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Fuzzy Semi-Markov Model with Weighted Fuzzy Transitions

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ABSTRACT

This paper deals with the web application defined through the weighted fuzzy transition possibilities by means of homogeneous fuzzy semi-Markov model. The results have been obtained through the weighted conditional possibilities of the process moving from one state to another state such that it has been stayed in the previous state with waiting time possibilities.

Keywords: Possibility space, Fuzzy semi-Markov model, Weighted fuzzy transitions.

I. INTRODUCTION

In this paper, the homogeneous fuzzy semi-Markov model is proposed as a useful tool for predicting the evolution of the web access during the specified period through possibilities, by assuming the state transitions as weighted fuzzy transition possibilities on the state space. Important theoretical results and applications for classical semi-Markov models can be found in [1 - 7] and fuzzy semi-Markov model in [8 – 11]. In this paper, the model is examined under the assumption of the weighted fuzzy transition between the states through the concept of possibility of events. Since the transition between the states of a system cannot be preciously measured due to the system that is intrinsically fuzzy, the decisions are associated with weighted fuzzy transition that can be defined as possibilities on the state space of a system.

This paper is constructed as follows. Section 2 recalls the basic definitions of possibility space. In section 3, deals with the homogeneous fuzzy semi Markov model with weighted fuzzy transitions, and the basic equations for the weighted fuzzy transition possibility. Section 4 illustrates the weighted fuzzy semi-Markov model. The conclusion is discussed in section 5. All the definitions and results are based on max – min operation which are more compatible when compared to usual addition and multiplication operation.

II. PRELIMINARIES

In this section, we recall some of the basic definitions of possibility space.

2.1 Possibility Space [4]

Let Γ be the universe of discourse and ψ be the power set of Γ. The possibility measure is a mapping

$\sigma:\psi\to[0,1]$ such that

$$(i)\sigma(\phi)=0,\sigma(\Gamma)=1$$
$$(ii)\sigma\left(\underset{i}{\cup}A_i\right)=\sup_i\left(\sigma(A_i)\right)$$

for every arbitrary collection A_i of ψ. Then (Γ,ψ,σ) is called as Possibility Space.

2.2 Conditional Possibility [4]

Let (Γ,ψ,σ) be a possibility space and $A,B\in\psi$. Then the possibility of A conditional on B is defined as

$$\sigma(A/B)=\begin{cases}1 & , \ if \quad \sigma(AB)=\sigma(B)\\ \sigma(AB) & , \ if \quad \sigma(AB)<\sigma(B)\end{cases}$$

2.3 Total Possibility Law [4]

Let (Γ,ψ,σ) be a possibility space and $\{A_i\}$ be the collection of sets such that $\underset{i}{\cup}A_i=\Gamma$ and $B\in\psi$. Then

$$\sigma(B)=\sup_i\sigma(A_iB)$$
$$=\sup_i\min\left[\sigma(B/A_i),\sigma(A_i)\right]$$

2.4 Possibility Variable [4]

Let (Γ,ψ,σ) be a possibility space and U be an arbitrary universe. A possibility variable X is a mapping from Γ to U.

If U is countable, then X is called discrete possibility variable and if U is uncountable, then it is called continuous possibility variable.

2.5 Possibility Distribution Function [4]

Let (Γ,ψ,σ) be a possibility space and U be an arbitrary universe. If $X:\Gamma\to U$ is a discrete possibility variable, then the possibility distribution function is

$$g(x)=\sigma(X=x),\forall x\in U.$$

III. HOMOGENEOUS FUZZY SEMI-MARKOV MODEL WITH WEIGHTED FUZZY TRANSITIONS

A classical semi-Markov process [1 - 7] is defined as a sequence of two dimensional random variable $\{(X_n, T_n), n \in I\}$ with the properties (1) X_n is a discrete time Markov chain taking values in a countable set S of the system and represents its state after transition n. (2) the holding times $T_{n+1} - T_n$ between two transitions are random variable, whose distribution depends on the present state and the state after the next transition and is given as

$$\begin{aligned} & p\left[X_{n+1} = j, T_{n+1} - T_n \le t / X_0, \ldots, X_n; T_0, \ldots, T_n\right] \\ & = p\left[X_{n+1} = j, T_{n+1} - T_n \le t / X_n\right] \end{aligned}$$

Since for many systems due to uncertainties and imprecision of data, the estimation of precise values of probability is very difficult. For this reason, we have used weighted possibilities defined on possibility space and since the computation using max-min operation are more robust to perturbation when compared to usual addition and multiplication operation; we have followed max-min operation throughout the paper. We now discuss about the discrete time homogeneous weighted fuzzy semi-Markov model, whose transitions are taken as weighted transition possibilities on the state space.

Let E={1, 2, …,m} be the state space and let (Γ, ψ, σ) be a possibility space. We define the following possibility variables:

$$J_n : \Gamma \to E, \;\; S_n : \Gamma \to N$$

where J_n represents the state at the n-th transition and S_n represents the time of the n-th transition.

The process $\left(J_n, S_n\right)_{n \in N}$ is called homogenous weighted fuzzy Markov renewal process if

$$\sigma\left[J_{n+1} = j, S_{n+1} \le t / J_n = i, S_n, J_{n-1}, S_{n-1}, \ldots, J_0, S_0\right] = \sigma\left[J_{n+1} = j, S_{n+1} \le t / J_n = i\right]$$

and the associated weighted homogeneous fuzzy semi-Markov kernel $\tilde{\tilde{Q}}$ =($\tilde{Q}$, w_Q) , where for $j \neq i$ $\tilde{\tilde{Q}} = (\tilde{\tilde{Q}}_{ij}(t))$ is defined as

$$\tilde{Q}_{ij}\left(t\right) = \sigma\left[J_{n+1} = j, S_{n+1} - S_n \le t / J_n = i\right]$$

These fuzzy semi-Markov kernels can be expressed in matrix form as

$$\tilde{Q}=\left(\tilde{Q}_{ij}(t)\right)=\begin{pmatrix} \tilde{Q}_{11}(t) & \tilde{Q}_{12}(t) & . & . & . & . & \tilde{Q}_{1m}(t) \\ \tilde{Q}_{21}(t) & \tilde{Q}_{22}(t) & . & . & . & . & \tilde{Q}_{2m}(t) \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ \tilde{Q}_{m1}(t) & \tilde{Q}_{m2}(t) & . & . & . & . & \tilde{Q}_{mm}(t) \end{pmatrix}_{m\times m}$$

called fuzzy semi-Markov kernel matrix.

As in classical semi-Markov process, here also we have $\ddot{p}_{ij} = (\tilde{p}_{ij}, w_p)$ where $\tilde{p}_{ij}$ is defined by

$$\tilde{p}_{ij} = \lim_{t\to\infty} \tilde{Q}_{ij}(t), i, j \in E, j \neq i$$

$$= \sigma\left[J_{n+1} = j / J_n = i\right]$$

represents the weighted possibility of a system making its next transition to state j, given that it entered state i at time t and $\ddot{P} = (\ddot{p}_{ij})$ is the m x m possibility matrix with weighted fuzzy transition of the embedded homogeneous fuzzy Markov chain $(J_n)_{n\in N}$.

3.1 Evolution Equation of a Homogeneous Weighted Fuzzy semi-Markov Model

In this section, we derive the evolution equation of a discrete time homogeneous weighted fuzzy semi-Markov model. The evolution equation represents the weighted fuzzy transition possibility from state i to reach state j with time duration 't'.

We now define the conditional cumulative weighted distribution function of the waiting time in each state, given the state subsequently occupied by,

$$\ddot{F}_{ij}(t) = (\tilde{F}_{ij}(t), w_F), \text{ where } \tilde{F}_{ij}(t) = \sigma\left[S_{n+1} - S_n \leq t / J_{n+1} = j, J_n = i\right]$$

Since the FSMK $\tilde{Q}_{ij}(t)$ is both characterized by a fuzzy Markov chain $(J_n)_{n\in N}$ and transition time $(S_n)_{n\in N}$ which depends on both the present state and the next state, we can rewrite $\tilde{Q}_{ij}(t)$ as

$$\tilde{Q}_{ij}(t) = \sigma\left[J_{n+1} = j, S_{n+1} - S_n \leq t / J_n = i\right]$$

$$= \min\left[\sigma\left[S_{n+1} - S_n \leq t / J_{n+1} = j, J_n = i\right], \sigma\left[J_{n+1} = j / J_n = i\right]\right]$$

$$= \min\left[\tilde{F}_{ij}(t), \tilde{p}_{ij}\right]$$

$$= \min\left[\tilde{p}_{ij}, \tilde{F}_{ij}(t)\right]$$

Without loss of generality, we denote the weighted duration time distributions as $\ddot{f}_{ij}(t) = (\tilde{f}_{ij}(t), w_f)$.

Let us introduce the weighted possibility that the process stays in state i for at least a duration time x, given state i is entered at time t as $\ddot{H}_i(t) = (\tilde{H}_i(t), w_H)$:

$$\tilde{H}_i(t) = \sigma\left[S_{n+1} - S_n \le x / J_n = i\right]$$

Obviously, it results that

$$\tilde{H}_i(t) = \max_{j \ne i} \tilde{Q}_{ij}(t) = \max_{j \ne i}\left\{\min\left[\tilde{p}_{ij}, \tilde{f}_{ij}(t)\right]\right\}, \quad i, j \in E.$$

Let us define the weighted possibility that the process has been in state i for duration time't' without transitioning to other state as $\ddot{S}_i(t) = (\tilde{S}_i(t), w_S)$, where

$$\tilde{S}_i(t) = \sigma\left[S_{n+1} - S_n > t / J_n = i\right]$$

Now it is possible to define the weighted homogeneous fuzzy semi-Markov process Z

$$Z = \left(Z_t, t \in R_0^+\right)$$

representing for each waiting time t, the state occupied by the process Z_t and this weighted fuzzy semi-Markov process is both characterized by a set of fuzzy transition matrices $\tilde{P}$ and a set of duration matrix $\tilde{D}$.

Now we define the weighted interval fuzzy transition possibilities $\ddot{\phi}_{ij}(t) = (\tilde{\phi}_{ij}(t), w_\phi)$ where $\tilde{\phi}_{ij}(t)$ is defined in the following way:

$$\tilde{\phi}_{ij}(t) = \sigma\left[Z_t = j / Z_0 = i\right]$$

By taking all the possible mutually exclusive ways in which it is possible for the event to take place, we could prove that for $\forall t, x \ge 0$

$$\tilde{\phi}_{ij}(t) = \max\left\{\min\left[\tilde{\delta}_{ij}, \tilde{S}_i(t)\right], \max_{l \in E}\left\{\max_{\tau = 0,1,\ldots,t}\left\{\min\left[\tilde{p}_{il}, \tilde{f}_{il}(\tau), \tilde{\phi}_{lj}(t-\tau)\right]\right\}\right\}\right\} \quad \text{----- (2)}$$

where

$$\tilde{\delta}_{ij} = \begin{cases} 0 & , \quad i \ne j \\ 1 & , \quad i = j \end{cases}$$

This equation represents the possibility of remaining in state i at time't' without any change from time 'x' and possibility of having changed state i and of having reached in some way state j and of staying in this state at time 'x' and this equation is called as the evolution equation of a discrete time homogeneous fuzzy semi-Markov model.

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IV. EXAMPLE

Consider the below web navigation model.

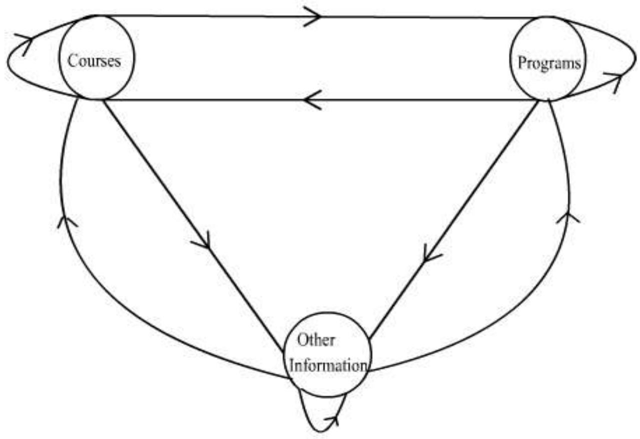

Fig 1: Web navigation Model

The operational units Courses, Programs, Other information are the set of states with the associated connections as the transitions. With each transition we associate a possibility based on the usage for the period of 10 days duration. These possibilities can be obtained from various sources containing information based on actual usage patterns to track usage and failures. As the web access increases the usage information will also increase. Hence, we have modeled weighted fuzzy semi-Markov model with state space S = {C, P, OI} for the above web navigation.

The corresponding weighted fuzzy state transition matrix $\ddot{P} = (\tilde{P}, 0.8)$where $\tilde{P}$is

$$\tilde{P} = \begin{array}{c} \\ C \\ P \\ OI \end{array} \begin{array}{c} \begin{array}{ccc} C & P & OI \end{array} \\ \begin{pmatrix} 0.98 & 0.98 & 0.973 \\ 0.98 & 0.776 & 0.98 \\ 0.782 & 0.99 & 0.969 \end{pmatrix} \end{array}$$

and the calculated weighted duration distribution for the given state space as $\ddot{D} = (\tilde{D}, 0.8)$ and $\tilde{D}$ is given by

$$\tilde{D} = \left(\tilde{f}_{ij}(10)\right) = \begin{array}{c} \\ C \\ P \\ OI \end{array} \begin{array}{c} \begin{array}{ccc} C & P & OI \end{array} \\ \begin{pmatrix} 0.981 & 0.982 & 0.9 \\ 0.89 & 0.964 & 0.841 \\ 0.964 & 0.872 & 0.78 \end{pmatrix} \end{array} \quad , i, j = C, P, OI.$$

Weighted Possibilities of staying in each state starting from day 1 up to 10 days are given as follows:

$(\tilde{S}_c, 0.8); (\tilde{S}_p, 0.8); (\tilde{S}_{OI}, 0.8)$, where

$$\tilde{S}_C(10) = 0.985\ ; \tilde{S}_P(10) = 0.982\ ; \tilde{S}_{OI}(10) = 0.964$$

Hence the weighted interval fuzzy transition possibility from one state to another state for the given period is given by $\tilde{\tilde{\phi}}_{ij}(t) = (\tilde{\phi}_{ij}(t), 0.85)$ where $\tilde{\phi}_{ij}(t)$ is obtained as

$$\tilde{\phi}_{ij}(t) = \begin{array}{c c} & \begin{array}{ccc} C & P & OI \end{array} \\ \begin{array}{c} C \\ P \\ OI \end{array} & \begin{pmatrix} 0.985 & 0.98 & 0.9 \\ 0.89 & 0.982 & 0.841 \\ 0.782 & 0.872 & 0.969 \end{pmatrix} \end{array}, \quad i, j = C, P, OI.$$

V. CONCLUSION

In this paper we have defined a weighted homogeneous fuzzy semi-Markov model for the dynamic evolution of web application defined by weighted interval fuzzy transition possibilities. By means of this approach, we can consider not only uncertainties in the possible stages of transitions, but also the uncertainties of the duration of the waiting time in each state which allows to do a further analysis.

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