

Reliability Characteristics of a Standby System with Priority for Preventive Maintenance and Server Failure

Jyoti Nandal*

**Department of Statistics, Govt. P.G. College for Women, Rohtak (India)*

ABSTRACT

Here, reliability characteristics of a two unit cold standby system are obtained by introducing the concepts of preventive maintenance, priority and server failure. The units are identical in nature and have two modes-normal and complete failures. There is a single server who attends the system immediately to tackle the faults whenever occur in the system. The preventive maintenance of the operating unit is conducted before its failure. The repair of the unit is done by the server at its complete failure. The server is subjected to failure while performing jobs. The failed server undergoes for treatment in order to resume the jobs. The random variables associated with failure time of the unit, preventive maintenance time and treatment time of the server are statistically independent with different probability distributions. Priority is given to the preventive maintenance over repair of the unit. The system model is analyzed by using semi-Markov process and regenerative point technique. The variation of some important reliability measures including mean time to system failure (MTSF), availability and profit function has been examined in steady state for arbitrary values of the parameters. The effect of the repair policy on profit of the system model is observed.

Keywords: *Preventive Maintenance, Priority, server failure, Repair and Reliability Characteristics*

I.INTRODUCTION

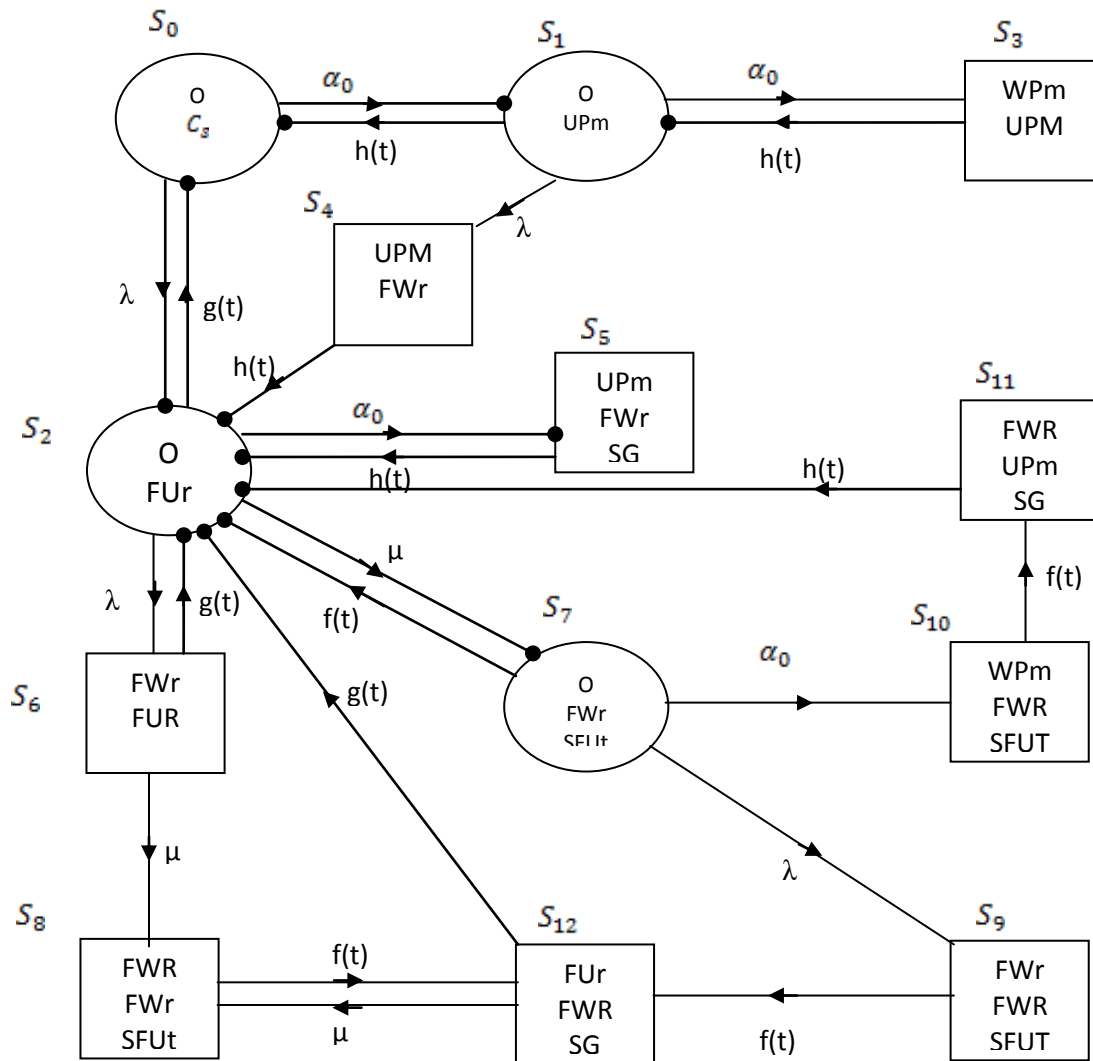
The breakdown of a service facility has been considered as a marked and unavoidable phenomenon in the queuing system. Until the failed facility is recovered, the waiting time for a customer in the system increases, thereby resulting a loss to the system. To offset this, an immediate treatment to the failed service facility is required. The systems with repairable server have been studied as queuing models by many authors such as Murari and Goyal (1984), Cao and Wu (1989) and Malik (2013). But, in most of these systems, it is assumed that repair facility neither fails nor deteriorates during jobs. In fact, this assumption seems to unrealistic when repair facility meets with an accident due to the reasons such as mishandling of the system, unskilled knowledge about the functioning of the systems, electric shocks and carelessness on the part of the server, etc. In such a situation, treatment may be given to the server in order to resume the job and also to reduce the down time of the system. Dhankar and Malik (2010) studied a single unit system with server failure during repair. Also, the existing literature on reliability modeling of repairable systems indicate that preventive maintenance is helpful in reducing the deterioration rate of the systems working in different environmental conditions. Malik (2013) discussed reliability model of a computer system with cold standby redundancy under preventive maintenance and repair.

In the light of above observations, this paper has been designed for determining reliability characteristics of a cold standby system with the ideas of preventive maintenance, server failure and priority in repair discipline. The units are identical in nature and have two modes-normal and complete failures. There is a single server who attends the system immediately to tackle the faults whenever occur in the system. The preventive maintenance of the operating unit is conducted before its failure. The repair of the unit is done by the server at its complete failure. The server is subjected to failure while performing jobs. The failed server undergoes for treatment in order to resume the jobs. The random variables associated with failure time of the unit, preventive maintenance time and treatment time of the server are statistically independent with different probability distributions. Priority is given to the preventive maintenance over repair of the unit. The system model is analyzed by using semi-Markov process and regenerative point technique. The expressions for some important reliability measures including mean time to system failure (MTSF), availability, busy period of the server due to preventive maintenance and repair, expected number of preventive maintenances of the units and treatments give to the server and profit function are derived in steady state and also for arbitrary values of the parameters. The effect of the repair policy on profit of the system model is observed. Graphs are drawn for a particular case to show the behavior of MTSF, availability and profit of the system model.

II.SYSTEM DESCRIPTION

A stochastic model for a system (cold standby) with priority to preventive maintenance over repair and server failure during repair has been developed. The block diagram of the system model is shown in Fig.:1

Fig.1: State Transition



The system mode (figure 1) has the following transition states:

- Regenerative States: S_0, S_1, S_2, S_5 and S_7
- Non-regenerative: $S_3, S_4, S_6, S_8, S_9, S_{10}, S_{11}$ and S_{12}

The following are the possible transition states of the system:

- | | | |
|--------------------------|-----------------------------|---------------------------|
| $S_0 = (O, C_s)$ | $S_1 = (O, UPm)$ | $S_2 = (O, FUR)$, |
| $S_3 = (WPm, UPM)$ | $S_4 = (UPM, FWr)$ | $S_5 = (UPm, FWr, SG)$ |
| $S_6 = (FWr, FUR)$ | $S_7 = (O, FWr, SFUt)$ | $S_8 = (FWr, FWr, SFUt)$ |
| $S_9 = (FWr, FWr, SFUT)$ | $S_{10} = (WPm, FWr, SFUT)$ | $S_{11} = (FWr, UPm, SG)$ |
| | $S_{12} = (FUR, FWr, SG)$ | |



$S_{12}=(FUr, FWR, SG)$

1. Transition Probabilities (TP)

Simple probabilistic considerations yield the following expressions for the non-zero transition probabilities as:

| | |
|---|--|
| $dQ_{01}(t) = \alpha_0 e^{-(\alpha_0+\lambda)t} dt$ | $dQ_{02}(t) = \lambda e^{-(\alpha_0+\lambda)t} dt$ |
| $dQ_{10}(t) = e^{-(\alpha_0+\lambda)t} h(t) dt$ | $dQ_{13}(t) = \alpha_0 e^{-(\alpha_0+\lambda)t} \overline{H(t)} dt$ |
| $dQ_{14}(t) = \lambda e^{-(\alpha_0+\lambda)t} \overline{H(t)} dt$ | $dQ_{20}(t) = e^{-(\mu+\alpha_0+\lambda)t} g(t) dt$ |
| $dQ_{25}(t) = \alpha_0 e^{-(\alpha_0+\lambda+\mu)t} \overline{G(t)} dt$ | $dQ_{26}(t) = \lambda e^{-(\alpha_0+\lambda+\mu)t} \overline{G(t)} dt$ |
| $dQ_{27}(t) = \mu e^{-(\alpha_0+\lambda+\mu)t} \overline{G(t)} dt$ | $dQ_{31}(t) = h(t) dt$ |
| $dQ_{42}(t) = h(t) dt$ | $dQ_{52}(t) = h(t) dt$ |
| $dQ_{62}(t) = e^{-\mu t} g(t) dt$ | $dQ_{68}(t) = \mu e^{-\mu t} \overline{G(t)} dt$ |
| $dQ_{62}(t) = e^{-\mu t} g(t) dt$ | $dQ_{72}(t) = e^{-(\alpha_0+\lambda)t} f(t) dt$ |
| $dQ_{7,9}(t) = \lambda e^{-(\alpha_0+\lambda)t} \overline{F(t)} dt$ | $dQ_{7,10}(t) = \alpha_0 e^{-(\alpha_0+\lambda)t} \overline{F(t)} dt$ |
| $dQ_{8,12}(t) = f(t) dt$ | $dQ_{9,12}(t) = f(t) dt$ |
| $dQ_{10,11}(t) = f(t) dt$ | $dQ_{11,2}(t) = h(t) dt$ |
| $dQ_{12,2}(t) = e^{-\mu t} g(t) dt$ | $dQ_{12,8}(t) = \mu e^{-\mu t} \overline{G(t)} dt$ |

The transition probabilities can be obtained as:

$$p_{ij} = Q_{ij}(\infty) = \int_0^{\infty} q_{ij}(t) dt$$

We have

| | | |
|---|--|------------------------------------|
| $p_{01} = \frac{\alpha_0}{\lambda+\alpha_0}$ | $p_{02} = \frac{\lambda}{\lambda+\alpha_0}$ | $p_{10} = h^*(\lambda + \alpha_0)$ |
| $p_{13} = \frac{\alpha_0}{\lambda+\alpha_0} [1 - h^*(\lambda + \alpha_0)]$ | $p_{14} = \frac{\lambda}{\lambda+\alpha_0} [1 - h^*(\lambda + \alpha_0)]$ | |
| $p_{20} = g^*(\mu + \lambda + \alpha_0)$ | $p_{25} = \frac{\alpha_0}{\mu+\lambda+\alpha_0} [1 - g^*(\mu + \lambda + \alpha_0)]$ | |
| $p_{26} = \frac{\lambda}{\mu+\lambda+\alpha_0} [1 - g^*(\mu + \lambda + \alpha_0)]$ | $p_{27} = \frac{\mu}{\mu+\lambda+\alpha_0} [1 - g^*(\mu + \lambda + \alpha_0)]$ | |
| $p_{31} = h^*(0)$ | $p_{42} = h^*(0)$ | |
| $p_{52} = h^*(0)$ | $p_{62} = g^*(\mu)$ | |
| $p_{68} = 1 - g^*(\mu)$ | $p_{72} = f^*(\lambda + \alpha_0)$ | |
| $p_{7,9} = \frac{\lambda}{\lambda+\alpha_0} [1 - f^*(\lambda + \alpha_0)]$ | $p_{7,10} = \frac{\alpha_0}{\lambda+\alpha_0} [1 - f^*(\lambda + \alpha_0)]$ | |
| $p_{8,12} = f^*(0)$ | $p_{9,12} = f^*(0)$ | |
| $p_{10,11} = f^*(0)$ | $p_{11,2} = h^*(0)$ | |
| $p_{12,2} = g^*(\mu)$ | $p_{12,8} = 1 - g^*(\mu)$ | |

For $h(t) = \gamma e^{-\gamma t}, g(t) = \beta e^{-\beta t}$ and $f(t) = \alpha e^{-\alpha t}$ (1)

For a perfect distribution



$$p_{01} + p_{02} = p_{10} + p_{13} + p_{14} = p_{20} + p_{25} + p_{26} + p_{27} = p_{31} = p_{42} = p_{52} = p_{62} + p_{68} =$$

$$p_{72} + p_{79} + p_{7,10} = p_{8,12} = p_{9,12} = p_{10,11} = p_{11,2} = p_{12,2} + p_{12,8} = 1$$

(2)

2. The Mean Sojourn Times (μ_i) is in the state S_i are

$$\mu_0 = \frac{1}{\alpha_0 + \lambda} \quad \mu_1 = \frac{1}{\square + \alpha_0 + \lambda} \quad \mu_2 = \frac{1}{\beta + \alpha_0 + \alpha + \mu}$$

$$\mu_3 = \mu_4 = \mu_5 = \mu_{11} = \frac{1}{\square} \quad \mu_6 = \mu_{12} = \frac{1}{\beta + \mu} \quad \mu_7 = \frac{1}{\alpha + \alpha_0 + \lambda}$$

$$\mu_8 = \mu_9 = \mu_{10} = \frac{1}{\alpha}$$

$$\mu_0 = m_{01} + m_{02} \quad \mu_1 = m_{10} + m_{13} + m_{14} \quad \mu_2 = m_{20} + m_{25} + m_{26} + m_{27}$$

$$\mu_3 = m_{31} \quad \mu_4 = m_{42} \quad \mu_5 = m_{52}$$

$$\mu_6 = m_{62} + m_{68} \quad \mu_7 = m_{72} + m_{7,9} + m_{7,10} \mu_8 = m_{8,12}$$

$$\mu_9 = m_{9,12} \quad \mu_{10} = m_{10,11} \quad \mu_{11} = m_{11,2}$$

$$\mu_{12} = m_{12,2} + m_{12,8}$$

$$\mu'_1 = m_{10} + m_{11,3} + m_{12,4}$$

$$\mu'_2 = m_{20} + m_{22,6} + m_{22,6(8,12)^n} + m_{25} + m_{27}$$

$$\mu'_7 = m_{72} + m_{72,10,11} + m_{72,9,12} + m_{72,9(12,8)^n} \tag{3}$$

IV. RELIABILITY CHARACTERISTICS

The following reliability characteristics have been evaluated for the system model:

4.1 Mean Time To System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$$\phi_0(t) = Q_{01}(t) \otimes \phi_1 + Q_{02}(t) \otimes \phi_2$$

$$\phi_1(t) = Q_{10}(t) \otimes \phi_0 + Q_{13}(t) + Q_{14}(t)$$

$$\phi_2(t) = Q_{20}(t) \otimes \phi_0 + Q_{27}(t) \otimes \phi_7 + Q_{25}(t) + Q_{27}(t)$$

$$\phi_7(t) = Q_{72}(t) \otimes \phi_2 + Q_{7,10}(t) + Q_{7,9}(t) \tag{4}$$

Taking LST of above relation (4) and solving for $\tilde{\phi}_0(s)$, we have

$$R^*(s) = \frac{1 - \tilde{\phi}_0(s)}{s} \tag{5}$$

And

$$MTSF = \lim_{s \rightarrow 0} \frac{1 - \tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1} \tag{6}$$

Where, $N_1 = (1 - p_{27}p_{72})(\mu_0 + p_{01}\mu_1) + p_{02}(\mu_2 + p_{27}\mu_7)$

$$D_1 = (1 - p_{01}p_{10})(1 - p_{27}p_{72}) - p_{02}p_{20}$$

4.2 Long Run Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at $t = 0$. The recursive relations for $A_i(t)$ are given as under:

$$A_0(t) = M_0(t) + q_{01}(t) \otimes A_1(t) + q_{02}(t) \otimes A_2(t)$$



$$\begin{aligned}
 A_1(t) &= M_1(t) + q_{10}(t) \odot A_0(t) + q_{11,3}(t) \odot A_1(t) + q_{12,4}(t) \odot A_2(t) \\
 A_2(t) &= M_2(t) + q_{20}(t) \odot A_0(t) + (q_{22,6}(t) + q_{22,6(8,12)^n}(t)) \odot A_2(t) + q_{25}(t) \odot A_5(t) \\
 &\quad + q_{27}(t) \odot A_7(t) \\
 A_5(t) &= q_{52}(t) \odot A_2(t) \\
 A_7(t) &= M_7(t) + (q_{72}(t) + q_{72,10,11}(t) + q_{72,9,12}(t) + q_{72,9(12,8)^n}(t)) \odot A_2(t)
 \end{aligned} \tag{7}$$

Where,

$$\begin{aligned}
 M_0(t) &= e^{-(\lambda+\alpha_0)t} & M_1(t) &= e^{-(\lambda+\alpha_0)t} \overline{H(t)} \\
 M_2(t) &= e^{-(\mu+\lambda+\alpha_0)t} \overline{G(t)} & M_7(t) &= e^{-(\lambda+\alpha_0)t} \overline{F(t)}
 \end{aligned}$$

Taking LT of above relations (7) and solving for $A_0^*(s)$.

The steady state availability is given by

$$A_0(\infty) = \lim_{s \rightarrow 0} s A_0^*(s) = \frac{N_2}{D_2}$$

Where, $N_2 = p_{20}(1 - p_{13})\mu_0 + p_{01}p_{20}\mu_1 + (p_{10}p_{02} + p_{14})(\mu_2 + \mu_7 p_{27})$

$D_2 = p_{20}(1 - p_{13})\mu_0 + p_{20}p_{01}\mu_1 + (p_{10}p_{02} + p_{14})(\mu_2 + p_{25}\mu_5 + p_{27}\mu_7)$

4.3 Busy Period of the Server Due to Repair in the Long Run

Let $B_i^R(t)$ be the probability that the server is busy in repairing the unit due to long run at an instant 't' given that the system entered state S_i at $t = 0$. The recursive relations $B_i^R(t)$ for are as follows:

$$\begin{aligned}
 B_0^R(t) &= q_{01}(t) \odot B_1^R(t) + q_{02}(t) \odot B_2^R(t) \\
 B_1^R(t) &= q_{10}(t) \odot B_0^R(t) + q_{11,3}(t) \odot B_1^R(t) + q_{12,4}(t) \odot B_2^R(t) \\
 B_2^R(t) &= W_2^R(t) + q_{20}(t) \odot B_0^R(t) + (q_{22,6}(t) + q_{22,6(8,12)^n}(t)) \odot B_2^R(t) + q_{25}(t) \odot B_5^R(t) \\
 &\quad + q_{27}(t) \odot B_7^R(t) \\
 B_5^R(t) &= q_{52}(t) \odot B_2^R(t) \\
 B_7^R(t) &= (q_{72}(t) + q_{72,10,11}(t) + q_{72,9,12}(t) + q_{72,9,11(12,8)^n}(t)) \odot B_2^R(t)
 \end{aligned} \tag{8}$$

where $W_i^R(t)$ be the probability that the server is busy in state S_i due to long run at an at time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

Where, $W_2^R(t) = e^{-(\alpha_0+\mu+\lambda)t} \overline{G(t)} + (\lambda e^{-\lambda t} \odot \mu e^{-\mu t} \odot f(t) \odot 1) \overline{G(t)}$

Let us take LT of relations (8) and solving for $B_0^{R*}(s)$.

We get, $B_0^R(\infty) = \lim_{s \rightarrow 0} s B_0^{R*}(s) = \frac{N_3}{D_2}$

$N_3 = W_2^*(0)(p_{14} + p_{02}p_{10})$ and D_2 is already specified.

4.4 Busy Period of the Server due to Preventive Maintenance (PM) in the Long Run

Let $B_i^P(t)$ be the probability that the server is busy in repairing the unit due to preventive maintenance in the long run at an instant 't' given that the system entered state i at $t = 0$. The recursive relations $B_i^P(t)$ for are as follows:

$$\begin{aligned}
 B_0^P(t) &= q_{01}(t) \odot B_1^P(t) + q_{02}(t) \odot B_2^P(t) \\
 B_1^P(t) &= W_1^P(t) + q_{10}(t) \odot B_0^P(t) + q_{11,3}(t) \odot B_1^P(t) + q_{12,4}(t) \odot B_2^P(t)
 \end{aligned}$$

$$\begin{aligned}
 B_2^P(t) &= q_{20}(t) \otimes B_0^P(t) + (q_{22.6}(t) + q_{22.6(8,12)^n}(t)) \otimes B_2^P(t) + q_{25}(t) \otimes B_5^P(t) \\
 &\quad + q_{27}(t) \otimes B_7^P(t) \\
 B_5^P(t) &= (q_{41}(t) + q_{41(8,12)^n}(t)) \otimes B_1^P(t) \\
 B_7^P(t) &= (q_{72}(t) + q_{72.10,11}(t) + q_{72.9,12}(t) + q_{72.9(12,8)^n}(t)) \otimes B_2^P(t) \tag{9}
 \end{aligned}$$

where $W_i^P(t)$ be the probability that the server is busy in state S_i due to preventive maintenance in long run at time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

Where, $W_1^P(t) = e^{-(\alpha_0 + \lambda)t} \overline{H(t)} + (\alpha_0 e^{-\alpha_0 t} \otimes 1) \overline{H(t)} + (\lambda e^{-\lambda t} \otimes 1) \overline{H(t)}$ and $W_5^P(t) = \overline{H(t)}$

Let us take LTof relations (9) and solving for $B_0^{P*}(s)$.

We get, $B_0^P(\infty) = \lim_{s \rightarrow 0} s B_0^{P*}(s) = \frac{N_4}{D_2}$

$N_4 = W_1^{P*}(0)p_{01}p_{20} + W_5^{P*}(0)p_{01}p_{14}p_{25}$ and D_2 is already specified.

4.5 Expected Number of Repairs (ENR) of the unit in the Long Run

Let $R_i(t)$ be the expected number of repair of the unit in the long run by the server in $(0, t]$ given that the system entered the regenerative state i at $t = 0$. The recursive relations for $R_i(t)$ are given as:

$$\begin{aligned}
 R_0(t) &= Q_{01}(t) \otimes R_1(t) + Q_{02}(t) \otimes R_2(t) \\
 R_1(t) &= Q_{10}(t) \otimes R_0(t) + Q_{11.3}(t) \otimes R_1(t) + Q_{12.4}(t) \otimes R_2(t) \\
 R_2(t) &= Q_{20}(t) \otimes (1 + R_0(t)) + (Q_{22.6}(t) + Q_{22.6(8,12)^n}(t)) \otimes (1 + R_1(t)) + Q_{25}(t) \otimes R_5(t) \\
 &\quad + Q_{27}(t) \otimes R_7(t) \\
 R_5(t) &= Q_{52}(t) \otimes R_2(t) \\
 R_7(t) &= (Q_{72}(t) + Q_{72.10,11}(t)) \otimes R_2(t) + (Q_{72.9,12}(t) + Q_{72.9(12,8)^n}(t)) \otimes (1 + R_2(t)) \tag{10}
 \end{aligned}$$

Taking LST of relations (10). And, solving for $\overline{PM}_i(s)$. The expected numbers of preventive maintenance of the unit in the long run are given by

Let us take LST of relations (10) and solving for $\widetilde{R}_0(s)$

We get, $R_0 = \lim_{s \rightarrow 0} s \widetilde{R}_0(s) = \frac{N_5}{D_2}$

Where, $N_5 = (p_{14} + p_{10}p_{02})(p_{20} + p_{26} + p_{79}p_{27})$ and D_2 is already specified.

4.6 Expected Number of Preventive Maintenance (PM) of the unit in the Long Run

Let $PM_i(t)$ be the expected number of preventive maintenance of the unit in the long run by the server in $(0, t]$ given that the system entered the regenerative state i at $t = 0$. The recursive relations for $PM_i(t)$ are given as

$$\begin{aligned}
 PM_0(t) &= Q_{01}(t) \otimes PM_1(t) + Q_{02}(t) \otimes PM_2(t) \\
 PM_1(t) &= Q_{10}(t) \otimes (1 + PM_0(t)) + Q_{11.3}(t) \otimes (1 + PM_1(t)) + Q_{12.4}(t) \otimes (1 + PM_2(t)) \\
 PM_2(t) &= Q_{20}(t) \otimes PM_6(t) + (Q_{22.6}(t) + Q_{22.6(8,12)^n}(t)) \otimes PM_2(t) + Q_{25}(t) \otimes PM_5(t) \\
 &\quad + Q_{27}(t) \otimes PM_7(t) \\
 PM_5(t) &= Q_{52}(t) \otimes (1 + PM_2(t))
 \end{aligned}$$



$$PM_7(t) = Q_{72}(t) \otimes PM_2(t) + (Q_{72.9,12}(t) + Q_{72.9(12,8)^n}(t)) \otimes PM_2(t) + Q_{72.10,11}(t) \otimes (1 + PM_2(t)) \tag{11}$$

Taking LST of relations (11). And, solving for $\widetilde{PM}_i(s)$. The expected numbers of preventive maintenance of the unit in the long run are given by

$$\text{We get, } PM_0 = \lim_{s \rightarrow 0} s \widetilde{PM}_i(s) = \frac{N_6}{D_2}$$

Where, $N_6 = p_{01}p_{20} + (p_{14} + p_{02}p_{10})(p_{25} + p_{7,10}p_{27})$ and D_2 is already specified.

4.7 Expected Number of Treatments (ENT) Given to the Server in the Long Run

Let $T_i(t)$ be the expected number of treatment given to the server in the long run in $(0, t]$ given that the system entered the regenerative state i at $t = 0$. The recursive relations for $T_i(t)$ are given as

$$\begin{aligned} T_0(t) &= Q_{01}(t) \otimes T_1(t) + Q_{02}(t) \otimes T_2(t) \\ T_1(t) &= Q_{10}(t) \otimes T_0(t) + Q_{11.3}(t) \otimes T_1(t) + Q_{12.4}(t) \otimes T_2(t) \\ T_2(t) &= Q_{20}(t) \otimes T_0(t) + Q_{22.6}(t) \otimes T_2(t) + Q_{22.6(8,12)^n}(t) \otimes (1 + T_2(t)) + Q_{25}(t) \otimes T_5(t) \\ &\quad + Q_{27}(t) \otimes T_7(t) \\ T_5(t) &= Q_{52}(t) \otimes T_2(t) \\ T_7(t) &= (Q_{72}(t) + Q_{72.10,11}(t) + Q_{72.9,12}(t) + Q_{72.9(12,8)^n}(t)) \otimes (1 + T_2(t)) \end{aligned} \tag{12}$$

Taking LST of relations (12). And, solving for $\widetilde{T}_0(s)$ The expected numbers of preventive maintenance of the unit in the long run are given by

Let us take LST of relations (12) and solving for $\widetilde{T}_0(s)$.

$$\text{We get, } T_0 = \lim_{s \rightarrow 0} s \widetilde{T}_0(s) = \frac{N_7}{D_2}$$

Where, $N_7 = (p_{14} + p_{02}p_{10})(p_{27} + p_{26}p_{68})$ and D_2 is already specified.

V.PROFIT ANALYSIS

$$P = K_0A_0 - K_1B_0^R - K_4B_0^P - K_2R_0 - K_5P_0 - K_3T_0$$

The profit of the system model can be obtained as:

Where

P = Profit of the system model

K_0 = Revenue per unit up-time of the system

K_1 = Cost per unit time for which server is busy due to repair

K_2 = Cost per unit time repair

K_3 = Cost per unit time for treatment of the server

K_4 = Cost per unit for which server is busy due to preventive maintenance

K_5 = Cost per unit time repair of the unit

VI.PARTICULAR CASES

$$\text{Let us take } g(t) = \beta e^{-\beta t} \quad f(t) = \alpha e^{-\alpha t} \quad h(t) = \square e^{-\square t}$$

$$p_{01} = \frac{\alpha_0}{\lambda + \alpha_0} \quad p_{02} = \frac{\lambda}{\lambda + \alpha_0} \quad p_{10} = \frac{\square}{\square + \lambda + \alpha_0}$$



$$\begin{aligned}
 p_{13} &= \frac{\alpha_0}{\square + \lambda + \alpha_0} & p_{14} &= \frac{\lambda}{\square + \lambda + \alpha_0} & p_{20} &= \frac{\beta}{\beta + \mu + \lambda + \alpha_0} \\
 p_{25} &= \frac{\alpha_0}{\beta + \mu + \lambda + \alpha_0} & p_{26} &= \frac{\lambda}{\beta + \mu + \lambda + \alpha_0} & p_{27} &= \frac{\mu}{\beta + \mu + \lambda + \alpha_0} \\
 p_{31} &= p_{42} = p_{52} = p_{8,12} = p_{9,12} = p_{10,11} = p_{11,2} = 1 \\
 p_{62} &= \frac{\beta}{\beta + \mu} & p_{68} &= \frac{\mu}{\beta + \mu} & p_{72} &= \frac{\alpha}{\alpha + \lambda + \alpha_0} \\
 p_{7,9} &= \frac{\lambda}{\alpha + \lambda + \alpha_0} & p_{7,10} &= \frac{\alpha_0}{\alpha + \lambda + \alpha_0} & p_{12,2} &= \frac{\beta}{\beta + \mu} \\
 p_{12,8} &= \frac{\mu}{\beta + \mu} & \mu_0 &= \frac{1}{\lambda + \alpha_0} & \mu_1 &= \frac{1}{\square + \lambda + \alpha_0} & \mu_2 &= \frac{1}{\beta + \mu + \lambda + \alpha_0} \\
 \mu_3 &= \mu_4 = \mu_5 = \mu_{11} = \frac{1}{\square} & \mu_6 &= \mu_{12} = \frac{1}{\beta + \mu} \\
 \mu_7 &= \frac{1}{\alpha + \lambda + \alpha_0} & \mu_8 &= \mu_9 = \mu_{10} = \frac{1}{\alpha} \\
 \mu'_1 &= \frac{1}{\square} & \mu'_2 &= \frac{\alpha\beta + \lambda(\alpha + \mu)}{\alpha\beta(\beta + \mu + \lambda + \alpha_0)} & \mu'_7 &= \frac{\beta\{\alpha\square + \alpha_0(\alpha + \square)\} + \lambda(\beta + \alpha + \mu)}{\alpha\beta\square(\alpha + \lambda + \alpha_0)}
 \end{aligned}$$

$$\begin{aligned}
 MTSF &= \frac{N_1}{D_1}, & \text{Availability}(A_0) &= \frac{N_2}{D_2} \\
 B_0^R &= \frac{N_3}{D_2} & B_0^P &= \frac{N_4}{D_2} & R_0 &= \frac{N_5}{D_2} \\
 P_0 &= \frac{N_6}{D_2} & T_0 &= \frac{N_7}{D_2}
 \end{aligned}$$

Where,

$$N_1 = (\gamma + \lambda + 2\alpha_0)[(\beta + \lambda + \mu + \alpha_0)(\alpha + \lambda + \alpha_0) - \mu\alpha] + \lambda(\gamma + \lambda + \alpha_0)(\alpha + \lambda + \mu + \alpha_0)$$

$$D_1 = [(\lambda + \alpha_0)(\gamma + \lambda + \alpha_0) - \gamma\alpha_0][(\beta + \lambda + \mu + \alpha_0)(\alpha + \lambda + \alpha_0) - \mu\alpha] - \lambda\beta(\gamma + \lambda + \alpha_0)(\alpha + \lambda + \alpha_0)$$

$$N_2 = \frac{\lambda\mu + (\beta + \lambda)(\alpha + \lambda + \alpha_0)}{(\lambda + \alpha_0)(\alpha + \lambda + \alpha_0)(\beta + \mu + \lambda + \alpha_0)}$$

$$D_2 = \frac{\alpha\beta^2\square(\alpha + \lambda + \alpha_0)(\square + \lambda) + \alpha_0\beta^2\alpha(\alpha + \lambda + \alpha_0)(\gamma + \lambda + \alpha_0) + \lambda(\gamma + \lambda + \alpha_0)[\square(\alpha + \lambda + \alpha_0)\{\beta\alpha + \lambda(\alpha + \mu)\} + \alpha_0\beta\alpha(\alpha + \lambda + \alpha_0) + \mu\{\alpha\beta\square + \alpha_0\beta(\square + \alpha) + \lambda\square(\alpha + \beta + \mu)\}]}{\alpha\beta\square(\lambda + \alpha_0)(\alpha + \lambda + \alpha_0)(\gamma + \lambda + \alpha_0)(\beta + \mu + \lambda + \alpha_0)}$$

$$N_3 = \frac{\lambda[\beta(\mu + \alpha_0) + \lambda(\beta + \mu + \lambda + \alpha_0)]}{\beta(\lambda + \alpha_0)(\mu + \lambda + \alpha_0)(\beta + \mu + \lambda + \alpha_0)}$$

$$N_4 = \frac{\alpha_0[(\gamma + \alpha_0 + \lambda)\square + \alpha_0\lambda]}{\square(\lambda + \alpha_0)(\gamma + \alpha_0 + \lambda)(\beta + \mu + \lambda + \alpha_0)}$$

$$N_5 = \frac{\lambda[\lambda\mu + (\beta + \lambda)(\alpha + \lambda + \alpha_0)]}{(\lambda + \alpha_0)(\alpha + \alpha_0 + \lambda)(\beta + \mu + \lambda + \alpha_0)}$$

$$N_6 = \frac{\alpha_0[\lambda\mu + (\beta + \lambda)(\alpha + \lambda + \alpha_0)]}{(\lambda + \alpha_0)(\alpha + \alpha_0 + \lambda)(\beta + \mu + \lambda + \alpha_0)}$$

$$N_7 = \frac{\lambda\mu(\beta + \mu + \lambda)}{(\lambda + \alpha_0)(\beta + \mu)(\beta + \mu + \lambda + \alpha_0)}$$

Numerical and Graphical Representation of Reliability Measures

Table 1: MTSF Vs Failure Rate

| λ | $\beta=3.1, \alpha=2.1, \mu=0.2, \gamma=2.1, \alpha_0=2.2$ | $\alpha=3.1$ | $\beta=4.1$ | $\mu=0.4$ | $\alpha_0=2.6$ | $\gamma=2.7$ |
|-----------|--|--------------|-------------|-----------|----------------|--------------|
| 0.1 | 1.2725 | 1.2727 | 1.2773 | 1.27179 | 1.0326 | 1.3820 |
| 0.2 | 1.2086 | 1.2088 | 1.2172 | 1.2073 | 0.9890 | 1.3058 |
| 0.3 | 1.1503 | 1.1506 | 1.1621 | 1.1486 | 0.9487 | 1.2369 |
| 0.4 | 1.0971 | 1.0974 | 1.1112 | 1.0950 | 0.9114 | 1.1746 |
| 0.5 | 1.0482 | 1.0485 | 1.0644 | 1.0459 | 0.8766 | 1.1178 |
| 0.6 | 1.0032 | 1.0036 | 1.0209 | 1.0008 | 0.8444 | 1.0659 |
| 0.7 | 0.9616 | 0.9621 | 0.9807 | 0.9592 | 0.8142 | 1.0184 |
| 0.8 | 0.9232 | 0.9236 | 0.9432 | 0.9206 | 0.7861 | 0.9747 |
| 0.9 | 0.8875 | 0.8879 | 0.9083 | 0.8849 | 0.7596 | 0.9344 |

Table 2: Availability Vs Failure Rate

| Λ | $\beta=3.1, \mu=0.2, \gamma=2.1, \alpha=2.1, \alpha_0=2.2$ | $\alpha=3.1$ | $\beta=4.1$ | $\mu=0.4$ | $\alpha_0=2.6$ | $\gamma=2.7$ |
|-----------|--|--------------|-------------|-----------|----------------|--------------|
| 0.1 | 0.638309857 | 0.638662 | 0.640246 | 0.637339 | 0.582459 | 0.718455 |
| 0.2 | 0.626125291 | 0.626811 | 0.629969 | 0.624225 | 0.571884 | 0.705116 |
| 0.3 | 0.614441143 | 0.615443 | 0.62016 | 0.611655 | 0.561737 | 0.692131 |
| 0.4 | 0.603214669 | 0.604515 | 0.610769 | 0.599587 | 0.551982 | 0.679492 |
| 0.5 | 0.592409414 | 0.593991 | 0.601758 | 0.587983 | 0.542589 | 0.667192 |
| 0.6 | 0.581994007 | 0.583841 | 0.59309 | 0.576812 | 0.533531 | 0.655225 |
| 0.7 | 0.571941228 | 0.574037 | 0.584737 | 0.566045 | 0.524785 | 0.643582 |
| 0.8 | 0.56222726 | 0.564557 | 0.576672 | 0.555658 | 0.516329 | 0.632255 |
| 0.9 | 0.552831108 | 0.555381 | 0.568873 | 0.545628 | 0.508146 | 0.621236 |

Table 3: Profit Vs Failure Rate

| λ | $\beta=3.1, \mu=0.2, \gamma=2.1, \alpha=2.1, \alpha_0=2.2$ | $\alpha=3.1$ | $\beta=4.1$ | $\mu=0.4$ | $\alpha_0=2.6$ | $\gamma=2.7$ |
|-----------|--|--------------|-------------|-----------|----------------|--------------|
| 0.1 | 8986.764272 | 8991.659 | 9072.41 | 8971.397 | 8123.667 | 10116.61 |
| 0.2 | 8692.01388 | 8701.391 | 8816.653 | 8662.471 | 7863.546 | 9791.326 |
| 0.3 | 8409.086428 | 8422.557 | 8571.125 | 8366.511 | 7613.585 | 9476.132 |
| 0.4 | 8137.066958 | 8154.262 | 8334.898 | 8082.55 | 7373.008 | 9170.712 |

| | | | | | | |
|-----|-------------|----------|----------|----------|--------------|--------------|
| 0.5 | 7875.166034 | 7895.739 | 8107.179 | 7809.744 | 7141.14 3 | 8874.75 1 |
| 0.6 | 7622.696191 | 7646.32 | 7887.285 | 7547.352 | 6917.4 | 8587.93 9 |
| 0.7 | 7379.053534 | 7405.422 | 7674.624 | 7294.715 | 6701.26 | 8309.97 1 |
| 0.8 | 7143.703237 | 7172.53 | 7468.679 | 7051.244 | 6492.26 4 | 8040.54 9 |
| 0.9 | 6916.168028 | 6947.184 | 7268.998 | 6816.41 | 6289.99 8 | 7779.37 9 |

VII.CONCLUSION

The numerical results considering a particular case are obtained for some reliability characteristics of the system model. It is observed that the MTSF (shown in table1) keeps on decline with the increase of the failure rate of the unit and server and the rate at which unit goes on Preventive Maintenance while it increases with the increase of treatment rate of the server (α), repair rate (β) and preventive maintenance rate of the unit (γ). The results given in tables 2 & 3 indicate that availability and profit go on decreasing when we increase of failure rate of the unit and server as well as the rate by which unit under goes for preventive maintenance. However, their values keep on increasing with the increase of repair rate of the unit, treatment rate of the server and preventive maintenance rate of the unit. The study also reveals that the provision of priority to preventive maintenance over repair is helpful in making the system more profitable and practicable to use.

REFERENCES

- [1.] 1.Malik, S.C., Sudesh,K. Barak and M. S. Barak (2014). Stochastic Analysis of a Cold Standby System with Priority to Preventive Maintenance over Repair. **Advances in Industrial Engineering and Management**, Vol. 3(2), pp. 33-38
- [2.] 2.Rathee, R. and Malik, S.C. (2014). A Parallel System with Priority to Repair over Preventive Maintenance subject to Maximum Operation and Repair Times. **International Journal of Statistics and Reliability Engineering**, Vol. 1(1), pp. 57-68
- [3.] 3.Upma and Malik, S.C. (2016). Cost-Benefit Analysis of a System of Non-Identical Units under Preventive Maintenance and Replacement Subject to Priority for Operation. **International Journal of Statistics and Systems**, Vol. 11(1), pp. 37-46
- [4.] 4.Nandal, J. and Malik, S.C. (2016). Reliability Measures of a Cold Standby System with Arrival Time of a Server Subject to Failure. **International Journal of Statistics and Systems**, Vol. 11(2), pp. 197-209
- [5.] 5.Nandal, J. and Rathee, R. (2015). (Stochastic Analysis of a Redundant System with Server Failure and Conditional Arrival Time. **International Journal of Statistics and Reliability Engineering**, Vol. 2(1), pp.94-102