

## **Efflux Time-Mini Review**

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### **ABSTRACT**

*Present paper discusses the literature available on efflux time. The mathematical equations used for arriving at efflux time for different geometries of storage vessels are also presented. Efflux time can be reduced by the addition of drag reducing polymers or surfactants etc. The literature available on reduction of efflux time by using these drag reducing agents is also discussed. The merits and demerits of these drag reducing agents are also presented in this review. The scope for future work is discussed.*

### **I INTRODUCTION**

Chemical Industries use different geometries of storage vessels for processing. The factors that can influence the selection of given geometry of a storage vessel includes but not limited to convenience, insulation and safety requirements. The time required to drain these storage vessels is of utmost importance and is referred to as efflux time [1]. This is of significant importance for increasing the productivity or under emergency situations and hence is of considerable interest in chemical, food and pharmaceutical industries [2]. When a liquid drains from a large tank, it drains so slowly that draining can be approximated to pseudo steady state. How large is the storage tank is the question that needs to be addressed. The liquid that needs to be drained can be Newtonian or Non-Newtonian. Majority of the work reported in the literature is centered around draining of Newtonian liquid and the literature available is sparse with respect to draining of Non-Newtonian liquid.

The liquid from the storage tank can be drained by either restricted orifice or by means of exit pipe(s). When the liquid is drained from a large tank by means of an exit pipe, the flow in the tank is laminar while the flow in the exit pipe can be laminar or turbulent depending upon the physical properties of the liquid to be drained. When the flow in the exit pipe is laminar, the efflux time so obtained is the maximum time required for draining the contents of the storage vessel and when the flow in the exit pipe is turbulent, the efflux time will be the minimum time required for emptying the contents of the storage vessel [3].

Hart and Sommerfeld [3] derived the equation for efflux time for a liquid ( below its bubble point) through restricted orifice located at the bottom of the tank for both horizontal and vertical annular containers. The

authors stated that draining problems can be solved using two fundamental equations. The two equations used are

$$A \frac{dh}{dt} = -VA_o \quad (1)$$

A and A<sub>o</sub> refer to the cross sectional area of tank and orifice respectively. V is linear velocity of liquid in the tank and  $\frac{dh}{dt}$  is the rate of variation of liquid level in the tank and h is the height of liquid in the tank.

The velocity of liquid in the tank is given by

$$V = C_o * \sqrt{2gh} \quad (2)$$

C<sub>o</sub> is the discharge coefficient. The authors derived the following generalized equation for efflux time

$$t = \frac{1}{C_o A_o \sqrt{2g}} \int_0^H \frac{A}{\sqrt{h}} \quad (3)$$

H is the height of liquid in the tank before draining. When C<sub>o</sub>=1, the equation boils down to Toricelli Equation.

Sommerfeld [4] derived equation for efflux time for different geometries of vessels. They include parallelepiped, vertical elliptical cylinder, regular tetrahedron, pyramid and paraboloid.

Foster [5] derived equation for efflux time for different geometries of vessels through restricted orifice. The authors mentioned that discharge coefficient for a Newtonian fluid for turbulent flow conditions is constant and depends upon the shaped of the container. The authors stated that the discharge coefficient for sharp edged orifice is 0.61 and rounded orifice is 0.98.

Vandogen and Roche.Jr [6] performed experiments on efflux time from cylindrical tank with exit pipe and fittings. Under conditions of turbulent flow in the exit pipe, the authors mentioned that the efflux time, t, could be related to the height of liquid (H<sub>i</sub>) relative to the bottom of the exit pipe, H by

$$t = k_1 (H_i^{3/7} - H^{3/7}) \quad (1)$$

The constant k<sub>1</sub> contains the relevant physical properties of the liquid and critical dimensions of the apparatus.

The friction factor equation

$$f = \frac{0.0316}{Re^{0.25}} \quad (2)$$

is used by authors for calculating the friction factor. The authors also mentioned that the one should not discount the possibility of pockets of air or vapour under some high flow-rate situations (Re>40,000 to 80,000). It has to be seen under what conditions the liquid needs to be drained under such high Reynolds number through an exit pipe.

Libii [7] presented his work on draining a Newtonian liquid from a cylindrical tank through restricted orifices of different diameters. The author mentioned that pseudo steady state assumption is valid when the ratio of cross sectional area of the tank to orifice is more than 100. Pseudo steady state assumption possibly refers to less deviation from theoretical and experimental values. In case of cylindrical tank, the cross section for flow suddenly changes at the exit pipe while the cross section gradually changes in case of cone, it has to be seen whether the pseudo steady state assumption is expected to be valid for much lower ratio of cross section of tank to exit pipe. It is also to be seen whether the ratio holds good when the liquid is drained by means of an exit pipe instead of an orifice plate.

Using computational tools, an alternative method of analysis for gravity draining of Newtonian liquid from a cylindrical tank through an exit pipe is presented [8]. Experiments are performed for a Reynolds number of 6000 with a fixed pipe length. The differences obtained between experimental and mathematical values were attributed to contraction coefficient. The maximum efflux time reported is 35 seconds. The ratio of tank cross section to pipe cross section was 228. Since this is larger than that suggested [7], the assumption of pseudo steady state is expected to be valid in this case.

Joye and Barret [9] reported their work on efflux time for draining a cylindrical tank through an exit pipe for both laminar and turbulent flow conditions in the exit pipe. They mentioned that turbulent flow (in the exit pipe) solutions are useful in many plant situations. While deriving the equation for efflux time, the authors made an assumption of constant friction factor in the exit pipe line for turbulent flow conditions in the exit pipe. The authors developed the following equation for efflux time when the exit pipe flow is turbulent.

$$t_{eff} = \frac{D^2}{d^2} (\sqrt{H_i + L_v} - \sqrt{H_f + L_v}) \sqrt{\frac{2 \left( 4f \frac{L}{d} + \sum K \right)}{g}} \quad (3)$$

$\sum K$  term includes the exit energy loss ( $K=1$ ) and the entrance loss ( $K=0.5$ ) from tank to pipe, which can also be considered as resistance. The friction factor equation used is same as that used for calculation of friction in turbulent flow.

Subbarao and other researchers [3,10] used macroscopic balances for arriving at the theoretical equation for efflux time. The authors mentioned that macroscopic balances are useful for making preliminary estimate of an engineering problem. The mathematical equation so developed based on macroscopic balances will be subsequently corrected with the experimental values for terms which have been omitted or about which there is insufficient information. They derived the following equation for efflux time

$t_{eff} = \sqrt{\frac{2}{g_m}} (\sqrt{H+L} - \sqrt{H'+L})$ , where  $g_m$  is the modified form of acceleration due to gravity and is given by

$$\frac{g_m}{g} = \frac{1}{(1 + 4f(L/d)(A_t/A_p))^2}$$

They named the simplified form of efflux time equation as modified form of Toricelli equation. They also reported that during draining of a liquid from a large cylindrical storage vessel through exit pipe, Froude number remains constant and is uninfluenced by diameter and length of the exit pipe. They made the same assumption of constant friction factor while deriving the above equation. However, the authors have not verified this. While deriving the equation, the authors neglected the fluid flow around the tank, the roughness of the walls and the friction in the tank. The authors arrived at combined friction factor to take into account the friction in the pipe lines, the fluid flow around the tank and the roughness of the walls. However, their work is confined to exit pipe of single diameter. Hence reduction in efflux time refers to increased Froude number.

Subbarao [ 11] developed equation for efflux time for cylindrical and conical tanks (of same exit pipe diameter and length) when a Newtonian liquid is drained from them through an exit pipe when the flow in the exit pipe is turbulent. To drain the same volume of liquid, the efflux time equations so developed are compared to find out which of the tanks considered drain faster. The authors concluded that for draining the same volume of liquid, cone drains faster than a cylinder.

Siva Kumar and Subbarao [12] compared the efflux time for cylinder and sphere ( of same exit pipe diameter and length) when a Newtonian liquid is drained from the tank(s) through an exit piping system for the case of turbulent flow in the exit pipe. The authors mentioned that for draining the same volume of liquid, sphere drains faster than a cylinder.

Efflux time comparison for cylinder, sphere and cone when drained by means of an exit pipe ( of same diameter and length) for laminar flow conditions in the exit pipe are derived by Subbarao and other researchers [13]. The authors stated that for draining the same volume of liquid, the efflux time for cylinder > sphere > cone.

Santosh Kumar et al [14] derived equation for efflux time for draining a Newtonian liquid by means of two exit piping system. The authors mentioned that during draining through two exit pipe system also, Froude number remains constant.

Subbarao and other researchers [15,16] also mentioned that Froude number can also be increased by either polymer additions. The authors used different concentration of polymer additions and mentioned that efflux time is minimum for a polyacrylamide polymer concentration of 10 ppm. The authors also mentioned that draining is faster with two exit pipe system compared to polyacryl amide polymer additions.

Raghav and other researchers [17] used flocculating agents as drag reducing agents. This work is based on the assumption that all drag reducing polymers are flocculating agents. Can all the flocculating agents be drag reducing. It has been concluded that all the flocculating agents need not be drag reducing.

Subbarao and other authors [18] carried out efflux time experiments using polythene oxide polymer. The authors concluded that as the diameter of exit pipe is increased, the addition of polymer solutions does not have any effect on reduction in efflux time. Hence the authors concluded that the addition of polymers is felt at contraction point between the tank and exit pipe only.

Subbarao and other researchers [19] conducted experiments on efflux time using surfactant solution (DBSA) and reported an optimum concentration of 900 ppm. However, this concentration is much higher than that of the polymer solutions.

Subbarao and other researchers [20] used surfactant solutions along with counter ion and concluded that the mixed solutions of surfactant and its counter ion (DBSA and NaCl at 1: 1 weight ratio) are better drag reducing agents than surfactant alone. The drag reduction achieved with mixed solutions is found to be more than even polymer solutions.

Subbarao and other researchers [21] used Cetyl Pyridinium Chloride (CPC) surfactant solutions as drag reducing agents for reducing the efflux time for draining a cylindrical tank through single exit pipe system and arrived at 600 ppm optimum concentration of surfactant. This suggests that CPC is better drag reducing agent than DBSA at lower concentrations.

Srinivasa Rao and other researchers [22] studied the effect of viscosity on drag using polymer solutions for draining a liquid from a spherical tank through single exit pipe for laminar flow conditions in the exit pipe. Laminar flow is maintained by using different concentrations of glycerine solutions. The authors concluded that drag reduction takes place only in presence of aqueous solutions of polymer and in case of glycerine solutions, instead of drag reduction, drag enhancement takes place.

Prasad and other researchers [23] developed an alternative equation for efflux time for a cylindrical tank drained through an exit pipe (for turbulent flow conditions in the exit pipe) and mentioned that the efflux time equation is valid in the Reynolds number of 4500 to 9000.

## **II CONCLUSIONS**

Based on the above literature, future work can be focussed on

1. Experiments on efflux time for different geometries of vessels using different polymer solutions.
2. Experiments on efflux time for different geometries of vessels using different surfactant solutions.
3. Experiments on efflux time for different ratios of surfactant and its counter ion.
4. Mechanism of drag reduction.
5. Verification of efflux time for different diameters of exit pipes.

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