_2| < \delta$, we have

$$M(Tt_1, Tt_2, \varepsilon/3) = 1 \text{ and}$$

$$N(Tt_1, Tt_2, \varepsilon/3) = 1$$

Finally, let $t_1 > t_2$ be such that $|t_1 - t_2| < \delta$ and τ be an $(\varepsilon/3)$ translation number of T contained in the interval $[-t_1, -t_1 + 1]$. Since $|t_2 - t_1| < \delta, 0 \le t_1 + \tau \le l$ it follows that $t_2 + \tau$ is situated on interval $[-1, 1 + l]$. Thus

$$M(Tt_2, Tt_1, \varepsilon) \ge M(Tt_2, T(t_2 + \tau), \varepsilon/3) * M[T(t_2 + \tau), \quad T(t_1 + \tau), \varepsilon/3] * M[T(t_1 + \tau), T(t_1), \varepsilon/3]$$

$$= 1 * 1 * 1 = 1$$

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Also,

$$N(Tt_2, Tt_1, \varepsilon) \leq N(Tt_2, T(t_2+\tau), \varepsilon/3) \lozenge M[T(t_2+\tau), T(t_1+\tau), \varepsilon/3] \lozenge N[T(t_1+\tau), T(t_1), \varepsilon/3]$$

$$= 1 \lozenge 1 \lozenge 1 = 1$$

This proves the result.

Theorem 2.2 If T_n is a sequence of almost periodic function with values in (X, M, N, $*$, $\lozenge$) and if

$$\lim_{n\to\infty} T_n(t) = T(t)$$

Uniformly in the sense of convergence of ε-norm, then T is almost periodic.

Proof. Continuity of T is evident. If $\varepsilon > 0$, then there exists a natural number $N(\varepsilon)$ with

$$M(T_n t, Tt, \varepsilon/3) = 1, t \in R, n \geq N(\varepsilon)$$

$$\text{and} \quad N(T_n t, Tt, \varepsilon/3) = 1, t \in R, n \geq N(\varepsilon) (1)$$

We fix n_0 , for which (1) is true and consider $l(\varepsilon/3)$ determined from the almost periodicity ofT_{n_0} and T an $(\varepsilon/3)$ translation number of f_{n_0}. For any $t \in R$, We have

$$M(T(t+\tau), T(t), \varepsilon) \geq M\left(T(t+\tau), \left(T_{n_0}(t+\tau), \varepsilon/3\right)\right) * M(T_n(t+\tau), T_n t, \varepsilon/3) * M(T_n t, Tt, \varepsilon/3)$$

$$= 1 * 1 * 1 = 1$$

Also

$$N(T(t+\tau), T(t), \varepsilon) \leq N\left(T(t+\tau), \left(T_{n_0}(t+\tau), \varepsilon/3\right)\right) \lozenge N(T_n(t+\tau), T_n t, \varepsilon/3) \lozenge N(T_n t, Tt, \varepsilon/3)$$

$$= 1 \lozenge 1 \lozenge 1 = 1$$

Which proves the almost periodicity of T.

III.CONCLUSION

Hence we have proved the result regarding the Almost periodic functions in an Intuitionistic fuzzy metric spaces. Examples are also given to support our main result.

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