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Inventory model with inventory-level-dependent demand rate without shortages

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ABSTRACT-

After the development of Harris-Wilson formula about economic order quantity in 1915, some of extensions to this model were studied by several researchers, who considered many practical situations. Demand for inventory modeling is the most significant property which needs analysis. There is the number of factors that affect the demand pattern. One of the major factors governing demand is inventory level. We have discussed a manufacturing model, in which demand during the production period does not change and during the depletion period, it depends on the on-hand inventory level down to a certain point, and then becomes constant for rest of the period.

Keywords-Demand, Inventory level, Replenishment level.

I.INTRODUCTION

It is common belief that the presence of inventory has a motivational effect on the people around it. Attempts have been made to consider inventory level dependent demand rate by researchers like Baker and Urban (1988), Mandal and Phaujdar (1989), who studied a situation of this type. One more paper by Mandal and Phaujdar (1989) included deterioration alongwith stock dependent demand rate. Dutta and Pal (1990) developed a model with same concept. In the same year Dutta and Pal (1990) extended their own model by providing shortages. Similarly Gupta and Vrat (1986) studied model with stock dependent demand rate, in another paper Gupta and Vrat (1986) covered the possibility of multi items. Recently Chao-Ton Su, Lee-Ing Ton and Hung-Chang Liao (1996) studied a deteriorating inventory model with inflation and stock dependent demand rate.

In the presented inventory model, it is assumed that demand during the production period remains constant, and during the depletion period demand depends on the on-hand inventory level down to a certain level. After that it becomes constant for rest of the period.

II.ASSUMPTIONS AND NOTATIONS

Following assumptions are made in this model:

1. Replenishment rate is finite (replenishment is not instantaneous).
2. Demand rate remains constant throughout the production period.

3. As soon as production stops demand on inventory level with the relation given by $R(Q) = \alpha Q^{\beta}$, ………(A)

where α and β are constants, $\alpha > 0$ and $0 < \beta < 1$,

4. Relation (A) is maintained till the level of inventory reaches a fixed point Q_0. From Q_0 onwards there is constant demand rate given by $R(Q) = \alpha Q_0^{\beta} = D$, ………(B)
5. Lead time is assumed to be zero.
6. Shortages are not allowed.
7. There is only one item in the inventory system.

Notations used in this model are as follows:

$\emptyset =$ Replenishment rate

$S =$ Demand rate

$Q =$ Inventory level at any time t

$Q_1 =$ Maximum level of inventory

$q =$ Lot size

$C_1 =$ Holding cost per item per unit time

$C_3 =$ Set-up cost per set-up

$0 - t_1 =$ Time duration in which inventory is built

$t_1 - t_2 =$ Time duration in which demand rate follows the relation αQ^{β}

$t_2 - T =$ Time duration in depletion period in which demand rate is constant

III.MATHEMATICAL ANALYSIS

Since $\emptyset$ is the production rate and S is the demand rate, therefore procuring rate of inventory is $\emptyset - S$.

Procuring period $t_1 = \frac{q}{\emptyset}$ ………. (i)

Now maximum level of inventory is $Q_1 = (\emptyset - S)t_1 = (\emptyset - S)\frac{q}{\emptyset}$ ……….(ii)

From (ii)

We have $t_1 = \frac{Q_1}{(\emptyset - S)}$ ……… (iii)

Level of inventory Q during depletion is as follows

$Q = Q_1$, when $t = t_1$

$= Q_0$, when $t = t_2$

$= 0$, when $t = T$

Differential equation for depletion of inventory is given by

$\frac{dQ}{dt} = -\alpha Q^{\beta}, \quad t_1 \le t \le t_2$ (1)

$= -D, \quad t_2 \le t \le T$ (2)

Using the condition $Q = Q_1$ at $t = t_1$, the solution of (1) becomes

$Q^{(1-\beta)} = Q_1^{(1-\beta)} + \alpha(1-\beta)t_1 - \alpha(1-\beta)t$ (3)

Since $Q = Q_0$ at $t = t_2$ so we have from (3)

$Q_0^{(1-\beta)} = Q_1^{(1-\beta)} + \alpha(1-\beta)t_1 - \alpha(1-\beta)t_2$ (4)

On Solving (2), using $Q = 0$ at $t = T$, we have

$Q = D(T - t_2), \quad t_2 \le t \le T$ (5)

At $t = t_2$, $Q = Q_0$. Therefore (5) can be written as

$Q_0 = D(T - t_2)$ (6)

$\Rightarrow t_2 = \frac{DT - Q_0}{D}$ (7)

Using t_2 from (7) in (4), we have

$Q_0^{(1-\beta)} = [Q_1^{(1-\beta)} + \alpha(1-\beta)t_1 - \alpha(1-\beta)\frac{DT-Q_0}{D}]$

$\Rightarrow T = \frac{\{Q_1^{(1-\beta)} - Q_0^{(1-\beta)}\}}{\{\alpha(1-\beta)\}} + t_1 + \frac{Q_0}{D}$ (8)

Total inventory 'I' during the complete cycle is calculated as

$I = \int_0^T Q\,dt = \int_0^{t_1} Q\,dt + \int_{t_1}^{t_2} Q\,dt + \int_{t_2}^{T} Q\,dt$

$$= \int_0^{t_1}(\emptyset - S)t\,dt + \int_{t_1}^{t_2}\left\{Q_1^{(1-\beta)} + \alpha(1-\beta)t_1 - \alpha(1-\beta)t\right\}^{1/(1-\beta)} dt$$

$$+ \int_{t_2}^{T} D(T-t)dt$$

$$= \frac{(\emptyset - S)t_1^2}{2} + \frac{\{Q_1^{(1-\beta)}\}^{(2-\beta)/(1-\beta)}}{\{\alpha(2-\beta)\}} - \frac{\{Q_1^{(1-\beta)} + \alpha(1-\beta)t_1 - \alpha(1-\beta)t_2\}^{(2-\beta)/(1-\beta)}}{\{\alpha(2-\beta)\}} + \frac{D(T-t_2)^2}{2}$$

Eliminating t_1, t_2 and T from (i), (4) and (6), we have

$$I = \frac{(\emptyset - S)q^2}{(2\emptyset^2)} + \frac{\{Q_1^{(2-\beta)} - Q_0^{(2-\beta)}\}}{\{\alpha(2-\beta)\}} + \frac{Q_0^2}{2D}$$

$$= \frac{(\emptyset - S)q^2}{(2\emptyset^2)} + \frac{[Q_0^2(2-\beta)\alpha + 2D\{Q_1^{(2-\beta)} - Q_0^{(2-\beta)}\}]}{\{2\alpha D(2-\beta)\}} \quad (9)$$

Eliminating q by using (ii) in (9), we have

$$I = \frac{Q_1^2}{\{2(\emptyset - S)\}} + \frac{\left[Q_0^2(2-\beta)\alpha + 2D\left\{Q_1^{(2-\beta)} - Q_0^{(2-\beta)}\right\}\right]}{\{2\alpha D(2-\beta)\}} \quad (10)$$

The cost function per unit time, $K_1(Q_1)$, is given by the relation

$$K_1(Q_1) = \frac{[C_1 I + C_3]}{T}$$

$$= \frac{\left[C_1\left\{\frac{Q_1^2}{\{2(\emptyset - S)\}} + \frac{\left[Q_0^2(2-\beta)\alpha + 2D\left\{Q_1^{(2-\beta)} - Q_0^{(2-\beta)}\right\}\right]}{\{2\alpha D(2-\beta)\}}\right\} + C_3\right]}{\left[\frac{\left\{Q_1^{(1-\beta)} - Q_0^{(1-\beta)}\right\}}{\{\alpha(1-\beta)\}} + t_1 + \frac{Q_0}{D}\right]} \quad (11)$$

Eliminating t_1 and D from (11) using equation (iii) and the relation $D = \alpha Q_0^{\beta}$, we get

$$K_1(Q_1) = \frac{\left[C_1\left\{\frac{Q_1^2}{\{2(\emptyset - S)\}} + \frac{\left[Q_0^2(2-\beta)\alpha + 2\alpha Q_0^{\beta}\left\{Q_1^{(2-\beta)} - Q_0^{(2-\beta)}\right\}\right]}{\{2\alpha D(2-\beta)\}}\right\} + C_3\right]}{\left[\frac{\left\{Q_1^{(1-\beta)} - Q_0^{(1-\beta)}\right\}}{\{\alpha(1-\beta)\}} + \frac{Q_1}{(\emptyset - S)} + \frac{Q_0}{\alpha Q_0^{\beta}}\right]}$$

Now for optimal $K_1(Q_1)$ we use the condition $\frac{dK_1(Q_1)}{dQ_1} = 0$

i.e., $A_1 Q_1^{(2-\beta)} + A_2 Q_1^{2(1-\beta)} + A_3 Q_1^2 + A_4 Q_1 + A_5 Q_1^{(1-\beta)} + A_6 Q_1^{-\beta} + A_7 = 0$ (12)

Where $A_1 = \alpha Q_0^{2\beta} C_1(\emptyset - S)\{2 + (1-\beta)(2-\beta)\}$,

$A_2 = 2C_1 Q_0^{2\beta}(\emptyset - S)^2$,

$A_3 = \alpha^2 Q_0^2 C_1(1-\beta)(2-\beta)$,

$A_4 = -2\alpha\beta Q_0^{(1-\beta)} C_1(\emptyset - S)(2-\beta)$,

$A_5 = -2\beta Q_0^{(1-\beta)} C_1(\emptyset - S)^2(2-\beta)$,

$A_6 = (1-\beta)(\emptyset - S)^2 Q_0^{\beta}\left\{\beta Q_0^2 C_1 - 2\alpha Q_0^{\beta} C_3(2-\beta)\right\}$ and

$A_7 = \alpha\beta C_1 Q_0^{(2+\beta)}(\emptyset - S)(1-\beta) - 2C_3\alpha^2 Q_0^{2\beta}(1-\beta)(2-\beta)(\emptyset - S)$.

Equation (12) can be solved by using a suitable numerical method such as Newton-Raphson Method.

As the criterion of optimality is based on minimum cost, the following condition should be satisfied.

$$\frac{d^2K_1(Q_1)}{dQ_1^2} > 0$$

q can be calculated by the formula $q = \frac{\emptyset Q_1}{(\emptyset - S)}$

In case Q_1 obtained from (12) is such that $Q_1 < Q_0$ then we proceed as follows:

Since demand follows linear relation as soon as level of inventory reaches the point Q_0 we have the following formula to calculate cost $K_2(Q_1)$.

Cost function = $K_2(Q_1) = \frac{C_1Q_1}{2} + \frac{C_3}{T}$

$$T = t_1 + t_2 = \frac{q}{\emptyset} + \frac{Q_1}{(\alpha Q_0^\beta)}$$

$$= \frac{q}{\emptyset} + \frac{(\emptyset - S)q}{(\alpha Q_0^\beta \emptyset)}$$

$$= \frac{q}{\emptyset}\frac{\{\alpha Q_0^\beta + (\emptyset - S)\}}{(\alpha Q_0^\beta)}$$

$$= \frac{Q_1}{(\emptyset - S)}\frac{\{\alpha Q_0^\beta + (\emptyset - S)\}}{(\alpha Q_0^\beta)}$$

Thus $K_2(Q_1) = \frac{C_1Q_1}{2} + \frac{C_3\alpha Q_0^\beta(\emptyset - S)}{[Q_1\{\alpha Q_0^\beta + (\emptyset - S)\}]}$

Applying the condition of optimality, we have

$$Q_1 = \left[\frac{2C_3\alpha Q_0^\beta(\emptyset - S)}{C_1\{\alpha Q_0^\beta + (\emptyset - S)\}}\right]^{1/2} \qquad (13)$$

It can be seen that if this $Q_1 < Q_0$ then Q_1 obtained from (12) is also less than Q_0.

IV.NUMERICAL EXAMPLES

Example 1 Assume $\alpha = 0.5$ units, $\beta = 0.4$, $\emptyset = 8$ units/unit time, $S = 6$ units/unit time, $C_1 = \$\ 1$/unit/ unit time, $C_3 = \$\ 45$/replenishment, $Q_0 = 6$ units.

Solution: Using the value of parameters in equation (13) we have optimal lot-size

$Q_1 = 7.806787712$

Since $Q_1 > Q_0$ formula (12) is applicable, from which Q_1 comes out to be 7.81.

Example 2 Assume $\alpha = 0.5$ units, $\beta = 0.4$, $\emptyset = 8$ units/unit time, $S = 6$ units/unit time, $C_1 = \$\ 1$/unit/ unit time, $C_3 = \$\ 26$/replenishment, $Q_0 = 6$ units.

Solution: Using the value of parameters in equation (13) we have optimal lot-size

$Q_1 = 7.806787712$

Since $Q_1 > Q_0$ formula (12) is applicable, from which Q_1 comes out to be 7.81.

Solution: Using the value of parameters in equation (13) we have optimal lot-size

$Q_1 = 5.934071665$

Since $Q_1 < Q_0$ so we need not to calculate Q_1 from equation (12).

V.DISCUSSION

It is a general tendency of customers to get attracted by huge stock of goods being sold. But customers continue to arrive for purchase inspite of the depleted stocks. This is mainly due to the goodwill, the maintenance of quality as well as the genuineness of the price of the articles.

Equation (12) reduces to the case of infinite replenishment when $(\emptyset - S) \to \infty$. If in addition $\beta \to 0$ the system reduces to simple lot-size formula and then equation (13) reduces to the well known squares root formula.

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