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# Thermoelastic Response on Thin Circular Plate with Heat Source

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## ABSTRACT

*The present paper deals with the determination of a thermal stresses in thin circular plate with heat source subjected to initial temperature with boundary surfaces under consideration dissipates heat by convection according to Newton's law of cooling to surrounding temperature ,the results are obtained in series form the term of Bessel's function are illustrated numerically.*

***Keywords*: Thermoelastic response, thin circular plate, stresses function, integral transform.**

## I.INTRODUCTIION

Many heat conduction problem encounters in engineering application involve time as independent variable. The high velocities of modern aircraft give rise to aerodynamics heating, which produces intense thermal stresses that reduce the strength of the aircraft structure. The purpose of analysis is to determine the variation in temperature, displacement and stresses with known boundary condition.

Deshmukh et al. [1,2] have discussed the non-homogeneous steady-state heat conduction problem in a thin circular plate and find its thermal stresses.

Nowacki [5] has determined steady-state thermal stresses in a circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and circular edge.

Roy Choudhuri, S. K. [6] has succeeded in determining the quasi-static thermal stresses in a thin circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated.

## II.STATEMENT OF THE PROBLEM

Consider a thin circular plate of thickness 2h occupying the space D: $0 \le r \le a$, $-h \le z \le h$. The differential equation governing the displacement function U (r, z, t) is

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1 + v)a_t T \tag{2.1}$$

With U = 0 at r=a (2.2)

$v$ and $a_t$ are the poisson's ratio and the linear coefficient of thermal expansion of the material of the disc respectively and T(r, z, t) is the temperature of the satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{\theta(r,z,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2.3}$$

Subject to the initial condition

$$T(r, z, 0) = F(r, z) \tag{2.4}$$

The boundary conditions

$$\left[T(r,z,t) + k_1 \frac{\partial T(r,z,t)}{\partial r}\right]_{r=a} = 0 \tag{2.5}$$

$$\left[T(r,z,t) + k_2 \frac{\partial T(r,z,t)}{\partial z}\right]_{z=h} = f_1(r,t) \tag{2.6}$$

$$\left[T(r,z,t) + k_3 \frac{\partial T(r,z,t)}{\partial z}\right]_{z=-h} = f_2(r,t) \tag{2.7}$$

Where $k_1, k_2, k_3$, are radiation constant

The interior condition

$T(\xi, z, t) = f(z,t)$ , at $r = \xi$ $(known)$ , $(0 < r < a)$

$T(a, z, t) = g(z, t)$ (unknown) (2.8)

The stress function $\sigma_{rr}$ and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr} = -2\mu \frac{1}{r}\frac{\partial U}{\partial r}$$

(2.9)

$$\sigma_{\theta\theta} = -\mu \frac{\partial^2 U}{\partial r^2} \tag{2.10}$$

Where $\mu$ is the Lame' constant, while each of the stress function are zero within the disc in the plane state of stress

The equation (2.1) to (2.10) constitute the mathematical formation of thee problem under consideration.

## III.SOLUTION OF THE PROBLEM

Apply Hankel transform to (2.3)

$$-\xi_m{}^2 \bar{T} + \frac{d^2\bar{T}}{dz^2} + \frac{\bar{\theta}}{k} = \frac{1}{\alpha}\frac{d\bar{T}}{dt} \tag{3.1}$$

Where α is thermal diffusivity of the material and k thermal conductivity of material

Apply Marchi- Fasulo transform to (3.1)

$$-\xi_m{}^2 \bar{T}^* - a_n{}^2 \bar{T}^* + \phi^* + \frac{\bar{\theta}^*}{k} = \frac{1}{\alpha}\frac{d\bar{T}^*}{dt} \tag{3.2}$$

$$\frac{d\bar{T}^*}{dt} + \alpha p^2 \bar{T}^* = \phi_1{}^* \tag{3.3}$$

$$\text{Where } \quad p^2 = \xi_m{}^2 + a_n{}^2 \quad , \phi_1{}^* = \alpha\left[\phi^* + \frac{\bar{\theta}^*}{k}\right] \tag{3.4}$$

Equation (3.3) is first order differential equation has solution

$$\bar{T}^* = e^{-\alpha p^2 t}\left[\int_0^t \phi_1{}^* e^{\alpha p^2 t} dt' + c\right] \quad , c = \bar{F}^*(m,n) \tag{3.5}$$

$$\bar{T}^{*} = \int_0^t \phi_1{}^{*} e^{-\propto\left(\xi_m{}^2 + a_n{}^2\right)(t-t')} dt' + e^{-\propto p^2 t}\, \bar{F}^{*}(m\,,n), \tag{3.6}$$

By taking inverse Hankel transform and inverse Marchi – Fasulo transform we obtain,

$$\mathrm{T}(\mathrm{r},\mathrm{z},\mathrm{t}) = \frac{2}{a^2}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n}\sum_{m=1}^{\infty}\frac{J_0(r\xi_m)}{J_1(a\xi_m)^2}\int_0^t \phi_1{}^{*} e^{-\propto\left(\xi_m{}^2 + a_n{}^2\right)(t-t')} dt' + e^{-\propto p^2 t}\mathbf{F}(\mathbf{r}\,,\mathbf{z}) \tag{3.7}$$

$$\mathrm{g}(\mathrm{z},\mathrm{t}) = \frac{2}{a^2}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n}\sum_{m=1}^{\infty}\frac{J_0(a\xi_m)}{J_1(a\xi_m)^2}\int_0^t \phi_1{}^{*} e^{-\propto\left(\xi_m{}^2 + a_n{}^2\right)(t-t')} dt' + e^{-\propto p^2 t}\mathbf{F}(\mathbf{r}\,,\mathbf{z}) \tag{3.8}$$

Where m, n is the positive integers, $\xi_m$ are the positive roots of the equation

$$J_0(r\xi_m) = 0 \tag{3.9}$$

## IV.DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substitute the value of $T\,(r\,,z,t)$ from (3.7) in (2.1) One obtains the thermo elastic displacement function $U(r\,,z\,,t)$ as

$$U(r\,,z\,,t) = -(1+v)a_t\,\frac{2}{a^2}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n}\sum_{m=1}^{\infty}\frac{J_0(r\xi_m)}{J_1\,(a\xi_m)}\int_0^t \phi_1{}^{*} e^{-\alpha\left(\xi_m{}^2 + a_n{}^2\right)(t-t')} dt' + e^{-\alpha p^2 t}\,\mathbf{F}(\mathbf{r}\,,\mathbf{z}) \tag{4.1}$$

## V.DETERMINATION OF STRESS FUNCTION

$$\sigma_{rr} = (1+v)a_t\,\frac{2\mu}{r}\frac{2}{a^2}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n}\sum_{m=1}^{\infty}\frac{J'_0(r\xi_m)}{J_1(a\xi_m)^2}\int_0^t \phi_1{}^{*} e^{-\propto\left(\xi_m{}^2 + a_n{}^2\right)(t-t')} dt' + e^{-\propto p^2 t}\,\mathbf{F}(\mathbf{r}\,,\mathbf{z}) \tag{5.1}$$

$$\sigma_{\theta\theta} = (1+v)a_t\,\frac{4\mu}{a^2}\sum_{n=1}^{\infty}\frac{P_n(x)}{\lambda_n}\sum_{m=1}^{\infty}\frac{J''_0(r\xi_m)}{J_1(a\xi_m)^2}\int_0^t \phi_1{}^{*} e^{-\propto\left(\xi_m{}^2 + a_n{}^2\right)(t-t')} dt' + e^{-\propto p^2 t}\,\mathbf{F}(\mathbf{r}\,,\mathbf{z}) \tag{5.2}$$

## VI.SPECIAL CASE AND NUMERICAL RESULT

Setting,

$$\theta(r\;z\;t) = \frac{\theta ci}{2\pi r}\delta(r-r_0)\delta(h)\;\;\delta(t)\;\;,F(r,z) = 0 \tag{6.1}$$

Apply Hankel transform and Marchi- Fasulo transform to (6.1) and their inverses we obtain, the temperature function $T(r\,,z,t) = 8\,\frac{(k_3+k_4)}{a^2}\;\times$

$$\sum_{n,m=1}^{\infty}\left[\frac{a_n h\cos^2(a_n h) - \cos(a_n h)\sin(a_n h)}{a_n{}^2}\right]\frac{P_n(z)}{\lambda_n}\,\frac{J_0(r\xi_m)J_0\left(r_0\xi_m\times\int_0^t(1-e^{-t})e^{-\propto\left(\xi_m{}^2+a_n{}^2\right)(t-t')dt'}\right)}{J_1\,(a\xi_m)^2} \tag{6.2}$$

$$g(z,t) = 8\frac{(k_3+k_4)}{a^2}\times$$

$$\sum_{n,m=1}^{\infty}\left[\frac{a_n h\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n{}^2}\right]\frac{P_n(z)}{\lambda_n}\,\frac{J_0(r\xi_m)J_0(r_0\xi_m)}{J_1\,(a\xi_m)^2}\times\int_0^t(1-e^{-t})e^{-\propto\left(\xi_m{}^2+a_n{}^2\right)(t-t')}dt' \qquad (6.3)$$

## VII.CONCLUSION

In this paper we discussed completely inverse unsteady state problem of the thermoelastic deformation of thin circular plate on outer surface with heat source. Where non homogeneous boundary condition of third kind on the plan surface of the circular plate .The finite Marchi-Fasulo, Hankel integral transform techniques are to be used to obtain numerical results. The temperature Displacement and Thermal stresses that are obtained can be applied to design of useful structure or machines in engineering application. Any particular case of special interest can be derived by assigning suitable value of the parameter and the function in the expression.

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