International Journal of Advance Research in Science and Engineering
Volume No.07, Issue No.03, March 2018
www.ijarse.com
IJARSE
ISSN: 2319-8354

A Critical Sketch of Sine and Cosine Functions of Trigonometry

Parmod Kumar[1] , Dr. A.K. Gupta[2], Dr. SBL Tripathi[3]

[1]*Research Scholar, Ph.D. In Mathematics, Bhagwant University Ajmer, Rajasthan (India)*

[2]*Research Guide,* [3]*External Guide Dr. SBL Tripathi (India)*

ABSTRACT

Trigonometry is an important school subject not only for mathematics but also for some other fields. A robust understanding of trigonometric functions requires different algebraic, geometric, and graphical aspects due to the complex nature of the topic. Despite limited, the literature on trigonometry learning and teaching reveals that it is a difficult topic for students, and that students develop fragmented understanding of trigonometric functions.

I.INTRODUCTION

Trigonometry, the branch of mathematics concerned with specific functions of angles and their application to calculations. There are six functions of an angle commonly used in trigonometry. Their names and abbreviations are sine (sin), cosine (cos), tangent (tan), cotangent (cot), secant (sec), and cosecant (cosec). These six trigonometric functions in relation to a right triangle are displayed in the figure. For example, the triangle contains an angle *A*, and the ratio of the side opposite to *A* and the side opposite to the right angle (the hypotenuse) is called the sine of *A*, or sin *A*; the other trigonometry functions are defined similarly. These functions are properties of the angle *A* independent of the size of the triangle, and calculated values were tabulated for many angles before computers made trigonometry tables obsolete. Trigonometric functions are used in obtaining unknown angles and distances from known or measured angles in geometric figures.

Trigonometry developed from a need to compute angles and distances in such fields as astronomy, mapmaking, surveying, and artillery range finding. Problems involving angles and distances in one plane are covered in plane trigonometry. Applications to similar problems in more than one plane of three-dimensional space are considered in spherical trigonometry.

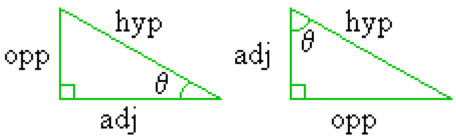
II.SINE AND COSINE FUNCTIONS

The **sine** (abbreviated "**sin**") and cosine ("**cos**") are the two most prominent trigonometric functions. All other trig functions can be expressed in terms of them. In fact, the sine and cosine functions are closely related and can be expressed in terms of each other.

III.DEFINITIONS OF SINE AND COSINE FUNCTIONS

The sine and cosine functions can be defined in a number of ways:

Definition I :- From a triangle



Given any angle θ (0 £ θ £ 90°), we can find the sine or cosine of that angle by constructing a right triangle with one vertex of angle θ. The sine is equal to the length of the side opposite to θ, divided by the length of the triangle's hypotenuse. The cosine is equal to the length of the side adjacent to θ, divided by the length of the triangle's hypotenuse. In this way, we can find the sine or cosine of any θ in the range 0 £ θ £ 90°.

This is the simplest and most intuitive definition of the sine and cosine function.

The sine definition basically says that, on a right triangle, the following measurements are related:

- the measurement of one of the non-right angles (θ)
- the length of the side opposite to that angle
- the length of the triangle's hypotenuse

Alternately, the cosine definition basically says that, on a right triangle, the following measurements are related:

- the measurement of one of the non-right angles (θ)
- the length of the side adjacent to that angle
- the length of the triangle's hypotenuse

Furthermore, Definition I gives exact equations that describe each of these relations:

sin(θ) = opposite / hypotenuse

cos(θ) = adjacent / hypotenuse

This first equation says that if we evaluate the sine of that angle q, we will get the exact same value as if we divided the length of the side **opposite** to that angle by the length of the triangle's hypotenuse. This second equation says that if we evaluate the cosine of that angle q, we will get the exact same value as if we divided the length of the side **adjacent** to that angle by the length of the triangle's hypotenuse. These relations hold for any right triangle, regardless of size.

The main result is this: If we *know* the values of any two of the above quantities, we can use the above relation to mathematically *derive* the third quantity. For example, the sine function allows us to answer any of the following three questions:

"Given a right triangle, where the measurement of one of the non-right *angles* (θ) is known and the length of the side *opposite* to that angle q is known, find the length of the triangle's *hypotenuse*."

"Given a right triangle, where the measurement of one of the non-right *angles* (θ) is known and the length of the triangle's *hypotenuse* is known, find the length of the side *opposite* to that angle q."

"Given a right triangle, where the length of the triangle's *hypotenuse* and the length of one of the triangle's other sides is known, find the measurement of the *angle* (θ) *opposite* to that other side."

The cosine is similar, except that the adjacent side is used instead of the opposite side.

The functions takes the forms **y = sin(θ)** and **x = cos(θ)**. Usually, q is an angle measurement and x and y denotes lengths.

The sine and cosine functions, like all trig functions, evaluate differently depending on the units on q, such as *degrees, radians, or grads*. For example, sin(90°) = 1, while sin(90)=0.89399.... explaination

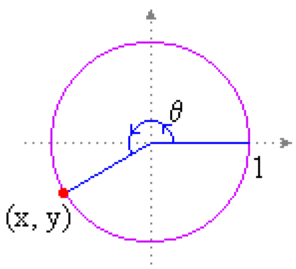
Both functions are trigonometric **cofunctions** of each other, in that function of the complementary angle, which is the "cofunction," is equal to the other function:

sin(x) = cos(90°-x) and

cos(x) = sin(90°-x).

Furthermore, sine and cosine are mutually **orthogonal**.

Definition II: From the unit circle



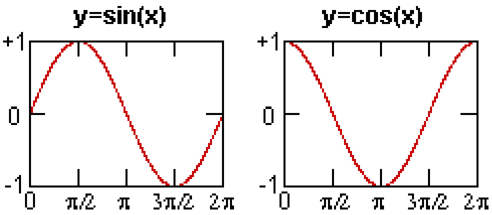
Draw a unit circle, in that a circle of radius 1, cantered at the origin of a 2-dimensional coordinate system. Given an angle θ, locate the point on the circle that is located at an angle θ from the origin. (According to standard convention, angles are measured counter-clockwise from the positive horizontal axis.) The Sin(θ) can be defined as the y-coordinate of this point. The Cos(θ) can be defined as the x-coordinate of this point. In this way, we can find the sine or cosine of **any** real value of θ (θ Î Â).

Definition III: Over the complex numbers

Given a complex number z = a + b **i**,

sin(z)=sin(a)cosh(b)+cos(a)sinh(b)**i**

cos(x) = cos(a)cosh(b) - sin(a)sinh(b) **i**



Some Properties of Sine and Cosine

- The sine function has a number of properties that result from it being **periodic** and **odd**. The cosine function has a number of properties that result from it being **periodic** and **even**. Most of the following equations should not be memorized by the reader; yet, the reader should be able to instantly derive them from an understanding of the function's characteristics.
- The sine and cosine functions are **periodic** with a period of 2p. This implies that
- sin(q) = sin(q + 2p)
- cos(q) = cos(q + 2p)

  or more generally,
- sin(q) = sin(q + 2pk)
- cos(q) = cos(q + 2pk),

  where k Î integers.

- The sine function is **odd**; therefore,
- sin(-q) = -sin(q)
- The cosine function is **even**; therefore,
- cos(-q) = cos(q)

Formulas of Sine and Cosine:

- sin(x + y) = sin(x)cos(y) + cos(x)sin(y)

It is then easily **derived** that

- sin(x - y) = sin(x)cos(y) - cos(x)sin(y)

Or more generally,

- sin(x ± y) = sin(x)cos(y) ± cos(x)sin(y)
- cos(x + y) = cos(x)cos(y) - sin(x)sin(y)

It is then easily **derived** that

- cos(x - y) = cos(x)cos(y) + sin(x)sin(y)

Or more generally,

- cos(x ± y) = cos(x)cos(y) (-/+) sin(x)sin(y)

From the above sine equation, we can derive that

- sin(2x) = 2sin(x)cos(x)

From the above cosine equation, we can derive that

- $\cos(2x) = \cos^2(x) - \sin^2(x)$

(The notation $\sin^2(x)$ is equivalent to $(\sin(x))^2$. Warning: $\sin^{-1}(x)$ stands for arcsine(x) not the multiplicative inverse of sin(x).)

By observing the graphs of sine and cosine, we can express the sine function in terms of cosine and vice versa:

- sin(x) = cos(90° - x)

and the cosine function in terms of sine:

- cos(x) = sin(90° - x)

Such a trig function (f) that has the property

f(q) = g(complement(q))

is called a **co function** of the function g, hence the names "sine" and "*co*sine."

The Pythagorean identity, $\sin^2(x) + \cos^2(x) = 1$, gives an alternate expression for sine in terms of cosine and vice versa

- $\sin^2(x) = 1 - \cos^2(x)$
- $\cos^2(x) = 1 - \sin^2(x)$

The **Law of Sines** relates various sides and angles of an arbitrary (not necessarily right) triangle:

- sin(A)/a = sin(B)/b = sin(C)/c = 2r.

where A, B, and C are the angles opposite sides a, b, and c respectively. Furthermore, r is the radius of the circle circumscribed in that triangle.

The **Law of Cosines** relates all three sides and one of the angles of an arbitrary (not necessarily right) triangle:

$c^2 = a^2 + b^2 - 2ab \cos(C)$.

where A, B, and C are the angles opposite sides a, b, and c respectively. It can be thought of as a generalized form of the Pythagorean Theorem. *Warning*: You must be careful when solving for one of the sides adjacent to the angle of interest, for there will often be two triangles that satisfy the given conditions. This can be understood from geometry. A triangle defined by SAS (side-angle-side) is unique, and, therefore, any triangle with the same SAS parameters must be congruent to it. A triangle defined by SSA, however, is not always unique, and two triangles with the same SSA parameters may or may not be congruent.

REFERENCES

[1].Blackett, N., & Tall, D. O. (1991). Gender and the versatile learning of trigonometry using computer software. In F. Furinghetti (Ed.), Proceedings of the 15th conferenceof the International Group for the Psychology of Mathematics Education(Vol.1,pp. 144–151).

[2].Assisi, Italy: PME.Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of theprocess conception of function. Educational Studies in Mathematics, 23, 247–285.

[3].Davis, R. B. (1992). Understanding “understanding”. Journal of Mathematical Behavior,11, 225–241.

[4].Dubinsky, E., & McDonald, M. (2001). APOS: A constructivist theory of learning in undergraduate mathematics education research. In D. Holton (Ed.), The teachingand learning of mathematics at university level: An ICMI study.(pp. 273–280).

[5].Dordrecht, The Netherlands: Kluwer.Gray, E. M., & Tall, D. O. (1994).

[6].Duality, ambiguity, and flexibility: A proceptual viewof elementary arithmetic. Journal for Research in Mathematics Education, 26(2),114–141.

[7].Hirsch, C. R., Weinhold, M., & Nichols, C. (1991). Trigonometry today.
Mathematics Teacher, 84(2), 98–106.

[8].Kendal, M., & Stacey, K. (1997). Teaching trigonometry. Vinculum, 34(1), 4–8.

[9].Lial, M. L., Hornsby, J., & Schneider, D. I. (2001). College algebra and trigonometry.

10].Menlo Park, CA: Addison Wellesley. Miller, S. (2001). Understanding transformations of periodic functions through art.Mathematics Teacher, 94(8), 632–635.

[11].National Council of Teachers of Mathematics. [NCTM] (1989). Curriculum and
evaluation standards for school mathematics. Reston, VA: NCTM.

[12].National Council of Teachers of Mathematics. [NCTM] (2000). Principles andstandards for school mathematics.

[13].Schoenfeld, A. (2000). Purposes and methods of research in mathematics education.
Notices of the AMS, 47, 641–649.