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Development of Set Theoretical Approach

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ABSTRACT

The Set theory is very impotent tool of mathematics. The researchers from various fields like mathematics, logics and computer science used the set theory concept. In this paper, the theoretical development of set theory and their properties has been presented.

Key Words:* *Fuzzy Set Theory, Rough Set, Set Theory, Soft Set.

I INTRODUCTION

Before studying any subject or specific topic of the subject, one question arises in our mind that what is the application of this topic in the real life or routine life. Same thing arises in set theory that the set theory have real life application or not if yes then what the application of set .The set is idea of storing out different objects in to similar groupings or sets or in other words set theory is collection of well defined objects or elements. Set theory is the very important tool of the modern era of mathematics. In daily life we use word like set of table and chair, set of cup and plate etc. Application of the set theory is commonly in the field of science and mathematics. It is also used in other areas like discrete structure, data structure. The mathematical objects like relations, functions and numbers can be considered as the set. The cardinality shows the number of elements in a set. Many of real life problems in engineering, medical sciences have different uncertainties and to resolve these uncertainties there are several theories like fuzzy set theory, rough set theory, soft set theory etc. To comparing the classical set, fuzzy set and rough set we see that classical set is a primitive notion and is defined intuitively. Fuzzy sets are defined by membership function and used advanced mathematical structures, numbers and functions. Rough Set is defined by approximations. [1], [2]

II HISTORICAL DEVELOPMENT OF SET THEORY:

Various researcher/ mathematicians have used the concept of set theory at very starting of the subject .The history of the set theory is something different from the other field of the mathematics. To develop the set theory the main contributions is of Cantor, but before Cantor several concepts about set theory around 450 BC, Greeks Zeno of Elea with the problems on the infinite was the starting but very important contribution in the development of the sets theory. The Aristotle in 384 – 322 B.C. says that infinite is imperfect, unfinished and so it is unthinkable.

Mathematician Bolzano considered a set with the definition "An embodiment of the idea or concept which we conceive when we regard the arrangement of its parts as a matter of indifference ". Bolzano defended the concept of an infinite set. He says that the element of the infinite sets is one- one correspondence with elements of one its proper subsets.

The Set Theory came in the proper manner after the work of Cantor. Cantor starts his work with number theory and he published a number of articles with high quality after this cantor moved from number theory to trigonometric series. These researches contain Cantor's first idea about the sets. Cantor is considered as the founder of set theory. But this result was not immediately accepted by his contemporaries and it also discovered that this definition of set to a contradictions and paradoxes. About these Russell Paradox give a concept known as Russell Paradox. To resolve these paradoxes Cantor's intuitive set theory came into existence. [1], [2]

Russell's Paradox: Most sets are not the element of themselves such that the set of all integers is not an integer. If we let S is the set of all sets which are not the elements of themselves:

$S = \{B \mid B \text{ is a set and } B \notin B\}$ Then S is the element of itself then answer of this problem is neither yes nor no. which is a contradiction and explain this Russell devised a puzzle, the barber puzzle.

Operation on the Sets:

1. Union of Sets: if $A \text{ and } B \text{ are two Sets then}$

$A \cup B = \{x : x \in A \text{ or } x \in B\}$ is called the union of two sets.

2. Intersection of sets: if $A \text{ and } B \text{ are two Sets then}$

$A \cap B = \{x : x \in A \text{ and } x \in B\}$ is called the union of two sets

3. Complement of Set: If A be any set and U be the universal set then

$A^c = \{x : x \in U \text{ and } x \notin A\}$

4. Set difference: if $A \text{ and } B \text{ are two Sets then}$

$A - B = \{x : x \in A \text{ and } x \notin B\}$

$B - A = \{x : x \in B \text{ and } x \notin A\}$

Properties on the Sets:

1. $(A^c)^c = A$
2. $A \cup B = B \cup A; A \cap B = B \cap A$
3. $(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$
4. $(A \cup B)^c = A^c \cap B^c$; $(A \cap B)^c = A^c \cup B^c$

Fuzzy Set:

In routine life we face many situations in which inclusion and non – inclusions in a set are not clearly defined like the classes of the tall people, the boundaries of such sets are vague. Fuzzy sets are the next development of set theory. The Fuzzy set theory is characterized by values from 0 to 1 where zero represent the no membership in a set and 1 represent complete relationship in a set. The Zadeh approach for the fuzzy set an element can belong to a set to a degree n where $0 \leq n \leq 1$, in contrast to classical set theory in which an element must belong to the set or not. The Fuzzy logic has many applications in different fields like Transportation, crime investigation, clinical decision support etc. The mathematical definition of the set fuzzy set theory is:

If X is a collection of the objects or elements, then a fuzzy set A in X is the ordered pairs

$A = \{(x, \mu_A(x)) : x \epsilon X, \mu_A(x) : X \to [0,1]\}$.

Here $\mu_A(x)$ is the member function of set A, defined from X into $[0, 1]$.

Fuzzy membership function has the following operations:

1. $\mu_{U-A}(x) = 1 - \mu_A(x)\ for\ any\ x \in U$
2. $\mu_{X \cup Y}(x) = \max(\mu_X(x), \mu_Y(x))\ for\ any\ x \in U$
3. $\mu_{X \cap Y}(x) = \min(\mu_X(x), \mu_Y(x))\ for\ any\ x \in U$

   So the membership of an element to be union and intersection of sets is uniquely determined by its membership to constituent sets. These properties are good and allow very simple operation on fuzzy sets.

Properties of Fuzzy set operations:

1. $(\mu_A^c(x))^c = \mu_A(x)$
2. $\mu_A(x) \cup \mu_B(x) = \mu_B(x) \cup \mu_A(x)$; $\mu_A(x) \cap \mu_B(x) = \mu_B(x) \cap \mu_A(x)$
3. $(\mu_A(x) \cup \mu_B(x)) \cup \mu_C(x) = \mu_A(x) \cup (\mu_B(x) \cup \mu_C(x))$;

$(\mu_A(x) \cap \mu_B(x)) \cap \mu_C(x) = \mu_A(x) \cap (\mu_B(x) \cap \mu_C(x))$ [3]

Rough Set:

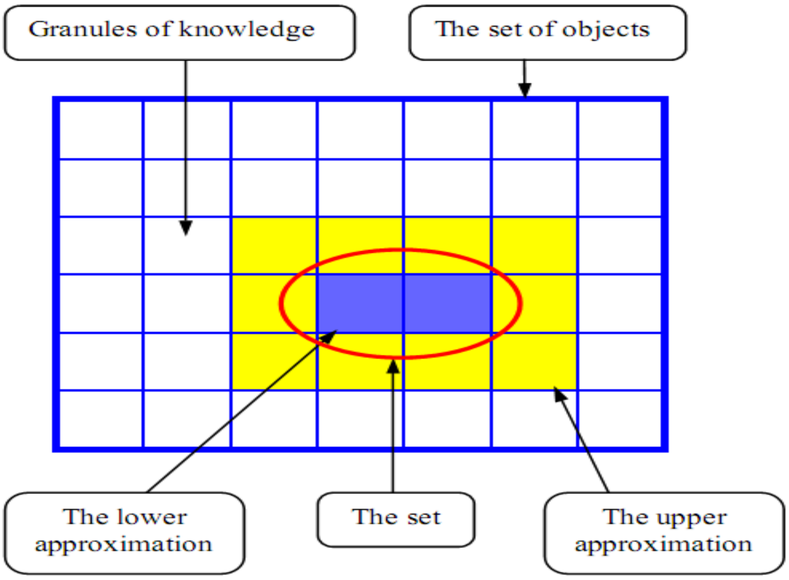
Rough Set is an approach of vagueness. It is the first non statistical approach for data analysis. Rough Set Theory concept can be quite generally by means a topological operation, interior and exterior called the approximation. The basic concept behind the rough set theory is approximation of lower and upper spaces of the set. The difference between upper approximation and lower approximation is called boundary region. If the boundary region is empty then the set is crisp otherwise the set is rough. The mathematical definition of the approximation and the boundary regions are

Lower Approximation of the set X: $R_*(x) = \cup \{R(x) : R(x) \subseteq X\}$

Upper Approximation of the set X: $R^*(X) = \cup \{R(x) : R(x) \cap X \neq \emptyset\}$

Boundary Region of X: $R = R^*(X) - R_*(x)$

The Rough Set Theory has much application in various fields like pattern recognition, feature selection, data mining etc.



Properties of Approximation:

1. $R_*(x) \subseteq X \subseteq R^*(X)$
2. $R_*(\emptyset) = R^*(\emptyset) = \emptyset$
3. $R_*(U) = R^*(U) = U$
4. $R^*(X \cup Y) = R^*(X) \cup R^*(Y)$; R^*(X∩Y)= $R^*(X) \cap R^*(Y)$
5. $R_*(X \cup Y) = R_*(X) \cup R_*(Y); R_*(X \cap Y) = R_*(X) \cap R_*(Y)$
6. $R_*(-X) = -R^*(X);\ R^*(-X) = -R_*(X)$

The member function also defines the approximation and boundary region of the set such as

1. $R_*(X) = \{x \in U: \mu_X^R(x) = 1\}$
2. $R^*(X) = \{x \in U: \mu_X^R(x)>1\}$
3. $RN_R(X) = \{x \in U: 0 < \mu_X^R(x) < 1\}$ [4], [5]

Soft Set:

There are lots of problems in economics, engineering and medical sciences in which data are not all crisp and we cannot successfully use classical methods due to various types of uncertainties which are present in the problem. Molodtsov gives the novel approach as the Soft Set theory. Soft Set Theory is a new mathematical approach for dealing with uncertainties and soft set theory associated with parameter and used in various directions. Molodtsov successfully use soft set theory into various fields like smoothness of functions, game theory and theory of measurement etc. Molodtsov defined the soft set by the following way: If U be the initial universe set and E be the set of parameters, P(U) be the power set of U and $A \subseteq E$. The order pair (F, A) is called the soft set where F is mapping given by $F: A \to P(U)$. Soft sets are different types like weighted soft set, Finite- multi soft set, Partitioned soft set, Basic Neighborhood soft set, Covering Soft Set, Neighborhood soft set, Fuzzy- Partitioned soft sets, and double fuzzy sets.

Properties of Soft Sets:

1. If (F, A) and (G, B) are two soft sets then they are said to be equal if (F, A) is soft subset of (G,B) and (G,B) are soft subset of (F,A).
2. If (F, A) and (G,B) are two soft sets on U, then
   $((F,A) \cup (G,B))^c = (F,A)^c \cup (G,B)^c$; $((F,A) \cap (G,B))^c = (F,A)^c \cap (G,B)^c$
3. If $\emptyset$ is a null soft set , A is an absolute soft set and (F,A) is a soft set on U , then
   i. $(F,A) \cup (F,A) = (F,A)$
   ii. $(F,A) \cap (F,A) = (F,A)$
   iii. $(F,A) \cap \emptyset = \emptyset$: $(F,A) \cup \emptyset = (F,A)$
   iv. $(F,A) \cap A = (F,A)$; $(F,A) \cup A = A$ [6], [7], [8],[9]

Conclusion:

Set Theory is a very important tool not only of mathematics but also in computer science. In this paper we present the theoretical development of set theory which are very useful in future for the researcher of mathematics and computer science.

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