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Normal Shock Wave diffraction for Sulphur Hexaflouride (SF_6) gas (Large Bends)

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ABSTRACT

Lighthill has considered the diffraction of normal shock wave past a small bend for $\gamma = 1.4$, γ *being the ratio of specific heats. Following Lighthill, Sakurai et al have obtained pressure distribution over the diffracted shock. Sakurai and Takayama further extended the work of Lighthill to large bends through singular perturbation technique. In the present paper pressure distribution over the diffracted shock wave has been obtained forSulphur Hexaflouride(SF_6) gas for larger bends.*

Keywords: Diffraction, Large bends, Pressure distribution, Sulphur Hexaflouride (SF6) gas, singular perturbation

I.INTRODUCTION

Lighthill (1949) considered the diffraction of normal shock wave past a small bend for $\gamma = 1.4$, γ being the ratio of specific heats. Following Lighthill (1949), Sakurai etal (2002) have obtained pressure distribution over the diffracted shock wave. The work of Lighthill (1949) was extended to larger bends by Sakurai and Takayama (2005) through singular bend perturbation technique. Srivastava (2016) used the work of Sakurai and Takayama (2005) theory for obtaining the pressure distribution over the diffracted shock wave for $\gamma = 1.4$. Srivastava (1963), (2016) extended the work of Lighthill (1949) for $\gamma = \frac{5}{3}$ and Srivastava (2017) carried forward the work of Sakurai and Takayama (2005) for $\gamma = \frac{5}{3}$. Pressure distribution over the diffracted shock has been obtained by Srivastava and Srivastava (2017,2017) both from Lighthill's theory and from Sakurai and Takayama theory for carbon dioxide gas (CO_2). Attempt has been made to obtain pressure distribution over diffracted shock for Sulphur Hexaflouride(SF_6) gas following Lighthill's theory which is applicable to lower bends. The work is ready for submission for publication.

In the present paper, pressure distribution over the diffracted shock wave has been obtained for Sulphur Hexaflouride(SF_6) gas following Sakurai and Takayama's(2005) theory which is applicable for larger bends. The Mach number of the shock wave is M=1.36 and $\gamma = 1.093$. It may be mentioned here that Srivastava (2011) has obtained the vorticity distribution over the diffracted shock wave for monoataomic gases. Reference may be made to the book by Srivastava (1994).

II.MATHEMATICAL FORMULATION

Let the velocity, pressure, density, sound speed behind the shock wave before it has crossed the bend be $q_1,\ \rho_1,\ p_1, a_1$ and ahead of the shock wave be $0,\ p_0,\ \rho_0,\ a_0$. Then applying the principle of conservation of mass, momentum and energy for general value of γ (γ being the ratio of specific heats)

$$q_1 = \frac{2U}{(\gamma+1)}\left(1 - \frac{a_0^2}{U^2}\right) \qquad - (1)$$

$$\rho_1 = \frac{\rho_0(\gamma+1)}{(\gamma-1) + 2\dfrac{a_0^2}{U^2}} \qquad - (2)$$

$$p_1 = \frac{\rho_0}{(\gamma-1)}\left[2U^2 - \frac{a_0^2(\gamma-1)}{\gamma}\right] \qquad - (3)$$

U being the velocity of shock wave, Mach number of the shock $M = \dfrac{U}{a_0},\ \ a_0 = \sqrt{\dfrac{\gamma\, p_0}{\rho_0}}$

For $\gamma = 1.093$ (Sulphur hexafluoride gas) and M = 1.36, the values of $q_1,\ \rho_1,\ p_1$ could be obtained from the above equations.

The wedge is made up of two walls having a small angle δ between them. After diffraction, the flow is two dimensional behind the shock wave. Let $\vec{q}_2,\ p_2,\ \rho_2$ and S_2 be the velocity vector, pressure, density and entropy at any point. We take the origin and Y axis lying on the leading edge of the wedge and X axis on the original wall produced.

Then the equations of conservation of mass, momentum and energy can be written as

$$\frac{D\rho_2}{Dt} + \rho_2 \operatorname{div} \vec{q}_2 = 0 \qquad - (4)$$

$$\frac{D\vec{q}_2}{Dt} + \frac{1}{\rho_2}\nabla \mathrm{p}_2 = 0 \qquad - (5)$$

$$\frac{DS_2}{Dt} = 0 \qquad - (6)$$

Now we introduce the following transformations

$$\frac{X - a_1 t}{a_1 t} = x \qquad - (7)$$

$$\frac{Y}{a_1 t} = y \qquad - (8)$$

$$\frac{\vec{q}_2}{q_1} = (1+u, v) \qquad - (9)$$

$$\frac{p_2 - p_1}{a_1\, q_1\, p_1} = p \qquad - (10)$$

By using small perturbation theory, the equations (4), (5), (6) and (7), (8), (9) and 10 yield an single second order partial equation in *p*. The equation is

$$\nabla^2 p = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1 \right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y} \right) \qquad - (11)$$

The characteristics of the differential equation (11) are tangents to the unit circle $x^2 + y^2 = 1$. The disturbed region behind the diffracted shock is therefore enclosed by the arc of the unit circle, the diffracted shock and wedge surface.

For such a boundary value problem, Lighthill (1949) has derived a function which satisfies all the boundary conditions $w(z_1)$ and is given as follows

$$w(z_1) = \frac{\partial p}{\partial y_1} + i\frac{\partial p}{\partial x_1} = \frac{C\delta\left[D(z_1 - x_0) - 1\right]}{\left(z_1^2 - 1\right)^{1/2}\left[\alpha - i\,(z_1 - 1)^{1/2}\right]\left[\beta - i\,(z_1 - 1)^{1/2}\right](z_1 - x_0)} \qquad - (12)$$

In (12), $z_1 = x_1 + iy_1$

In the final z_1-plane, the imaginary part on the left-hand side of (12) will be equal to $\frac{\partial p}{\partial x_1}$ which after integration will give pAfter carrying this exercise, one obtains

$$\frac{\partial p}{\partial x_1} = \frac{C\delta}{\left(x_1^2 - 1\right)^{1/2}}\left[D - \frac{1}{(x_1 - x_0)}\right]\frac{(\alpha + \beta)(x_1 - 1)^{1/2}}{\left[\alpha^2 + (x_1 - 1)\right]\left[\beta^2 + (x_1 - 1)\right]} \qquad - (13)$$

In (13), all the quantities are functions of the Mach number of the shock wave M except x_1 which runs from 1 to ∞ on the diffracted shock in the transformed plane and is connected to *y* in the physical plane through the relation

$$\frac{y}{k'} = \left(\frac{x_1 - 1}{x_1 + 1}\right)^{1/2}, k' = \sqrt{1 - k^2} \qquad - (14)$$

When $x_1 = 1$, $\frac{y}{k'} = 0$ (wall surface), when $x_1 \to \infty$, $\frac{y}{k'} = 1$ (point of intersection of shock and unit circle).

The theory of Lighthill (1949) was extended by Sakurai and Takayama (2005) to higher δ by considering second order terms through singular perturbation techniques. Sakurai and Takayama (2005) assumed y on the diffracted shock and computed $\bar{y}$ and $\bar{x}_1$. The relationship between $\bar{y}$ and $\bar{x}_1$ is the same as given by (14) in which y is replaced by $\bar{y}$ and x_1 by $\bar{x}_1$. The new $\bar{y}_1$ and $\bar{x}_1$ are used to calculate pressure distribution from equation (13). The new results of Sakurai and Takayama (2005) required for calculations are given below.

$$\bar{y} = \sqrt{r^2 - k^2}\ ,\ r = \xi + \delta r_1 , \xi = \sqrt{y^2 + k^2}$$

$$r_1 = K(\phi)\bar{R}\log\bar{R}\ ,\ \bar{R} = \left[\rho^2 + \frac{2\bar{X}k\rho}{\xi} + X_0^2\right]^{1/2}$$

$$\rho = \frac{1-\sqrt{1-\xi^2}}{\xi}\ ,\ \phi = \tan^{-1}\frac{y}{M_1 + k}$$

$$K(\phi) = \frac{1}{\pi}.\frac{M_1^4}{1-\sqrt{1-M_1^2}}\frac{1}{\cos\phi\cos 2\phi}\left[1+\frac{\gamma+1}{2}\frac{M_1^2}{1-M_1^2}\cos 2\phi\right]$$

where, ϕ in $K(\phi)$ is a variable

$$X_0 = \frac{1-\sqrt{1-M_1^2}}{M_1}$$

y is the y coordinate on the diffracted shock and ξ is the strained variable and others are connected within themselves.

III.NUMERICAL RESULTS

The pressure distribution over the diffracted shock is obtained by integrating equation (13). The pressure is zero at $x_1 = \infty$ i.e. at $\frac{y}{k'} = 1$ (the point of intersection of shock wave and unit circle) and so pressure at other points could be known by integration of equation (13) between the intervals.

The points chosen over the diffracted shock are

$$y/k' = 0,\ y/k' = 0.2,\ y/k' = 0.4,\ y/k' = 0.6,\ y/k' = 0.8$$

The equations (13) and (14) have been used to get the results. The following table gives the results after integration. The table is for y/k' versus $-\frac{p}{k\delta}$ M is 1.36 and $\gamma = 1.093.$

Table-1

y/k'	0	0.2	0.4	0.6	0.8	1
$-\frac{p}{k\delta}$	3.67	3.66	3.45	2.76	2.04	0

The table shows that $-\frac{p}{k\delta}$ is maximum at $y/k' = 0$ i.e. at the point of intersection of the wall and shock. The value of $-\frac{p}{k\delta}$ decreases from there and attains the value zero at $y/k' = 1$ i.e. at the point of intersection of shock and unit circle. On physical grounds the results are quite consistent.

IV.CONCLUSION

It is difficult to obtain pressure distribution over diffracted shock due to several complexities. The results would be useful in aeronautics and to aeronautical engineers.

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