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Lateral Hyperbases and Covered Lateral Hyperideals of Ordered Ternary Semihypergroups

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ABSTRACT

In this paper, we define lateral hyperbases and covered lateral hyperideals of ordered ternary semihypergroups. Here we study their related properties and explore the relationship between lateral hyperbases and covered lateral hyperideals of ordered ternary semihypergroups.

Keywords:Ordered ternary semihypergroup, Lateral hyperbase, Covered lateral hyperideal.

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I. INTRODUCTION AND PRELIMINARIES

The idea of investigation of n-ary algebras i.e. the sets with one n-ary operation was given by Kasner's [15]. In particular, n-ary semigroups are known as ternary semigroups for n=3 with one associative operation [17]. Ideal theory in ternary semigroup was studied by Sioson [21]. He also defined regular ternary semigroups.

Marty [18] proposed the notion of algebraic hyperstructures as an extension of the branch of classical algebraic structures. In algebraic structures, the composition of two elements is an element, while in algebraic hyperstructure the composition of two elements is a non-empty set. Ternary semihypergroups are known as n-ary semihypergroup for $n = 3$ with one ternary associative hyperoperation. Davvaz and Leoreanu [10] studied binary relations on ternary semihypergroups and studied some basic properties of compatible relations on them. Hila and Naka [20] defined the notion of regularity in ternary semihypergroups and characterize them by using various hyperideals of ternary semihypergroups. In [19] they gave some properties of left (right) and lateral hyperideals in ternary semihypergroups. Hila et al.[15] introduced some classes of hyperideals in ternary semihypergroups.

The concept of ordering hypergroups investigated by Chavlina [4] as a special class of hypergroups. Heidari and Davvaz [13] studied a semihypergroup (S, o) besides a binary relation $\leq$, where $\leq$ is a partial order relation such that it satisfies the monotone condition. Polygroups [3] which are partially ordered are introduced with some properties and related results. Yaqoob and Gulistan [25] introduced the concept of partially ordered left almost semihypergroups and studied their related properties. Iampan [14] introduces the notion of an ordered ternary semigroup and characterized the minimality and maximality of ordered lateral ideals in ordered ternary semigroups. Daddi and Pawar [7] introduced the concepts of ordered quasi-ideals, ordered bi-ideals in an ordered ternary semigroup and study their properties. He also defined regular ordered ternary semigroup. Abbasi et.al. [2] defined hyperideals in ordered ternary semihypergroups and characterized regular and intra-regular ordered ternary semihypergroup via hyperideals.

Tamura [23] introduced the notion of (right)left base of semigroup. Later, Fabrici [18] described the structure of semigroup containing one-sided bases. Summaprad and Changphas [22] defined the relationship between right bases and maximal right ideals.

Fabrici [14] showed some other properties and the mutual relation between covered ideals and bases of semigroups. Changphas and Summaprab [23] studied ordered semigroup containing covered ideals. They generalised the results on semigroup studied by Fabrici [12]. Basar et. al. [1] studied some properties of covered γ-ideals in po-γ-semigroups. Thongkam and Changphas [24] introduced the notion of left bases and right bases of a ternary semigroup

First, we recall some basic terms and definitions:

Definition 1.1 *[17] A non-empty set T is called a ternary semigroup if there exists a ternary operation $T \times T \times T \to T$, written as $(x_1, x_2, x_3) \to [x_1 x_2 x_3]$, satisfying the following identity for any $x_1, x_2, x_3, x_4, x_5 \in T$,*

$[[x_1 x_2 x_3] x_4 x_5] = [x_1 [x_2 x_3 x_4] x_5] = [x_1 x_2 [x_3 x_4 x_5]]$.

For non-empty subsets A, B and C of a ternary semigroup T,

$[ABC] := \{[abc] : a \in A, b \in B \text{ and } c \in C\}$.

If $A = \{a\}$, then we write $[\{a\}BC]$ as $[aBC]$ and similarly if $B = \{b\}$ or $C = \{c\}$, we write $[AbC]$ and $[ABc]$, respectively. Throughout the paper, we denote $[x_1 x_2 x_3]$ by $x_1 x_2 x_3$ and $[ABC]$ as ABC.

For any positive integers m and n with $m \le n$ be any elements $x_1, x_2, x_3 ....x_{2n}$ and x_{2n+1} of a ternary semigroup [21], written as

$[x_1 x_2 x_3 ....x_{2n+1}] = [x_1, x_2, x_3 ..[[x_m x_{m+1} x_{m+2}] x_{m+3} x_{m+4}]...x_{2n+1}]$.

Definition 1.2 *A ternary hypergroupoid is called the pair (H, f) where f is a ternary hyperoperation on the set H.*

If A, B, C are non-empty subsets of H, then we define

$$f(A, B, C) = \bigcup_{a \in A, b \in B, c \in C} f(a, b, c).$$

Definition 1.3 *[10] A ternary hypergroupoid (H, f) is called a ternary semihypergroup if for all $a_1, a_2, ..., a_3 \in H$, we have*

$f(f(a_1, a_2, a_3), a_4, a_5) = f(a_1, f(a_2, a_3, a_4), a_5) = f(a_1, a_2, f(a_3, a_4, a_5))$.

Since the set $\{x\}$ can be indentified with the element x, any ternary semigroup is a ternary semihypergroup.

It is clear that due to the associative law in ternary semihypergroup H, for any elements $x_1, x_2, ..., x_{2n+1} \in H$ and positive integers m, n with $m \leq n$, one may write,

$$\begin{aligned} f(x_1, x_2, ..., x_{2n+1}) &= f(x_1, ..., x_m, x_{m+1}, x_{m+2}, ..., x_{2n+1}) \\ &= f(x_1, ..., f(f(x_m, x_{m+1}, x_{m+2}), x_{m+3}, x_{m+4}), ..., x_{2n+1}). \end{aligned}$$

Definition 1.4 *[10] Let* (H, f) *be a ternary semihypergroup. A binary relation* ρ *is called:*

• compatible on the left if $a \ \rho \ b$ and $x \in f(x_1, x_2, a)$ imply that there exists $y \in f(x_1, x_2, b)$ such that $x \ \rho \ y$;

• compatible on the right if $a \ \rho \ b$ and $x \in f(a, x_1, x_2)$ imply that there exists $y \in f(b, x_1, x_2)$ such that $x \ \rho \ y$;

• compatible on the lateral if $a \ \rho \ b$ and $x \in f(x_1, a, x_2)$ imply that there exists $y \in f(x_1, b, x_2)$ such that $x \ \rho \ y$;

• compatible on the two-sided if $a_1 \ \rho \ b_1$, $a_2 \ \rho \ b_2$, and $x \in f(a_1, z, a_2)$ imply that there exists $y \in f(b_1, z, b_2)$ such that $x \ \rho \ y$;

• compatible if $a_1 \ \rho \ b_1$, $a_2 \ \rho \ b_2$, $a_3 \ \rho \ b_3$ and $x \in f(a_1, a_2, a_3)$ imply that there exists $y \in f(b_1, b_2, b_3)$ such that $x \ \rho \ y$.

Definition 1.5 *[2] A ternary semihypergroup* (H, f) *is called a partially ordered ternary semihypergroup if there exits a partially ordered relation* $'\leq'$ *on* H *such that* $'\leq'$ *are compatible on left, compatible on right, compatible on lateral and compatible.*

Example 1.1 *Let* $H = \{ \begin{pmatrix} a & 0 & 0 & b \\ c & 0 & d & e \\ f & g & 0 & h \\ i & 0 & 0 & j \end{pmatrix} : a, b, c, d, e, f, g, h, i, j \in N_0 \}$, *where* N_0 *i.e. the set of all non-negative integers is an ordered ternary semigroup under the ordinary multiplication of numbers with partial ordered relation* $\leq_N$ *is "less than or equal to".*

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Now, let $A_1 = \{ \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b \end{pmatrix} : a, b \in N_0 \}$, $A_2 = \{ \begin{pmatrix} 0 & 0 & 0 & a \\ 0 & 0 & b & 0 \\ 0 & c & 0 & 0 \\ d & 0 & 0 & 0 \end{pmatrix} : a, b, c, d \in N_0 \}$.

Then H is a ternary semihypergroup under ternary hyperoperation $'f'$ defined as follows:

$$f(X, Y, Z) = X \cdot A_1 \cdot Y \cdot A_2 \cdot Z \text{ for all } X, Y, Z \in \mathrm{H},$$

where $'\cdot'$ is the usual multiplication of matrices over N_0.

Now we define partial order relation $\leq_{\mathrm{H}}$ on H by, for any $A, B \in \mathrm{H}$

$A \leq_{\mathrm{H}} B$ if and only if $a_{ij} \leq_N b_{ij}$, for all i and j.

Then it is easy to verify that H is an ordered ternary semihypergroup with partial order relation $\leq_{\mathrm{H}}$.

Throughout the paper, we denote $(\mathrm{H}, f, \leq)$ as an ordered ternary semihypergroup.

Lemma 1.1 *[2] For subsets* A, B *and* C *of* $(\mathrm{H}, f, \leq)$, *the following statements hold:*

- $\mathrm{A} \subseteq (\mathrm{A}]$ for every $\mathrm{A} \subseteq \mathrm{H}$.
- If $\mathrm{A} \subseteq \mathrm{B}$, then $(\mathrm{A}] \subseteq (\mathrm{B}]$ for every $\mathrm{A}, \mathrm{B} \subseteq \mathrm{H}$.
- $((\mathrm{A}]] = (\mathrm{A}]$ for every $\mathrm{A} \subseteq \mathrm{H}$.
- $f((\mathrm{A}], (\mathrm{B}], (\mathrm{C}]) \subseteq (f(\mathrm{A}, \mathrm{B}, \mathrm{C})]$.
- $(f((\mathrm{A}], (\mathrm{B}], (\mathrm{C}])] \subseteq (f(\mathrm{A}, \mathrm{B}, \mathrm{C})]$ for all $\mathrm{A}, \mathrm{B}, \mathrm{C} \subseteq \mathrm{H}$.

Definition 1.6 *[2] Let* $(\mathrm{H}, f, \leq)$ *be an ordered ternary semihypergroup and* $\varnothing \neq T \subseteq \mathrm{H}$. *Then* T *is called an ordered ternary sub-semihypergroup of* H *if and only if* $f(T, T, T) \subseteq T$ *and* $(T] \subseteq T$.

Definition 1.7 *[2] An element* a *of an ordered ternary semihypergroup* $(\mathrm{H}, f, \leq)$ *is called regular if there exists an element* x *in* H *such that* $a \in (f(a, x, a)]$. H *is called regular ordered ternary semihypergroup if every element of* H *is regular.*

Definition 1.8 *[2] A non-empty subset* I *of an ordered ternary semihypergroup* $(\mathrm{H}, f, \leq)$ *is called a right hyperideal(resp., lateral hyperideal, left hyperideal) of* H *if*

- $f(I, \mathrm{H}, \mathrm{H}) \subseteq I$ ($f(\mathrm{H}, I, \mathrm{H}) \subseteq I$, $f(\mathrm{H}, \mathrm{H}, I) \subseteq I$);
- If $i \in I$ and $h \leq i$, then $h \in I$ for every $h \in \mathrm{H}$.

Definition 1.9 *[2] A non-empty subset I of $(\mathrm{H}, f, \leq)$ is called a two sided hyperideal of H if it is a left, right hyperideal of H and I is called hyperideal of H if it is a left, right and lateral hyperideal of H.*

Lemma 1.2 *[2] Let $(\mathrm{H}, f, \leq)$ be an ordered ternary semihypergroup. For any $\varnothing \neq A \subseteq \mathrm{H}$,*

- $(f(A, \mathrm{H}, \mathrm{H}) \cup A]$ is the smallest right hyperideal of H containing A;
- $(f(\mathrm{H}, \mathrm{H}, A, \mathrm{H}, \mathrm{H}) \cup f(\mathrm{H}, A, \mathrm{H}) \cup A]$ is the smallest lateral hyperideal of H containing A;
- $(f(\mathrm{H}, \mathrm{H}, A) \cup A]$ is the smallest left hyperideal of H containing A;
- $(f(A, \mathrm{H}, \mathrm{H}) \cup f(\mathrm{H}, \mathrm{H}, A, \mathrm{H}, \mathrm{H}) \cup f(\mathrm{H}, A, \mathrm{H}) \cup f(\mathrm{H}, \mathrm{H}, A) \cup A]$ is the smallest hyperideal of H containing A.

Lemma 1.3 *[2] Let $(\mathrm{H}, f, \leq)$ be an ordered ternary semihypergroup. For any $\varnothing \neq A \subseteq \mathrm{H}$,*

- $(f(A, \mathrm{H}, \mathrm{H})]$ is a right hyperideal of H;
- $(f(\mathrm{H}, \mathrm{H}, A, \mathrm{H}, \mathrm{H}) \cup f(\mathrm{H}, A, \mathrm{H})]$ is a lateral hyperideal of H;
- $(f(\mathrm{H}, \mathrm{H}, A)]$ is a left hyperideal of H;
- $(f(A, \mathrm{H}, \mathrm{H}) \cup f(\mathrm{H}, \mathrm{H}, A, \mathrm{H}, \mathrm{H}) \cup f(\mathrm{H}, A, \mathrm{H}) \cup f(\mathrm{H}, \mathrm{H}, A)]$ is a hyperideal of H.

Definition 1.10 *[2] Let $(\mathrm{H}, f, \leq)$ be an ordered ternary semihypergroup. A lateral hyperideal M of H is called a minimal lateral hyperideal of H if there is no lateral hyperideal M' of H such that $M' \subset M$.*

Definition 1.11 *[2] Let $(\mathrm{H}, f, \leq)$ be a ternary semihypergroup. A right hyperideal R of H is called a maximal right hyperideal of H if for every right hyperideal A of H such that $R \subset A$, we have $A = \mathrm{H}$.*

Let H be an ordered ternary semihypergroup and a, b be any non-zero elements of H.
Then the equivalence relation $\mathbf{M}$ is defined by:

$$a \,\mathbf{M}\, b \text{ if } a \text{ and } b \text{ generate the same principal lateral hyperideal of } T.$$

$\mathbf{M}$-class containing a is denoted by M_a. Now let a, b be any non-zero elements f H, then we define a quasi-ordering $\preccurlyeq$ on the set of all equivalence classes as: $\mathrm{M}_a \preccurlyeq \mathrm{M}_b$ if donly if $\mathrm{M}(a) \subseteq \mathrm{M}(b)$, where $\mathrm{M}(a)$ and $\mathrm{M}(b)$ are the principal lateral hyperideals of H generated by a and b.

II. LATERAL HYPERBASES OF ORDERED TERNARY SEMIHYPERGROUPS

Definition 2.1 *Let* $(\mathrm{H}, f, \leq)$ *be an ordered ternary semihypergroup. A non-empty subset* A *of* H *is called a lateral hyperbase of* H *if,*

(1) $(\mathrm{A} \cup f(\mathrm{H},\mathrm{A},\mathrm{H}) \cup f(\mathrm{H},\mathrm{H},\mathrm{A},\mathrm{H},\mathrm{H})] = \mathrm{H}$, and

(2) $(\mathrm{B} \cup f(\mathrm{H},\mathrm{B},\mathrm{H}) \cup f(\mathrm{H},\mathrm{H},\mathrm{B},\mathrm{H},\mathrm{H})] \neq \mathrm{H}$, for any proper subset B of A.

Example 2.1 *Let* $\mathrm{H} = \left\{ \begin{pmatrix} 0 & a & 0 \\ b & 0 & c \\ 0 & d & 0 \end{pmatrix} : a,b,c,d \in N_0 \right\}$, *where* N_0 *i.e. the set of all non-negative integers is an ordered ternary semigroup under the ordinary multiplication of numbers with partial ordered relation* $\leq_N$ *is "less than or equal to".*

Then H is a ternary semihypergroup under ternary hyperoperation $'f'$ defined as follows:

$f(X,Y,Z) = X \cdot Y \cdot Z$ for all $X,Y,Z \in \mathrm{H}$,

where $'\cdot'$ is the usual multiplication of matrices over N_0. Also, H is an ordered ternary semihypergroup under f over N_0 with partial order relation $\leq_H$, as defined in Example 1.1.

Now consider a set $\mathrm{M} = \left\{ \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right\}$. Then, we can easily verify that M is lateral hyperbase of H as

$(\mathrm{M} \cup f(\mathrm{H},\mathrm{M},\mathrm{H}) \cup f(\mathrm{H},\mathrm{H},\mathrm{M},\mathrm{H},\mathrm{H})] = \mathrm{H}$.

Lemma 2.1 *Let* $(\mathrm{H}, f, \leq)$ *be an ordered ternary semihypergroup.. If two elements* a, b *belongs to lateral hyperbase* A *of* H *such that* $a \in (f(\mathrm{H},b,\mathrm{H}) \cup f(\mathrm{H},\mathrm{H},b,\mathrm{H},\mathrm{H})]$. *Then* $a = b$.

Proof. Let A be a lateral hyperbase of an ordered ternary semihypergroup H and a, $b \in \mathrm{A}$ such that $a \in (f(\mathrm{H},b,\mathrm{H}) \cup f(\mathrm{H},\mathrm{H},b,\mathrm{H},\mathrm{H})]$. Suppose $a \neq b$. Take a proper subset B of A as $\mathrm{B} = \mathrm{A} \setminus \{a\}$. Then b belongs to B. Now $a \in (f(\mathrm{H},b,\mathrm{H}) \cup f(\mathrm{H},\mathrm{H},b,\mathrm{H},\mathrm{H})]$, it implies

$$\begin{aligned} \mathrm{M}(a) &\subseteq (f(\mathrm{H},b,\mathrm{H}) \cup f(\mathrm{H},\mathrm{H},b,\mathrm{H},\mathrm{H})] \\ &\subseteq \mathrm{M}(b) \\ &\subseteq \mathrm{M}(B). \end{aligned}$$

Hence, $\mathrm{M}(A) \subseteq \mathrm{M}(B)$. Thus we get $\mathrm{M}(B) = \mathrm{H}$, which is contradiction because A is a lateral hyperbase of

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H. Therefore, $a = b$.

Lemma 2.2 Let A be a subset of an ordered ternary semihypergroup $(\mathsf{H}, f, \leq)$. Then A is a lateral hyperbase of H if and only if it satisfies the following conditions:

(1) For any $t \in \mathsf{H}$ there exists a in A such that $\mathsf{M}_t \preccurlyeq \mathsf{M}_a$;

(2) If a_1, $a_2 \in \mathsf{A}$ such that $a_1 \neq a_2$, then neither $\mathsf{M}_{a_1} \preccurlyeq \mathsf{M}_{a_2}$ nor $\mathsf{M}_{a_2} \preccurlyeq \mathsf{M}_{a_1}$.

Proof. Suppose A is a lateral hyperbase of an ordered ternary semihypergroup $(\mathsf{H}, f, \leq)$. Let $t \in \mathsf{H}$. As A is a lateral hyperbaseof H, we have $(\mathsf{A} \cup f(\mathsf{H},\mathsf{A},\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},\mathsf{A},\mathsf{H},\mathsf{H})] = \mathsf{H}$. Then $t \in (\mathsf{A} \cup f(\mathsf{H},\mathsf{A},\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},\mathsf{A},\mathsf{H},\mathsf{H})]$, so $t \in (\mathsf{A}]$ or $t \in (f(\mathsf{H},\mathsf{A},\mathsf{H})]$ or $t \in (f(\mathsf{H},\mathsf{H},\mathsf{A},\mathsf{H},\mathsf{H})]$. If $t \in (\mathsf{A}]$, then $t \leq a$, for some $a \in \mathsf{A}$, it follows $\mathsf{M}_t \preccurlyeq \mathsf{M}_a$. If $t \in (f(\mathsf{H},\mathsf{A},\mathsf{H})]$, then $t \leq f(t_1, a', t_2)$ for some $t_1, t_2 \in \mathsf{H}$ and $a' \in \mathsf{A}$, we have $\mathsf{M}_t \preccurlyeq \mathsf{M}_{a'}$. If $t \in (f(\mathsf{H},\mathsf{H},\mathsf{A},\mathsf{H},\mathsf{H})]$, then $t \leq f(t_3, t_4, a'', t_5, t_6)$ for some $t_3, t_4, t_5, t_6 \in \mathsf{H}$ and $a'' \in \mathsf{A}$, we obtain $\mathsf{M}_t \preccurlyeq \mathsf{M}_{a''}$. Hence, the result (i) is proved.

Now, suppose $a_1 \leq a_2$ i.e. $\mathsf{M}_{a_1} \preccurlyeq \mathsf{M}_{a_2}$, $\mathsf{M}(a_1) \subseteq \mathsf{M}(a_2)$. If $a_1 \neq a_2$, then $a_1 \in (f(\mathsf{H}, a_2, \mathsf{H}) \cup f(\mathsf{H},\mathsf{H}, a_2, \mathsf{H},\mathsf{H})]$. So, by Lemma 2.1, $a_1 = a_2$. Thus for $a_1 \neq a_2$, $\mathsf{M}_{a_1} \not\preccurlyeq \mathsf{M}_{a_2}$. Analogously, we can prove $\mathsf{M}_{a_2} \not\preccurlyeq \mathsf{M}_{a_1}$.

Conversely, suppose that conditions (i) and (ii) holds. From (i), $\mathsf{M}(\mathsf{A}) = \mathsf{H}$. Now, if possible $\mathsf{M}(\mathsf{B}) = \mathsf{H}$, where B is a proper subset of A. Let $a_1 \in \mathsf{A} \setminus \mathsf{B}$, then there exists $a_2 \in \mathsf{B}$ such that $a_1 \in (a_2 \cup f(\mathsf{H}, a_2, \mathsf{H}) \cup f(\mathsf{H},\mathsf{H}, a_2, \mathsf{H},\mathsf{H})]$. Hence $\mathsf{M}(a_1) \subseteq \mathsf{M}(a_2)$, i.e. $\mathsf{M}_{a_1} \preccurlyeq \mathsf{M}_{a_2}$, which is a contradiction to the assumption (2). Therefore, $\mathsf{M}(\mathsf{B}) \neq \mathsf{H}$. Hence, A is a lateral hyperbase of H.

Theorem 2.1 *Let an ordered ternary semihypergroup $(\mathsf{H}, f, \leq)$ contains a lateral hyperideal. Then M is a maximal lateral hyperideal of H if and only if $\mathsf{H} \setminus \mathsf{M}$ is a maximal $\mathbf{M}$-class.*

Proof. Suppose M is a maximal lateral hyperideal of an ordered ternary semihypergroup $(\mathsf{H}, f, \leq)$. Let t_1, $t_2 \in \mathsf{H} \setminus \mathsf{M}$. As $\mathsf{M} \subseteq \mathsf{M} \cup \mathsf{M}(t_1) \subseteq \mathsf{H}$, it implies $\mathsf{M} \cup \mathsf{M}(t_1) = \mathsf{H}$. Thus, $t_2 \in \mathsf{M}(t_1)$. Similarly $t_1 \in \mathsf{M}(t_2)$. Therefore, $\mathsf{M}(t_1) = \mathsf{M}(t_2)$. This shows that $\mathsf{H} \setminus \mathsf{M}$ is in $\mathbf{M}$-class. Now, for $\mathsf{H} \setminus \mathsf{M} \prec \mathsf{M}_t$ for some $t \in \mathsf{H}$, then there exists $s \in \mathsf{H}$ such that $\mathsf{H} \setminus \mathsf{M} \subseteq \mathsf{M}(s)$, which is a contradiction. Hence, $\mathsf{H} \setminus \mathsf{M}$ is a

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maximal $\mathbf{M}$-class.

Conversely, suppose that $\mathrm{H} \setminus \mathrm{M}$ is a maximal $\mathbf{M}$-class such that $\mathrm{H} \setminus \mathrm{M} = \mathrm{M}_t$ for some $t \in \mathrm{H}$. We have to show that M is maximal lateral hyperideal of H. Firstly, we will prove that M is a lateral hyperideal of H. i.e. $f(\mathrm{H,M,H}) \cup f(\mathrm{H,H,M,H,H}) \subseteq \mathrm{M}$ and $(\mathrm{M}] = \mathrm{M}$. On contrary suppose that M is not a lateral hyperideal of H. Then there exists $m \in f(\mathrm{H,M,H}) \cup f(\mathrm{H,H,M,H,H})$ such that $m \notin \mathrm{M}$. It implies $m \in \mathrm{H} \setminus \mathrm{M} = \mathrm{M}_t$, then we have $\mathrm{M}(m) = \mathrm{M}(t)$. As $m \in f(\mathrm{H,M,H}) \cup f(\mathrm{H,H,M,H,H})$, there exists $t_1, t_2, ..., t_6 \in \mathrm{H}$ and $m_1, m_2 \in \mathrm{M}$ such that $m \in f(t_1, m_1, t_2)$ or $m \in f(t_3, t_4, m_2, t_5, t_6)$. Hence, we have $\mathrm{M}_t \prec \mathrm{M}_{m_1}$ or $\mathrm{M}_t \prec \mathrm{M}_{m_2}$. Therefore, $\mathrm{M}_t \prec \mathrm{M}_m$, which is a contradiction to the assumption that $\mathrm{H} \setminus \mathrm{M}$ is a maximal $\mathbf{M}$-class. It implies $f(\mathrm{H,M,H}) \cup f(\mathrm{H,H,M,H,H}) \subseteq \mathrm{M}$. Now, let $m \in \mathrm{M}$ and $s \in \mathrm{H}$ such that $s \le m$. Suppose $s \in \mathrm{H} \setminus \mathrm{M} = \mathrm{M}_t$. Then $s < m$, hence $\mathrm{M}_t \prec \mathrm{M}_m$. This is a contradiction. Thus, we have $s \in \mathrm{M}$. Therefore, M is a lateral hyperideal of H. Let M' be a proper lateral hyperideal of H such that M is properly contained in M'. Then there exists $a \in \mathrm{H} \setminus \mathrm{M}'$. Thus, $\mathrm{M}_t = \mathrm{M}_a$. Also there exists $m' \in \mathrm{M}' \setminus \mathrm{M}$ such that $\mathrm{M}_{m'} = \mathrm{M}_t$. It follows that $a \in \mathrm{M}_a = \mathrm{M}_t = \mathrm{M}_{m'} \subseteq \mathrm{M}'$, which is also a contradiction. Therefore, M is a maximal lateral hyperideal of H.

III. COVERED LATERAL HYPERIDEALS OF ORDERED TERNARY SEMIHYPERGROUPS

Definition 3.1 *A proper lateral hyperideal* L *of an ordered ternary semihypergroup* $(\mathrm{H}, f, \le)$ *is said to be covered lateral hyperideal (CLt. ideal) if*

$$\mathrm{L} \subset (f(\mathrm{H},(\mathrm{H}-\mathrm{L}),\mathrm{H}) \cup f(\mathrm{H},\mathrm{H},(\mathrm{H}-\mathrm{L}),\mathrm{H},\mathrm{H})].$$

Remark 3.1 *By definition of CLt.-hyperideal, ternary semihypergroup itself is not a CLt.-hyperideal.*

Example 3.1 *Let* $\mathrm{H} = \left\{ \begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix} : a,b,c,d,e,f \in N_0 \right\}$ *excluding null matrix, where* N_0 *is an ordered ternary semigroup under the ordinary multiplication of numbers with partial ordered relation* $\le_{N_0}$ *is "less than or equal to".*

Then H is a ternary semihypergroup under ternary hyperoperation $'f'$ defined as follows:

$$f(X,Y,Z) = X \cdot Y \cdot Z \text{ for all } X,Y,Z \in H,$$

where '$\cdot$' is the usual multiplication of matrices over N_0. Then H is an ordered ternary semihypergroup under 'f' over N_0 with partial order relation $\leq_H$, as defined in the Example 1.1.

Consider $L = \{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ b & 0 & 0 \end{pmatrix} : b \in N \}$. Then, it easy to verify that L is lateral hyperideal of H and $H - L$ =

$\{ \begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ 0 & d & e \end{pmatrix} : a,b,c,d,e \in N_0 \}$.

Now, $L = \{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ b & 0 & 0 \end{pmatrix} : b \in N \} \subset \{ \begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix} : a,b,c,d,e,f \in N_0 \}$ =

$(f(H,(H-L),H) \cup f(H,H,(H-L),H,H)]$. Therefore, L is a CLt.-hyperideal of H.

Lemma 3.1 *Let* $(H, f, \leq)$ *be an ordered ternary semihypergroup and let* H *contains two different lateral hyperideals* L_1 *and* L_2 *such that* $L_1 \cup L_2 = H$*, then none of the lateral hyperideals* L_1, L_2 *is a CLt.-hyperideal.*

Proof. Suppose $(H, f, \leq)$ is an ordered ternary semihypergroup and L_1, L_2 are two different lateral hyperideals of H such that $L_1 \cup L_2 = H$. It implies $H - L_1 \subset L_2$ and $H - L_2 \subset L_1$. Suppose that L_2 is a covered lateral hyperideal of H, then $L_2 \subset (f(H,(H-L_2),H) \cup f(H,H,(H-L_2),H,H)]$ which implies $L_2 \subset (f(H,(H-L_2),H) \cup f(H,H,(H-L_2),H,H)] \subset (f(H,L_1,H) \cup f(H,H,L_1,H,H)] \subset$ $(f(H,L_1,H) \cup f(H,L_1,H)] \subseteq L_1$. Similarly $L_1 \subset L_2$. Therefore, $L_1 = L_2$. But L_1 and L_2 are different. Thus, our assumption was wrong. Hence, neither L_1 nor L_2 is a CLt.-hyperideal.

Corollary 3.1 *Let* $(H, f, \leq)$ *be an ordered ternary semihypergroup. If an ordered ternary semihypergroup* H *contains more than one maximal lateral hyperideal, then none of the maximal lateral hyperideal is CLt.-hyperideal.*

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Proof. Suppose that the ordered ternary semihypergroup H contains two maximal lateral hyperideals M_1 and M_2. We know that union of lateral hyperideals is a lateral hyperideal. Then $\mathsf{M}_1 \cup \mathsf{M}_2$ is a lateral hyperideal of H and $\mathsf{M}_1 \subset \mathsf{M}_1 \cup \mathsf{M}_2$. As M_1 is a maximal lateral hyperideal of H. It implies $\mathsf{M}_1 \cup \mathsf{M}_2 = \mathsf{H}$. Hence, by Lemma 3.1, neither M_1 nor M_2 is a CLt.-hyperideal of H.

Lemma 3.2 *Let $(\mathsf{H}, f, \leq)$ be an ordered ternary semihypergroup. If L is a lateral hyperideal of H such that $\mathsf{L} \subset (f(\mathsf{H},t,\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},t,\mathsf{H},\mathsf{H})]$ and $\mathsf{L} \neq (f(\mathsf{H},t,\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},t,\mathsf{H},\mathsf{H})]$ for some $t \in \mathsf{H}$. Then L will be a CLt. hyperideal of H.*

Proof. Suppose that $(\mathsf{H}, f, \leq)$ is an ordered ternary semihypergroup. Let L be a lateral hyperideal of H such that $\mathsf{L} \subset (f(\mathsf{H},t,\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},t,\mathsf{H},\mathsf{H})]$ and $\mathsf{L} \neq (f(\mathsf{H},t,\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},t,\mathsf{H},\mathsf{H})]$ for some $t \in \mathsf{H}$. Here $t \notin \mathsf{L}$, otherwise $(f(\mathsf{H},t,\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},t,\mathsf{H},\mathsf{H})] \subseteq (f(\mathsf{H},\mathsf{L},\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},\mathsf{L},\mathsf{H},\mathsf{H})] \subseteq \mathsf{L}$ and we assume that $\mathsf{L} \neq (f(\mathsf{H},t,\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},t,\mathsf{H},\mathsf{H})]$. Hence
$\mathsf{L} \subset (f(\mathsf{H},t,\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},t,\mathsf{H},\mathsf{H})] \subset (f(\mathsf{H},(\mathsf{H}-\mathsf{L}),\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},(\mathsf{H}-\mathsf{L}),\mathsf{H},\mathsf{H})]$. This shows that L is a CLt. hyperideal of H.

Corollary 3.2 *An ordered ternary semihypergroup H in which t does not belongs to $(f(\mathsf{H},t,\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},t,\mathsf{H},\mathsf{H})]$ contains some CLt.-hyperideal.*

Proof. Let $\mathsf{L} = (f(\mathsf{H},t,\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},t,\mathsf{H},\mathsf{H})]$. Then L is a lateral hyperideal of H. If $t \notin \mathsf{L}$, we have

$$\mathsf{L} = (f(\mathsf{H},t,\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},t,\mathsf{H},\mathsf{H})]$$
$$\subset (f(\mathsf{H},(\mathsf{H}-\mathsf{L}),\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},(\mathsf{H}-\mathsf{L}),\mathsf{H},\mathsf{H})].$$

This implies L is a CLt.-hyperideal of H.

Lemma 3.3 *Let $(\mathsf{H}, f, \leq)$ be an ordered ternary semihypergroup and L_1 and L_2 be two covered lateral hyperideal of H. Then $\mathsf{L}_1 \cup \mathsf{L}_2$ is a CLt.-hyperideal of H.*

Proof. Assume $(\mathsf{H}, f, \leq)$ be an ordered ternary semihypergroup and L_1 and L_2 are two covered lateral hyperideal of H. Now to prove $\mathsf{L}_1 \cup \mathsf{L}_2$ is a CLt.-hyperideal of H, we have to show that
$\mathsf{L}_1 \cup \mathsf{L}_2 \subset (f(\mathsf{H},[\mathsf{H}-(\mathsf{L}_1 \cup \mathsf{L}_2)],\mathsf{H}) \cup f(\mathsf{H},\mathsf{H},[\mathsf{H}-(\mathsf{L}_1 \cup \mathsf{L}_2)],\mathsf{H},\mathsf{H})]$. As L_1 is a CLt.-hyperideal i.e. $\mathsf{L}_1 \subset (f(\mathsf{H},(\mathsf{H}-\mathsf{L}_1)\mathsf{H} \cup f(\mathsf{H},\mathsf{H},(\mathsf{H}-\mathsf{L}_1),\mathsf{H},\mathsf{H})]$, which implies for any $m \in \mathsf{L}_1$, there exists

$m_1, m_2 \in H - L_1$ such that $m \in (f(H, m_1, H) \cup f(H, H, m_2, H, H)]$. Now we have following four cases:

Case1: If $m_1, m_2 \in H - L_1 - L_2$.

Then

$m \in (f(H, (H - L_1 - L_2), H) \cup f(H, H, (H - L_1 - L_2), H, H)] \subseteq (f(H, [H - (L_1 \cup L_2)], H) \cup f(H, H, [H - (L_1 \cup L_2)], H, H)]$.

Case2: If $m_1, m_2 \in (H - L_1) \cap L_2$. It implies

$m_1, m_2 \in L_2 \subset (f(H, (H - L_2), H) \cup f(H, H, (H - L_2), H, H)]$. Then, there exists

$m_3, m_4, m_5, m_6 \in H - L_2$ s.t. $m_1 \in (f(H, m_3, H) \cup f(H, H, m_4, H, H)]$ and

$m_2 \in (f(H, m_5, H) \cup f(H, H, m_6, H, H)]$. Here $m_3, m_4 \notin L_1$, otherwise

$m_1 \in (f(H, m_3, H) \cup f(H, H, m_4, H, H)] \subseteq (f(H, L_1, H) \cup f(H, H, L_1, H, H)] \subseteq L_1$. Hence $m_1 \in L_1$,

which is contradiction as $m_1 \in H - L_1$. Thus we have $m_3, m_4 \in H - L_1$. Therefore

$m_3, m_4 \in H - L_1 \cap H - L_2 = H - (L_1 \cup L_2)$. Similarly $m_5, m_6 \in H - (L_1 \cup L_2)$. Now

$$
\begin{aligned}
m & \in (f(H, m_1, H) \cup f(H, H, m_2, H, H)] \\
& \subset (f(H, (f(H, m_3, H) \cup f(H, H, m_4, H, H)], H) \cup f(H, H, (f(H, m_5, H) \cup f(H, H, m_6, H, H)], H, H)] \\
& \subseteq (f((H], (f(H, m_3, H) \cup f(H, H, m_4, H, H)], (H]) \cup f((H], (H], (f(H, m_5, H) \cup f(H, H, m_6, H, H)], (H], (H])] \\
& \subseteq ((f(H, f(H, m_3, H) \cup f(H, H, m_4, H, H), H) \cup f(H, H, f(H, m_5, H) \cup f(H, H, m_6, H, H), H, H)]] \\
& = (f(H, f(H, m_3, H) \cup f(H, H, m_4, H, H), H) \cup f(H, H, f(H, m_5, H) \cup f(H, H, m_6, H, H), H, H)] \\
& \subseteq (f(H, H, m_3, H, H) \cup f(H, m_4, H) \cup f(H, m_5, H) \cup f(H, H, m_6, H, H)] \\
& \subset (f(H, [H - (L_1 \cup L_2)], H) \cup f(H, H, [H - (L_1 \cup L_2)], H, H)].
\end{aligned}
$$

Case 3: If $m_1 \in H - L_1 - L_2$ and $m_2 \in (H - L_1) \cap L_2$. As $m_1 \in H - L_1 - L_2$, it implies

$m_1 \in H - (L_1 \cup L_2)$. From case 2, $m_2 \in H - (L_1 \cup L_2)$. Therefore

$m \in (f(H, m_1, H) \cup f(H, H, m_2, H, H)] \subset (f(H, [H - (L_1 \cup L_2)], H) \cup f(H, H, [H - (L_1 \cup L_2)], H, H)]$.

Case 4: If $m_2 \in H - L_1 - L_2$ and $m_1 \in (H - L_1) \cap L_2$. Then this is similar to case 3.

Therefore, in all these cases $m \in (fH, [H - (L_1 \cup L_2)], H) \cup f(H, H, [H - (L_1 \cup L_2)], H, H)]$. Similarly, we can prove this for $m \in L_2$.

Thus $L_1 \cup L_2 \subset (f(H, [H - (L_1 \cup L_2)], H) \cup f(H, H, [H - (L_1 \cup L_2)], H, H)]$ and hence, $L_1 \cup L_2$ is a

CLt.-hyperideal of H.

Lemma 3.4 *Let* $(H, f, \leq)$ *be an ordered ternary semihypergroup. If* L_1 *is a covered lateral hyperideal of* H *and* L_2 *is any lateral hyperideal of* H. *Then,* $L_1 \cap L_2$ *is a CLt.-hyperideal of* H, *provided* $L_1 \cap L_2 \neq \emptyset$.

Proof. Assume that L_1 is a covered lateral hyperideal and L_2 is a lateral hyperideal of H such that $L_1 \cap L_2 \neq \emptyset$. Then $L_1 \subset (f(H,(H-L_1),H) \cup f(H,H,(H-L_1),H,H)]$. It implies

$$\begin{aligned} L_1 \cap L_2 &\subset (f(H,(H-L_1),H) \cup f(H,H,(H-L_1),H,H)] \\ &\subset (f(H,[H-(L_1 \cap L_2)],H) \cup f(H,H,[H-(L_1 \cap L_2)],H,H)]. \end{aligned}$$

Therefore, $L_1 \cap L_2$ is a CLt.-hyperideal of H.

Lemma 3.5 *Let* $(H, f, \leq)$ *be an ordered ternary semihypergroup. If* L_1 *and* L_2 *are two covered lateral hyperideal of* H. *Then,* $L_1 \cap L_2$ *is a CLt.-hyperideal of* H, *provided* $L_1 \cap L_2 \neq \emptyset$.

Proof. Proof is similar to the above lemma.

Definition 3.2 *A proper lateral hyperideal* L *is said to be the greatest lateral hyperideal of on ordered ternary semihypergroup* H *if* L *contains any proper lateral hyperideal of* H. *We shall denote it by* L^*.

Theorem 3.1 *Let* $(H, f, \leq)$ *be an ordered ternary semihypergroup. If* H *have only one maximal lateral hyperideal* M *and* M *is a CLt.-hyperideal, then* $M = M^*$

Proof. Suppose that M is a maximal lateral of an ordered ternary semihypergroup H and M is also a CLt.-hyperideal of H. Let M_1 be lateral hyperideal of H such that $M_1 \cup M$. As $M_1 \cup M$ is a lateral hyperideal of H and $M \subset M_1 \cup M$. It follows that $M_1 \cup M = H$. Therefore, by Lemma 3.1, H cannot contains any CLt.-hyperideals, which is contradiction. Thus $M_1 \subseteq M$. Hence, $M = M^*$.

Theorem 3.2 *Let* $(H, f, \leq)$ *be an ordered ternary semihypergroup. If* H *is not a simple such that there is no any two proper lateral hyperideals in which there intersection is empty. Then* H *contains at least one CLt.-hyperideal.*

Proof. Let M be a proper lateral hyperideal of H. Then $M_1 = (f(H,(H-M),H) \cup f(H,H,(H-M),H,H)]$ is also a lateral hyperideal of H. By assumption $M \cap M_1 \neq \emptyset$. Thus, $M_c = M \cap M_1$ is a lateral hyperideal of H and $\mathrm{M}_c \subset \mathrm{M}$, it implies $\mathrm{H}-\mathrm{M}_c \supset \mathrm{H}-\mathrm{M}$.

Now, we have $\mathrm{M}_c \subset \mathrm{M}_1$ =

$(f(\mathrm{H},(\mathrm{H}-\mathrm{M}),\mathrm{H}) \cup f(\mathrm{H},\mathrm{H},(\mathrm{H}-\mathrm{M}),\mathrm{H},\mathrm{H})] \subset (f(\mathrm{H},(\mathrm{H}-\mathrm{M}_c),\mathrm{H}) \cup f(\mathrm{H},\mathrm{H},(\mathrm{H}-\mathrm{M}_c),\mathrm{H},\mathrm{H})]$. This shows that M_c is a CLt. hyperideal of H.

Theorem 3.3 *Let* $(\mathrm{H}, f, \leq)$ *be an ordered ternary semihypergroup containing maximal lateral hyperideals. If the intersection of maximal lateral hyperideal is empty or a covered lateral hyperideal, then* H *contains a lateral hyperbase.*

Proof. Let $\{\mathrm{M}_i : i \in I\}$ be the set of all maximal lateral hyperideals of an ordered ternary semihypergroup H. By Theorem 2.1, for each $i \in I$, $\mathrm{H}-\mathrm{M}_i$ is a maximal $\mathbf{M}$-class. Set $\mathrm{H}-\mathrm{M}_i = \mathrm{M}_{m_i}$, for each $i \in I$. Then

$\mathrm{M}_{int} = \bigcap_{i\in I} \mathrm{M}_i = \bigcap_{i\in I} (\mathrm{H}-\mathrm{M}_{m_i}) = \mathrm{H}-\bigcup_{i\in I}\mathrm{M}_{m_i}$.

Construct C as, for every M_{m_i}, put into C only one element. We will prove that C is a lateral hyperbaseof H. Now we consider two cases:

*Case*1: If $\mathrm{M}_{int} = \varnothing$. Then $\mathrm{H} = \bigcup_{i\in I}\mathrm{M}_{m_i}$. If $m \in \mathrm{H}$, then $m \in \mathrm{M}_{m_i}$ for some $i \in I$ and so $\mathrm{M}(m) = \mathrm{M}(m_i)$. Then $\mathrm{M}_m \preccurlyeq \mathrm{M}_{m_i}$. As M_{m_i} is a maximal $\mathbf{M}$-class for all $i \in I$, it implies for different i, $j \in I$, neither $\mathrm{M}_{m_i} \preccurlyeq \mathrm{M}_{m_j}$ nor $\mathrm{M}_{m_j} \preccurlyeq \mathrm{M}_{m_i}$. Then, by Lemma 2.2, C will be a lateral hyperbase.

*Case*2: If M_{int} is a covered lateral hyperideal of H, i.e.

$\mathrm{M}_{int} \subseteq (f(\mathrm{H},(\mathrm{H}-\mathrm{M}_{int}),\mathrm{H}) \cup f(\mathrm{H},\mathrm{H},(\mathrm{H}-\mathrm{M}_{int}),\mathrm{H},\mathrm{H})]$. Now if, $m \in \mathrm{H}-\mathrm{M}_{int}$, then $m \in \bigcup_{i\in I}\mathrm{M}_{m_i}$ and so $m \in \mathrm{M}_{m_{i_0}}$ for some $i_0 \in I$. Then $\mathrm{M}(m) = \mathrm{M}(m_{i_0}) \subseteq (\mathrm{C} \cup f(\mathrm{H},\mathrm{C},\mathrm{H}) \cup f(\mathrm{H},\mathrm{H},\mathrm{C},\mathrm{H},\mathrm{H})]$. Thus, we have $m \in (\mathrm{C} \cup f(\mathrm{H},\mathrm{C},\mathrm{H}) \cup f(\mathrm{H},\mathrm{H},\mathrm{C},\mathrm{H},\mathrm{H})]$. It implies

$\mathrm{H}-\mathrm{M}_{int} \subseteq (\mathrm{C} \cup f(\mathrm{H},\mathrm{C},\mathrm{H}) \cup f(\mathrm{H},\mathrm{H},\mathrm{C},\mathrm{H},\mathrm{H})]$.

Also, we have

$$\begin{aligned}
\mathrm{M}_{int} &\subseteq (f(\mathrm{H},(\mathrm{H}-\mathrm{M}_{int}),\mathrm{H})\cup f(\mathrm{H},\mathrm{H},(\mathrm{H}-\mathrm{M}_{int}),\mathrm{H},\mathrm{H})] \\
&\subseteq (f(\mathrm{H},(\mathrm{C}\cup f(\mathrm{H},\mathrm{C},\mathrm{H})\cup f(\mathrm{H},\mathrm{H},\mathrm{C},\mathrm{H},\mathrm{H})],\mathrm{H})\cup f(\mathrm{H},\mathrm{H},(\mathrm{C}\cup f(\mathrm{H},\mathrm{C},\mathrm{H})\cup f(\mathrm{H},\mathrm{H},\mathrm{C},\mathrm{H},\mathrm{H})]),\mathrm{H},\mathrm{H})] \\
&\subseteq (f((\mathrm{H}],(\mathrm{C}\cup f(\mathrm{H},\mathrm{C},\mathrm{H})\cup f(\mathrm{H},\mathrm{H},\mathrm{C},\mathrm{H},\mathrm{H})],(\mathrm{H}])\cup f((\mathrm{H}],(\mathrm{H}],(\mathrm{C}\cup f(\mathrm{H},\mathrm{C},\mathrm{H})\cup f(\mathrm{H},\mathrm{H},\mathrm{C},\mathrm{H},\mathrm{H})],(\mathrm{H}],(\mathrm{H}])] \\
&\subseteq ((f(\mathrm{H},(\mathrm{C}\cup f(\mathrm{H},\mathrm{C},\mathrm{H})\cup f(\mathrm{H},\mathrm{H},\mathrm{C},\mathrm{H},\mathrm{H})),\mathrm{H})\cup f(\mathrm{H},\mathrm{H},(\mathrm{C}\cup f(\mathrm{H},\mathrm{C},\mathrm{H})\cup f(\mathrm{H},\mathrm{H},\mathrm{C},\mathrm{H},\mathrm{H})),\mathrm{H},\mathrm{H})]] \\
&= (f(\mathrm{H},(\mathrm{C}\cup f(\mathrm{H},\mathrm{C},\mathrm{H})\cup f(\mathrm{H},\mathrm{H},\mathrm{C},\mathrm{H},\mathrm{H})),\mathrm{H})\cup f(\mathrm{H},\mathrm{H},(\mathrm{C}\cup f(\mathrm{H},\mathrm{C},\mathrm{H})\cup f(\mathrm{H},\mathrm{H},\mathrm{C},\mathrm{H},\mathrm{H})),\mathrm{H},\mathrm{H})] \\
&\subseteq (f(\mathrm{H},\mathrm{C},\mathrm{H})\cup f(\mathrm{H},\mathrm{H},\mathrm{C},\mathrm{H},\mathrm{H})].
\end{aligned}$$

It follows that $\mathrm{H}=\mathrm{M}_{int}\cup(\mathrm{H}-\mathrm{M}_{int})\subseteq(\mathrm{C}\cup f(\mathrm{H},\mathrm{C},\mathrm{H})\cup f(\mathrm{H},\mathrm{H},\mathrm{C},\mathrm{H},\mathrm{H})]$. It implies that $m\in\mathrm{H}$, then there exists $m_i\in\mathrm{C}$ such that $\mathrm{M}_m\preccurlyeq\mathrm{M}_{m_i}$. Hence by Lemma 2.2, C is a lateral hyperbase of H.

IV. CONCLUSION

This paper is a contribution to the study of covered hyperideals and hyperbases. In this paper, we have introduced lateral hyperbases and covered lateral hyperideals of ordered ternary semihypergroups and defined the relation between them. Some further work can be done on hyperbases and covered hyperideals based on the results of this paper.

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