



## ISOMORPHISM IN PLANAR KINEMATIC CHAINS-A CASE STUDY IN GRAPH THEORY ALGORITHM

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### ABSTRACT

*In structural synthesis of Planar kinematic chains(PKC), Detection of isomorphism is an interesting area since many years. Enumeration of planar and geared kinematic chains becomes easy only when isomorphism problem is resolved effectively. Many researchers proposed algorithms based on topological characteristics, code based, genetic algorithm, fuzzy logic, graph theory etc. which need lot of computations and comparisons. Graph theory is an effective tool in dealing with the structural synthesis of planar kinematic chains in an effective manner. In this work, 'net distance' based algorithm developed recently is tested for the distinct kinematic chains of 8-link single degree of freedom. All the results obtained justified the validity of the 'net distance' algorithm.*

**Keywords:** *Kinematic chain, Isomorphism, Net distance, Graph*

### I. INTRODUCTION

In the structural synthesis of planar kinematic chains, detection of isomorphism is a critical area to be studied. Various researchers proposed methods to identify isomorphism. Many attempts have been made in the past to solve this problem using the graph theory. Woo presented all the 230 isomers of 10-link 1-DOF K-chains [1]. Mruthyunjaya developed computerized methods for structural synthesis of K-chains. In addition, he presented a complete review of kinematic structure of mechanisms [2-4]. Other approaches are theory of finite groups by Tuttle [5], link's adjacent chain table by Kui and C.W.Qing [6]. Ambekar and Agrawal proposed the Max Code and Min Code methods for the detection of isomorphism [7, 8]. Yadav et al. presented 'distance' concept in modified version in checking K-chains for isomorphism and finding inversions [9, 10]. Rao A.C and his co-researchers contributed many concepts in testing isomorphism and structural synthesis of kinematic chains, namely, hamming number approach [11], pseudo hamming values [12], fuzzy logic [13], loop based methods in multi-degree of freedom K-chains to check isomorphism and finding distinct inversions [14]. Bedi and Sanyal had presented Joint connectivity approach for detection of isomorphism and distinct inversions of planar kinematic chains [15-17]. Xiao et al. proposed novel evolutionary algorithms for isomorphism detection [18]. Ashok Dargar et al. used link adjacency values – first and second to identify the distinct inversions. The method has the potential of identifying the isomorphic kinematic chains also [19]. Jaspal Singh Bal et al. [20] used link invariant functions based on distance matrix and kinematic chain loops to detect isomorphism and inversions.



Yanhua Zou et al. proposed an automatic topological structural synthesis algorithm for planar-simple-joint-kinematic-chains [21]. Ding et al. presented computerized methods for automatic synthesis of all the 2, 3-DOF kinematic chains [22]. Lohumi et al. [23, 24] proposed a method based on interaction effect of connecting links and computerized loop based algorithm to detect isomorphism in K-chains. Xue et al. presented a thorough review of graph theory applications in synthesis of geared K-chains [25]. In 2017, Huang et al. [26] presented an algorithm for the connectivity calculation in planar closed kinematic chains. Rizvi et al. developed an algorithm for detection of isomorphism and distinct mechanisms of kinematic chains [27]. Varadaraju and Mohankumar applied split hamming string as an isomorphism test for one degree-of-freedom planar simple-jointed kinematic chains containing sliders [28]. Kamesh et al. developed a novel method to detect isomorphism in epicyclic gear trains using adjacency of links and level of dependency over other links. He also introduced two new parameters namely 'Vertex Incidence Polynomial' and 'Rotation Index' to identify displacement isomorphism and rotational isomorphism in epicyclic gear trains. He also developed another method to detect degenerate structures in planetary gear trains [29-31]. In 2017, Kamesh et al. developed an innovative method based on graph theory [32]. In that work, author proposed a new parameter called 'Net distance' to check isomorphism in planar kinematic chains as well as geared kinematic chains.

In the present work, 8-link 1-DOF distinct kinematic chains are tested on the 'Net distance' algorithm developed by the author earlier [32].

## II. ALGORITHM TO DETECT ISOMORPHISM

The developed algorithm is based on the fundamentals of the graph theory. The steps in the algorithm are:

**Step 1:** For each link or vertex in a graph, calculate the 'distance' from all other links or vertices.

**Step 2:** Sum of all the distances will result 'total distance' for the link.

**Step 3:** Sum of total distance of all the links will result 'Net distance' of the kinematic chain.

**Step 4:** Arrange the link distance components in an array or string in ascending or descending order.

**Step 5:** 'Net distance' of a kinematic chain along with string of link components is used as a quantitative measure to compare with any other kinematic chain.

**Step 6:** If any two kinematic chains have same 'Net distance' along with the string components, they are said to be 'isomorphic' else 'distinct'.

The proposed algorithm has less computational complexity. The total number of calculations needed for a kinematic chain with 'n' links is ' $n(n-1)/2$ '.

## III. TESTING OF ALGORITHM

The developed algorithm is tested on distinct kinematic chains of 8-link 1-DOF (16 no.) in Appendix I to validate its efficiency in checking the isomorphism. The calculations are shown in the following.



CHAIN NO. 1	Net distance from every link to all other links								Total
	LINK	1	2	3	4	5	6	7	
1	0	1	2	3	2	1	2	1	12
2	1	0	1	2	1	2	3	2	12
3	2	1	0	1	2	3	4	3	16
4	3	2	1	0	1	2	3	4	16
5	2	1	2	1	0	1	2	3	12
6	1	2	3	2	1	0	1	2	12
7	2	3	4	3	2	1	0	1	16
8	1	2	3	4	3	2	1	0	16
	<b>Total</b>								<b>112</b>

The string for the Chain No. 1 is 112-4(16)-4(12). In the same way, all the 16 chains of 8-link 1-DOF are calculated the net distance and the corresponding string.

#### IV. RESULTS AND CONCLUSION

All the distinct kinematic chains of 8-link 1-DOF are tested on the 'Net distance' algorithm. The results are as follows.

**TABLE A: NET DISTANCE CALCULATION FOR ALL THE DISTINCT CHAINS OF 8-LINK 1-DOF**

CHAIN NO.	NET DISTANCE	STRING
1	112	112-4(16)-4(12)
2	100	100-4(14)-4(11)
3	100	100-3(14)-13-12-3(11)
4	106	106-4(14)-2(13)-2(12)
5	104	104-2(15)-2(14)-2(12)-2(11)
6	104	104-4(14)-13-2(12)-11
7	100	100-14-3(13)-3(12)-11
8	104	104-4(14)-4(12)
9	96	96-4(13)-4(11)
10	102	102-7(13)-1(11)
11	98	98-14-3(13)-2(12)-11-10
12	102	102-15-14-4(13)-11-10
13	102	102-15-2(14)-2(13)-12-11-10
14	108	108-2(16)-3(14)-2(12)-10
15	104	104-6(14)-2(10)
16	96	96-4(13)-2(12)-2(10)

All the 16 distinct kinematic chains of 8-link 1-dof exhibit distinct strings of net distance.

From the above calculations, all the 'Net distance' strings of 8-link 1-DOF kinematic chains (16 no.) are unique thus distinct, thus validating the efficiency of the developed method. 'Net distance' concept using Graph theory is proved its efficiency in checking of isomorphism in planar kinematic chains. The concept can be extended to validate distinct kinematic mechanisms, estimation of parallelism, compactness, rigidity etc. with necessary modifications.

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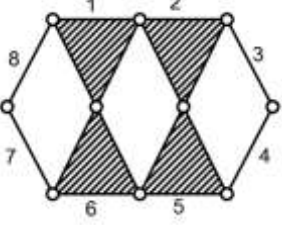
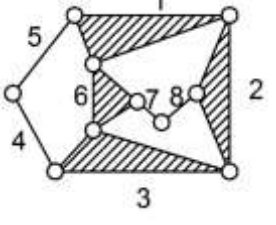
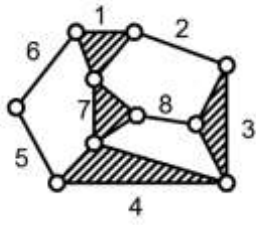
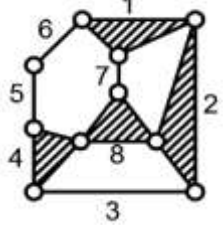
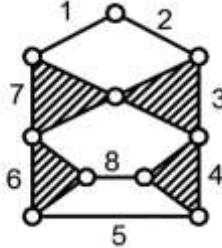
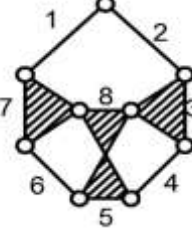
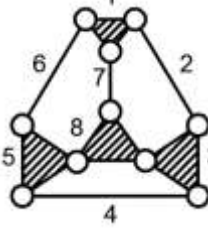
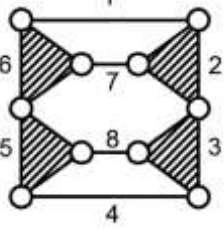
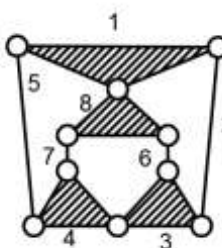
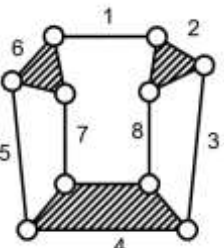
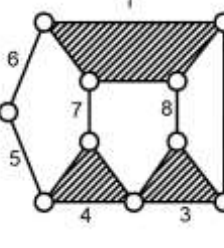
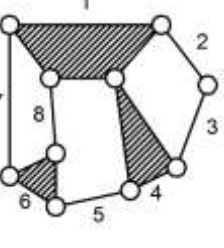
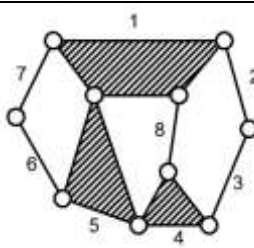
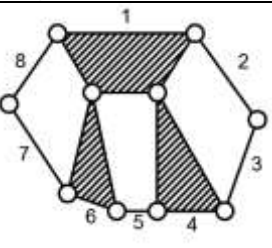
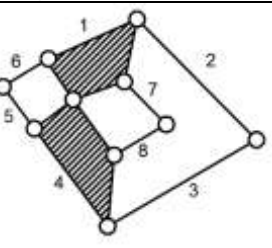
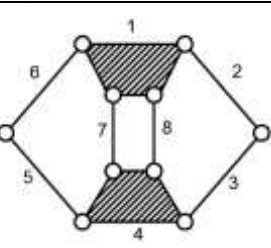


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APPENDIX I: 8-LINK 1-DOF KINEMATIC CHAINS

			
<p><b>Fig. 1: Chain No.1</b></p>	<p><b>Fig. 2: Chain No.2</b></p>	<p><b>Fig. 3: Chain No.3</b></p>	<p><b>Fig. 4: Chain No.4</b></p>
			
<p><b>Fig. 5: Chain No.5</b></p>	<p><b>Fig. 6: Chain No.6</b></p>	<p><b>Fig. 7: Chain No.7</b></p>	<p><b>Fig. 8: Chain No.8</b></p>
			
<p><b>Fig. 9: Chain No.9</b></p>	<p><b>Fig. 10: Chain No.10</b></p>	<p><b>Fig. 11: Chain No.11</b></p>	<p><b>Fig. 12: Chain No.12</b></p>
			
<p><b>Fig.13: Chain No.13</b></p>	<p><b>Fig. 14: Chain No.14</b></p>	<p><b>Fig. 15: Chain No.15</b></p>	<p><b>Fig. 16: Chain No.16</b></p>