

New approaches for Minimum Spanning Tree

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ABSTRACT

In computer science minimum spanning tree plays a significant role in graph algorithm.MST is widely used in data networks and communication to transfer data between nodes. It is used to find minimum cost applied on networks nodes. In this paper we minimizing the cost of the tree using three different approaches named as column minima and row minima in matrix and quick sort approach.

Keywords: MST, COLOUMN MINIMA, ROW MINIMA, QUICK SORT.

I. INTRODUCTION

Minimum-cost spanning trees are used to implement many applications such as building cable networks that connect n locations ,consist of minimum cost and building a road network that connects n cities ,consist of minimum cost. A minimum spanning tree is a connected sub graph without cycles.

Minimum spanning trees consists of various applications in the design of networks, including computer networks , TV cable, telecommunications networks (likes the Internet, telephone network, the global Telex network, the aeronautical ACARS network.), transportation networks, (like airline routes, Road) ,water supply networks, and electrical grids.

The major application of minimum spanning tree is to solve a problem like phone network design. You have a business with several offices; You want to lease phone lines to connect them up with each other; And the phone company charges different amounts of money to connect different pairs of cities. You want a set of lines that connects all your offices with a minimum total cost. It should be a spanning tree, since if a network isn't a tree you can always remove some edges and save money.

A less obvious application is that the minimum spanning tree can be applied to approximately solve the traveling salesman problem. A convenient formal way of defining this problem is to find the shortest path that visits each point at least once.

Minimum Spanning Tree plays a vital role in the design of a computer network. MST can be used to connect a group of individual Computers over a wired network which is separated by varying distances .MST can be used to calculate the minimum costly paths which does not form any cycle in the network, So minimum spanning tree is used to connecting all the Computer node at a minimum cost.

MST is also used to determine airline path. The vertices of the graph denote cities, and the edges denotes path between the cities. Apparently, cost goes up when one has to travel more, so MST can be implemented to optimize airline paths by determining the minimum costly paths that does not contain any cycle.

II. PROCEDURES FOR IMPLEMENTING MST

2.1 .1 Sorting approach For Directed Graph

- i) Make set of all edges with their weights.
- ii) Put the weights into an array.
- iii) Apply quick sort algorithm to sort the array which contains weight in increasing order.
- iv) Take first element of the array and draw their respective edge up to n-1 elements where n denotes number of nodes exist in a graph.
- v) If any elements form a cycle then skip that weight and take next element from the sorted array but not exceed from n-1 elements

Example:

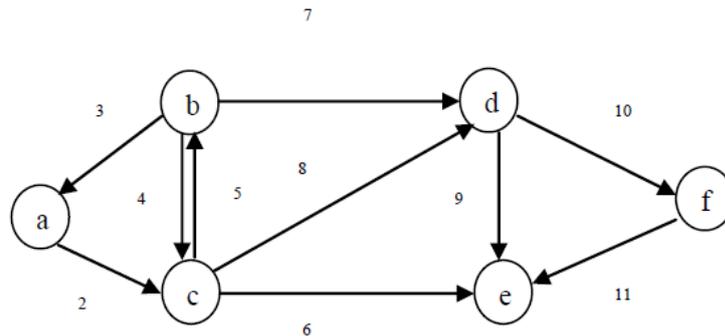


Fig 1: Directed graph

Step1: Put all edges into an array

3	7	10	11	6	2	4	5	8	9
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Step2: Apply Quick sort method to sort given array. Now after sorting, the array is

2	3	4	5	6	7	8	9	10	11
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Step3: Take smallest element up to (n-1) node available in given graph and draw minimum spanning tree. If any element creates a cycle then element will be removed and select next element from the sorted array. But selection of element should not be exceeding by (n-1) node.

Now draw minimum spanning tree from given array

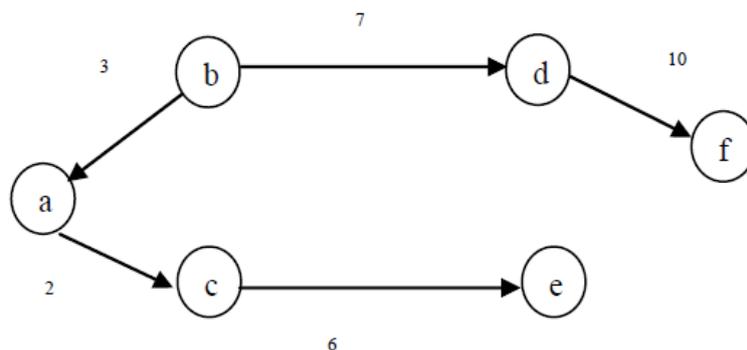


Fig2: minimum spanning tree using sorting approach.

Minimum spanning tree cost using sorting approach is=2+3+7+6+10=28

Edge Weight 4 and 5 will be eliminated because weight 5 is creating a cycle and weight 4 is pointing the node that is covered already by another node i.e. weight 2. Repeat same process on weight 8 and 9. Weight 8 and 9 will be eliminated because both weight are pointing those nodes that are already covered. Our self-generated approach can be applied on undirected graph.

2.1.2 Sorting Approach applied on an undirected Graph

Example: Consider an undirected graph

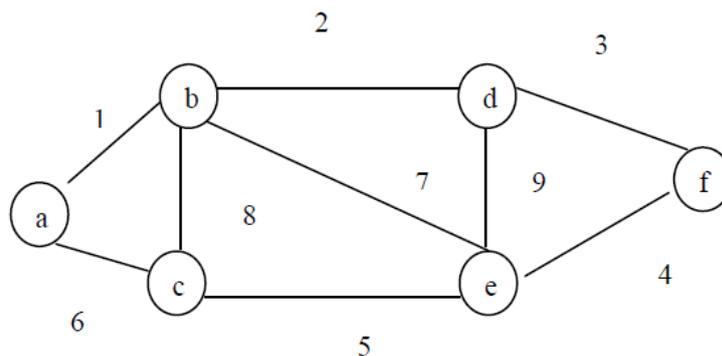


Fig 3: Undirected graph

Step1: Put all edges into an array

1	6	8	2	7	5	9	3	4
---	---	---	---	---	---	---	---	---

Step2: Apply Quick sort method to sort given array. Now after sorting ,the array is

1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---

Step3: Take smallest element up to (n-1) node available in given graph and draw minimum spanning tree. If any element creates a cycle then element will be removed and select next element from the sorted array. But selection of element should not be exceeding by (n-1) node. Now draw minimum spanning tree from given array.

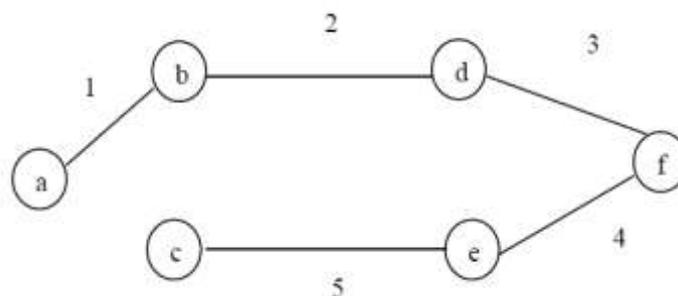


Fig 4: minimum spanning tree

MST cost: 1+2+3+4+5=15

2.2. Row Minima approach

- i) Consider the undirected graph and make it adjacency matrix.
- ii) Starts from first row and circle minimum non-zero element of the row and cut all element of that row.
- iii) If the weights are similar is same row then take any one and cut another.
- iv) Repeat steps ii) and iii) for the each row of the matrix.
- v) If any element creates cycle then cut that element and choose next smallest element in same row. If (n-1) elements are considered then leave that row without any element.

2.2.1 Row Minima approach for undirected graph

Step 1: Consider an undirected graph

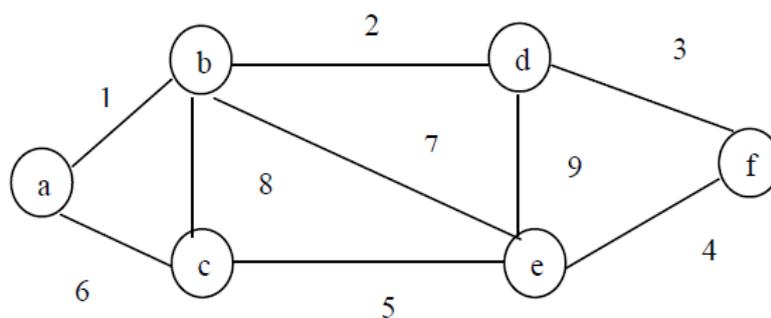


Figure 5: Undirected graph

Step 2: Make adjacency Matrix from Undirected Graph

$$\begin{pmatrix} 0 & 1 & 6 & 0 & 0 & 0 \\ 1 & 0 & 8 & 2 & 7 & 0 \\ 6 & 8 & 0 & 0 & 5 & 0 \\ 0 & 2 & 0 & 0 & 9 & 3 \\ 0 & 7 & 5 & 9 & 0 & 4 \\ 0 & 0 & 0 & 3 & 4 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \cancel{0} & 1 & \cancel{6} & \cancel{0} & \cancel{0} & \cancel{0} \\ 1 & \cancel{0} & \cancel{8} & \cancel{2} & \cancel{7} & \cancel{0} \\ \cancel{6} & \cancel{8} & \cancel{0} & \cancel{0} & \cancel{5} & \cancel{0} \\ \cancel{0} & \cancel{2} & \cancel{0} & \cancel{0} & \cancel{9} & \cancel{3} \\ \cancel{0} & \cancel{7} & \cancel{5} & \cancel{9} & \cancel{0} & \cancel{4} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{3} & \cancel{4} & \cancel{0} \end{pmatrix}$$

Step 3: Choose Minimum element in each row and remove all elements in the matrix.

Step 4: element 1 is two times selected but it points same edge. Edge (a, b) is selected and (b, a) is skipped. Edge can be skipped if (n-1) elements are already selected otherwise next non-zero smallest element can be considered in same row.

Now make minimum spanning tree of given graph using matrix.

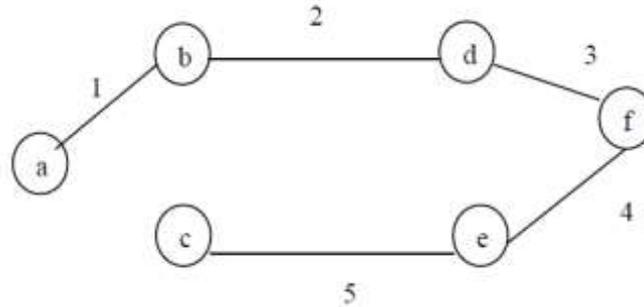


Figure 6: Minimum spanning tree from undirected graph

MST COST: 1+2+3+4+5=15

2.2.2 Row Minima approach for directed graph

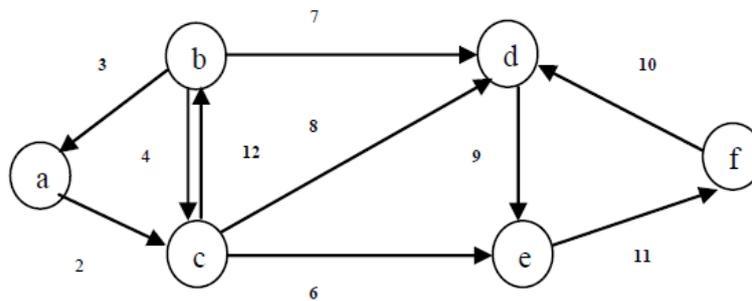


Fig 7: Directed graph

Step1: Make adjacency Matrix from directed Graph

0	0	2	0	0	0	0	0	2	0	0	0
3	0	4	7	0	0	3	0	4	7	0	0
0	12	0	8	6	0	0	12	0	8	6	0
0	0	0	0	9	0	0	0	0	0	9	0
0	0	0	0	0	11	0	0	0	0	0	11
0	0	0	10	0	0	0	0	0	10	0	0

Step 3: Choose Minimum element in each row and remove all elements in the matrix.

Step 4: In this matrix 9 forms cycle .so that element is removed .for making MST, only (n-1) nodes are needed. Where n are the number of nodes. Now make minimum spanning tree of given graph using matrix.

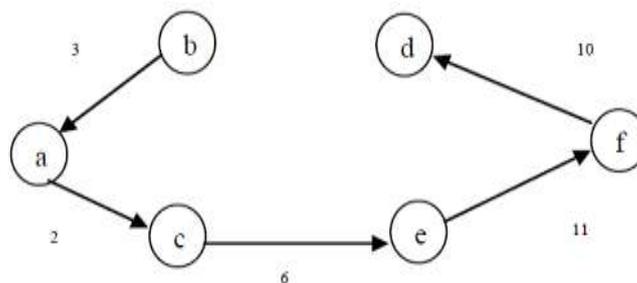


Figure 8: Minimum spanning tree from directed graph

MST Cost = 3+2+6+11+10=32

Weight 9 will be eliminated because it is forming a cycle.

2.3 Column minima approach

- i) Consider the undirected graph and make it adjacency matrix.
- ii) Starts from first column and select minimum non-zero element of the column and cut all element of respective column.
- iii) If the weights are similar in same column then take any one and cut another.
- iv) If cycle is formed .choose another minimum non-zero element in same column.
- v) Repeat steps ii) and iii) until visit all columns of the matrix.

2.3.1 Column minima approach for undirected graph

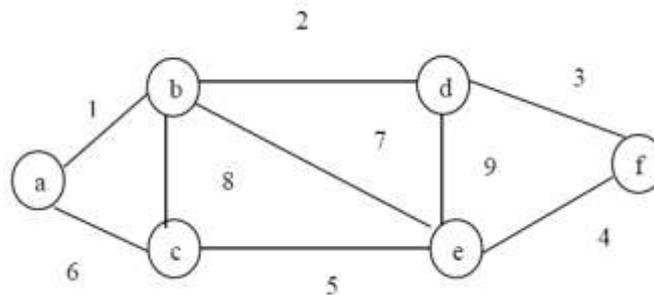


Fig 9: Undirected graph

Step 2: Make adjacency Matrix from Undirected Graph.

$$\begin{pmatrix} 0 & 1 & 6 & 0 & 0 & 0 \\ 1 & 0 & 8 & 2 & 7 & 0 \\ 6 & 8 & 0 & 0 & 5 & 0 \\ 0 & 2 & 0 & 0 & 9 & 3 \\ 0 & 7 & 5 & 9 & 0 & 4 \\ 0 & 0 & 0 & 3 & 4 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 6 & \emptyset & 0 & 0 \\ 1 & 0 & 8 & 2 & 7 & 0 \\ 6 & 8 & 0 & \emptyset & 5 & 0 \\ 0 & 2 & \emptyset & 0 & 9 & 3 \\ 0 & 7 & 5 & 9 & 0 & 4 \\ 0 & 0 & \emptyset & 3 & 4 & 0 \end{pmatrix}$$

Step 3: Choose Minimum non-zero element in each column and cut all elements in their respective columns.

Step 4: If cycle is formed. Choose another minimum element in row. Now make minimum spanning tree of given graph using matrix. Here edge b to a is repeated so 1 is removed and consider 2.

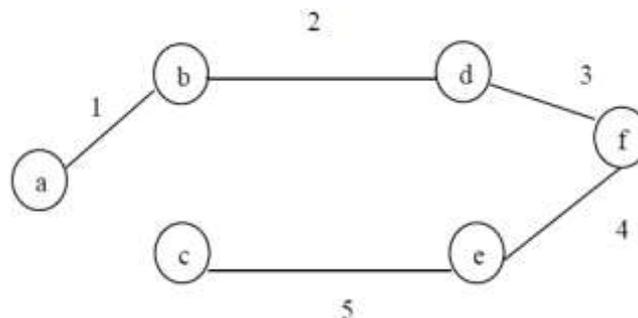


Fig10: minimum spanning tree from undirected graph

MST COST =1+2+3+4+5=15

2.3.2 Column minima approach for directed graph

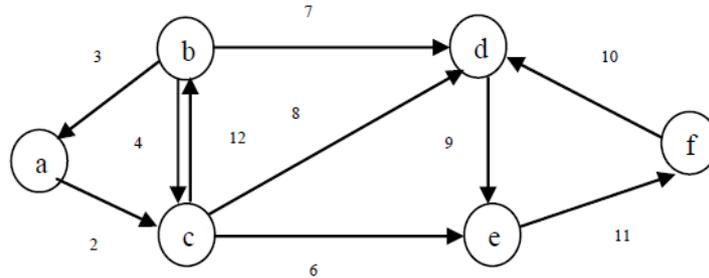


Fig 11: Directed graph

Step 2: Make adjacency Matrix from directed Graph

$$\begin{pmatrix} 0 & 0 & 2 & 0 & 0 & 0 \\ 3 & 0 & 4 & 7 & 0 & 0 \\ 0 & 12 & 0 & 8 & 6 & 0 \\ 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 11 \\ 0 & 0 & 0 & 10 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} \emptyset & \emptyset & 2 & \emptyset & \emptyset & \emptyset \\ 3 & \emptyset & 4 & 7 & \emptyset & \emptyset \\ \emptyset & 12 & \emptyset & 8 & 6 & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & 9 & \emptyset \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & 11 \\ \emptyset & \emptyset & \emptyset & 10 & \emptyset & \emptyset \end{pmatrix}$$

Step 3: Choose Minimum non –zero element in each column and cut all elements in their respective columns.

Step 4: If cycle is formed .choose another minimum element in row. Now make minimum spanning tree of given graph using matrix. Element 12 creates a cycle. So remove it.

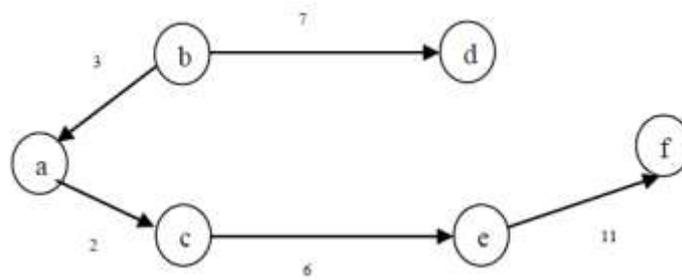


Fig12: minimum spanning tree from directed graph

Weight 12 and 8 is forming cycles so these weight are eliminated from minimum spanning tree.

MST COST IS =3+2+6+7+11=29

III. ANALYSIS

In every graph, there are two components –edges (e) and vertices (v).Each and every graph G contains a set of edges and vertices and their weights. $G \in (e, v)$

3.1 Sorting approach

In this approach, all weights of the edges are sorted using quick sort, and then take only $v-1$ edges correspond to their weights to make MST. To make minimum spanning tree $O(E \log E)$ time is taken by this approach in best case and average case and $O(E^2)$ time is taken in worst case.

3.2 Row minima and column minima approach

In row minima and column minima, to find out the minimum cost edges, adjacency matrix is constructed. To construct adjacency matrix, each node finds edges with every node. This takes $O(V^2)$ time. We take minimum element from the matrix using row minima or column minima. Both approaches take $O(V^2)$ time in all cases to make minimum spanning tree.

IV. CONCLUSION

MST plays significant role in various fields. With the help of MST, minimum cost can be calculated to send data from one node to another node. In this paper, three approaches are proposed to find minimum spanning tree from directed or undirected graph. Minimum spanning tree is used to solve travelling salesman problem. MST is used in various applications directly or indirectly. It is used in design of networks like computer networks, telecommunication network, transportation network, water supply network, and electric grid. It is also used in Image registration, segmentation. It is used to construct trees for broadcasting in computer networks. MST is also used in regionalization of socio-graphic areas and grouping of areas into homogeneous, continuous regions. The proposed three approaches give different time complexities- sorting approach takes $O(E \log E)$, row-minima and column – minima approaches take $O(V^2)$. All three approaches are used for both directed and undirected graph.

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