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Normal Shock Wave diffraction for Sulfur Hexaflouride (SF_6) gas

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ABSTRACT

Lighthill has considered the diffraction of normal shock wave past a bend of small angle δ for $\gamma = 1.4$, γ being the ratio of specific heats. Srivastava extended the work of Lighthill to monoatomic gases for which $\gamma = \frac{5}{3}$. Srivastava and Srivastava further extended the work for carbon dioxide (CO_2) gas. In the present paper, diffraction of normal shock waves has been solved for Sulphur Hexafluoride (SF_6) gas.

Keywords: Diffraction, Normal shock, Sulphur Hexafluoride (SF_6) gas, Pressure Distribution.

I.INTRODUCTION

Lighthill (1949) considered the diffraction of normal shock wave past a small bend for $\gamma = 1.4$, γ being the ratio of specific heats. This work has been extended for $\gamma = \frac{5}{3}$ by Srivastava (1963) and Srivastava (2016) and further treated by Srivastava and Srivastava (2017) for $\gamma = 1.29$ Carbon dioxide (CO_2 gas). In the present paper, the pressure distribution over the diffracted shock has been obtained for Sulphur Hexafluoride (SF_6) gas.The Mach number of the shock wave has been assumed to be 1.36. It may be mentioned here that Srivastava (2011) has obtained the vorticity distribution over the diffracted shock for monoatomic gases. Reference may be made to the book by Srivastava (1994) for more details.

Let the velocity, pressure, density, sound speed behind the shock wave before it has crossed the bend be q_1, ρ_1, p_1, a_1 and ahead of the shock wave be 0, p_0, ρ_0, a_0. Then applying the principle of conservation of mass, momentum and energy for general value of γ (γ being the ratio of specific heats)

$$q_1 = \frac{2U}{(\gamma+1)}\left(1 - \frac{a_0^2}{U^2}\right) \qquad - (1)$$

$$\rho_1 = \frac{\rho_0(\gamma+1)}{(\gamma-1) + 2\frac{a_0^2}{U^2}} \qquad - (2)$$

$$p_1 = \frac{\rho_0}{(\gamma - 1)}\left[2U^2 - \frac{a_0^2(\gamma-1)}{\gamma}\right] \qquad - (3)$$

U being the velocity of shock wave, Mach number of the shock $M = \frac{U}{a_0}$, $a_0 = \sqrt{\frac{\gamma\, p_0}{\rho_0}}$

The wedge is made up of two walls having a small angle δ between them. The shock wave gets diffracted after it has crossed the corner and the flow is two dimensional behind the diffracted shock wave. Let $\vec{q}_2$, p_2, ρ_2 and S_2 be the velocity vector, pressure, density and entropy at any point behind the diffracted wave shock. We take the X-axis along the original wall produced, the origin on the leading edge of the wedge and Y axis normal to the leading edge.

In this coordinate system, the equations of ofconservation of mass, momentum and energy can be written as

$$\frac{D\rho_2}{Dt} + \rho_2 \operatorname{div} \vec{q}_2 = 0 \qquad - (4)$$

$$\frac{D\vec{q}_2}{Dt} + \frac{1}{\rho_2}\nabla \mathrm{p}_2 = 0 \qquad - (5)$$

$$\frac{DS_2}{Dt} = 0 \qquad - (6)$$

Now we introduce the following transformations

$$\frac{X - q_1 t}{a_1 t} = x \qquad - (7)$$

$$\frac{Y}{a_1 t} = y \qquad - (8)$$

$$\frac{\vec{q}_2}{q_1} = (1 + u, v) \qquad - (9)$$

$$\frac{p_2 - p_1}{a_1\, q_1\, p_1} = p \qquad - (10)$$

We assume that $\vec{q}_2, p_2, \rho_2$ differ by small quantities from the values $(q_1, 0)$, p_1, ρ_1 which they had before diffraction, then using the equations (4), (5), (6) and (7), (8), (9), (10) we obtain a single second order partial differential equation in *p*. This equation is

$$\nabla^2 p = \left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + 1\right)\left(x\frac{\partial p}{\partial x} + y\frac{\partial p}{\partial y}\right) \qquad - (11)$$

The characteristics of the differential equation (11) are tangents to the unit circle $x^2 + y^2 = 1,$ signifying that the disturbed region is enclosed by the arc of unit circle $x^2 + y^2 = 1,$, the diffracted shock and the wedge surface.

The position of the straight portion of the shock wave in *x, y* coordinates is given by $x = k,$ where $k = \frac{U - q_1}{a_1}$.The coordinates of the corner is $(-M_1, 0)$ where $\left(M_1 = \frac{q_1}{a_1}\right)$.

Using Busemann transformation and complex variable techniques, Lighthill (1949) worked out a function which satisfies all the boundary conditions. The function $w(z_1)$ is given by

$$w(z_1) = \frac{\partial p}{\partial y_1} + i\frac{\partial p}{\partial x_1} = \frac{C\delta[D(z_1 - x_0 - 1)]}{(z_1^2 - 1)^{1/2}\left[\alpha - i(z_1 - 1)^{1/2}\right]\left[\beta - i(z_1 - 1)^{1/2}\right](z_1 - x_0)} \qquad - (12)$$

In (12), $z_1 = x_1 + iy_1$

In the final z_1-plane, the imaginary part on the left hand side of (17) gives the pressure derivative which determines the pressure distribution over the diffracted shock. If one does that, then the expression for pressure derivative is given by

$$\frac{\partial p}{\partial x_1} = \frac{C\delta}{(x_1^2 - 1)^{1/2}}\left[D - \frac{1}{(x_1 - x_0)}\right]\frac{(\alpha + \beta)(x_1 - 1)^{1/2}}{[\alpha^2 + (x_1 - 1)][\beta^2 + (x_1 - 1)]} \qquad - (13)$$

In(13),all the quantities are functions of the Mach number of the shock wave M except x_1 which runs from 1 to ∞ on the diffracted shock in the transformed plane and is connected to *y* in the physical plane through the relation

$$\frac{y}{k'} = \left(\frac{x_1 - 1}{x_1 + 1}\right)^{1/2}, \quad k' = \sqrt{1 - k^2} \qquad - (14)$$

When $x_1 = 1,\ \frac{y}{k'} = 0$ (wall surface), when $x_1 \to \infty$, $\frac{y}{k'} = 1$ (point of intersection of shock and unit circle).

II.NUMERICAL SOLUTION

The pressure distribution over the diffracted shock is obtained by integrating equation(13).The pressure *p*is zero at $x_1 = \infty$ i.e. at $\frac{y}{k'} = 1$ (the point of intersection of shock wave and unit circle) and so pressure at other points could be known by integrating in intervals. The points chosen over the diffracted shock are

$$y/k' = 0,\ y/k' = 0.2,\ y/k' = 0.4,\ y/k' = 0.6,\ y/k' = 0.8$$

The equations(13) and (14)have been used to get the results. The following table gives the results after integration. The table is for y/k', versus $-\frac{p}{k\delta}$. For this table M is 1.36 and $\gamma = 1.093$.

Table-1

y/k'	0	0.2	0.4	0.6	0.8	1
$-\frac{p}{k\delta}$	3.67	3.62	3.44	2.68	1.86	0

The table shows that $-\frac{p}{k\delta}$ is maximum at $y/k' = 0$ i.e. at the point of intersection of the wall and shock. The value of $-\frac{p}{k\delta}$ falls from there and attain the value zero at $y/k' = 1$ i.e. at the point of intersection of shock and unit circle. The results are physically consistent.

The value of $-\frac{p}{k\delta}$ here are lower than those of the CO_2 gas. For the paper of Sakurai (2017) et al may be referred.

III.CONCLUSION

The results are quite useful in the area of aeronautics.The problem is complex and difficult. The solution will be available to aeronautical engineers for future work,

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