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Free free scattering theory in circularly polarized laser field for polarized potential

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ABSTRACT

In this paper work, we have investigated scattering of an electron by hydrogen atoms in the presence of the Circularly Polarized (CP) laser field. In our present work we have included the polarization effect of laser field on hydrogen atom and effect of the resulted polarized potential on differential scattering cross section is studied. Since we assumed the scattered electrons to have initially 20 eV kinetic energy, this permitted us to treat the scattering process in first order Born Approximation. The scattering electron was described by Volkov wave function. The differential scattering cross section decreases with the increase in scattering angle, for a fixed value of a laser parameters and kinetic energy of an incident electron. So from this study we concluded that non linear differential scattering cross section depends upon the scattering angle and kinetic energy of the incident electrons also we calculated the differential scattering cross-section area for scattering electron.

Key word: Volkov wave function, differential scattering cross-section, circular polarization, polarized potential and photon

I. INTRODUCTION

Electron atom interaction in the presence of a laser field attracted considerable theoretical attention in the recent years not only because of the importance in applied areas (such as plasma heating or laser driven fusion), but also in view of their interest in fundamental atomic theory. The problem of this process, is in general, very complex, since in addition to the difficulties associated with the treatment of electron atom collision, the presence of the laser introduces new parameters (for example, the laser photon energy $\hbar\omega$ and intensity I) which may influence the collision. Moreover, the laser photon can play the role of a "third body" during the collision, and "dressed" the atomic states [1]. It is therefore of interest to begin the theoretical analysis by considering the simpler problem of the scattering of an electron by a potential in the presence of a laser field. A fully realistic description of the target atom is quite difficult. We shall represent it here by a potential model. The laser field of a monochromatic infinite plane wave, linearly polarized, in the dipole approximation, the plane wave assumption is not critical, as the extension to the single mode laser pulse of adiabatically varying intensity can subsequently be made. At high intensities atomic transition abundantly involves multi-photon absorption and emission. The description by perturbation theory is no longer valid, and new method of solution of the Schrödinger equation are needed [2]. A non perturbative theory was developed earlier by Kroll and Watson for low frequency regime [3] is well suited for the range of the intense IR laser. In the following we shall present our theory for the low frequency regime. We shall mainly deal with the case of electron atom collision in the radiation field also termed free-free transitions. It is the purpose of the present work to investigate free-free transition on a hydrogen atom for a CP laser field and the target dressing by radiation field is ignored. We shall first describe the formalism and then apply it to the case of a polarized potential.

The free -free process can theoretically be studied at various levels. As the target does not change states in this process, its own energy spectrum can be ignored and a simple potential can mimic the electron atom interaction. Furthermore, the collision can be treated as occurred at such intensities of electromagnetic field that the electron-field coupling is the dominant process and the target is transparent to the field such that photon-target coupling can be ignored. Here we discussed such intensity of the electromagnetic field where the photon-field interaction can be neglected.[4]

II.MATERIALS AND METHODS

We consider free-free transition for scattering of an electron by the potential [5]

$$V(\mathbf{r},t) = \mathrm{V(r)} + \propto_s \frac{\vec{r}.\vec{E}}{\mathrm{r}^3} \tag{1}$$

which describes a hydrogen atom in a laser field. $V(r)$ denotes the potential:

$$V(r) = -e^{-2r}\left(1+\frac{1}{r}\right) \tag{2}$$

And $\propto_s$ is the static polarizability ($\propto_s = 4.5$ a.u. for hydrogen in its ground state). The second term in equation (1) describes approximately the interaction between the electron and the atomic dipole moment induced by the field.

For circularly polarized electric field in the dipole approximation,

$$\vec{E}(t) = i\frac{E_0}{2}[\exp(-i\omega t)\ \vec{\epsilon} - \exp\ (\ i\omega t)\ \vec{\epsilon}^*] \tag{3}$$

where $\vec{\epsilon}$ and $\vec{\epsilon}^*$ are polarization vectors. On expanding the exponential functions

$$\vec{E}(t) = \frac{E_0}{\sqrt{2}}\left\{\cos(\omega t)\frac{i[\vec{\epsilon} - \vec{\epsilon}^*]}{\sqrt{2}} + \sin(\omega t)\frac{[\vec{\epsilon} + \vec{\epsilon}^*]}{\sqrt{2}}\right\} \tag{4}$$

Defining two unit vectors along two orthogonal directions in the polarization plane:

$$\hat{e_j} = -\frac{i[\vec{\epsilon} - \vec{\epsilon}^*]}{\sqrt{2}} \quad \text{and} \quad \hat{e_i} = \frac{[\vec{\epsilon} + \vec{\epsilon}^*]}{\sqrt{2}} \tag{5}$$

So, equation (4.), can be written as:

$$\vec{E}(t) = \frac{E_0}{\sqrt{2}}[\hat{e_i}(\sin\omega t) - \hat{e_j}(\cos\omega t)] \tag{6}$$

where E_0 is the amplitude and ω is the frequency of the electric field, $\hat{e_i}$ and $\hat{e_j}$ are the unit vectors along two orthogonal directions in the polarization plane and $\vec{A}$ is the vector potential representing the laser field for a circularly polarized wave. Here, we take $\vec{A}$ in spatially independent form because we have taken Coulomb gauge. In Coulomb gauge,

$$\vec{E} = -\frac{1}{c}\frac{\partial\vec{A}}{\partial t} \Rightarrow \frac{\partial\vec{A}}{\partial t} = -\frac{1}{c}\vec{E}(t)$$

Substituting for $\vec{E}(t)$ and integrating, we get

$$\vec{A} = A_0[\hat{e_i}(\cos\omega t) + \hat{e_j}(\sin\omega t)] \tag{7}$$

where, $A_0 = cE_0/(\omega\sqrt{2})$. Here, we assume that the laser field is circularly polarized. Using equation (7), the Volkov wave function becomes

$$\chi(\vec{r},t) = (2\pi)^{\frac{-3}{2}}\exp(i\vec{k}\cdot\vec{r})$$

$$\times\ \exp\left(-\frac{i}{\hbar}\left\{\int_{-\infty}^{t}\frac{P^2}{2m} + \frac{e}{mc}A_0[\hat{e_i}(\cos\omega t) + \hat{e_j}(\sin\omega t)]\cdot\vec{P} + V(\vec{r},t)dt\right\}\right)$$

$$= (2\pi)^{\frac{-3}{2}}\exp(i\vec{k}\cdot\vec{r}) \times\ \exp\left(-i\frac{E_k}{\hbar}t - \frac{i}{\hbar}\frac{eA_0}{mc}\left\{\int[\hat{e_i}(\cos\omega t) + \hat{e_j}(\sin\omega t)]\cdot\vec{P}\,dt\right\} - \frac{i}{\hbar}\int V(\vec{r},t)dt\right)$$

For an electron of kinetic energy E_k and momentum $\vec{k}$ the volkov solution reads as

$$= (2\pi)^{\frac{-3}{2}}\exp\left\{i\vec{k}\cdot\vec{r} - \frac{iE_k t}{\hbar}\right\}\times\ \exp\left\{-i\alpha_o[\hat{e_i}\sin\omega t - \hat{e_j}\cos\omega t]\cdot\vec{k} - \frac{i}{\hbar}\int V(\vec{r},t)dt\right\}$$

where,

$$\alpha_0 = \frac{eA_0}{mc\omega} = \frac{e}{mc\omega}\frac{cE_0}{\omega\sqrt{2}} = \frac{1}{\sqrt{2}}\frac{eE_0}{m\omega^2}$$

Now, defining

$$\vec{\alpha}(t) = \alpha_0[\hat{e_i}\sin\omega t - \hat{e_j}\cos\omega t] \quad (8$$

Where $\vec{\alpha}(t)$ represents the classical oscillation of the electron in the electric field $\vec{E}(t)$ and $\alpha_0 = \frac{E_0}{\omega^2}$

$$\chi(\vec{r},t) = (2\pi)^{\frac{-3}{2}} \exp\left[\left(i\vec{k}.\vec{r} - i\vec{k}\cdot\vec{\alpha}(t)\right) - \frac{iE_k t}{\hbar} - \frac{i}{\hbar}\int V(\vec{r},t)dt\right] \tag{9}$$

This is volkov wave function.

In the first-order Born approximation, the S-matrix element corresponding to the scattering of the electron is

$$S = \frac{-i}{\hbar}\int_{-\infty}^{\infty} dt < \chi^{*}_{\vec{k}_f} | V(\vec{r},t) | \chi_{\vec{k}_i} > \tag{10}$$

Where $\chi_{\vec{k}_i}$and $\chi^{*}_{\vec{k}_f}$ are volkov solution

Substituting for $\chi_{\vec{k}_f}$ and $\chi_{\vec{k}_i}$, we get

$$S = \frac{-i}{\hbar(2\pi)^3}\int_{-\infty}^{\infty}\int \exp\left(-i\vec{k}_f\cdot\vec{r} + i\vec{k}_f\cdot\vec{\alpha}(t) + iE_{kf}\frac{t}{\hbar}\right)\left\{V(r) + \propto_s \frac{\vec{r}\cdot\vec{E}}{r^3}\right\}$$
$$\times \exp\left(i\vec{k}_i\cdot\vec{r} - i\vec{k}_i\cdot\vec{\alpha}(t) - iE_{ki}\frac{t}{\hbar}\right) d^3r dt$$

$$= \frac{-i}{\hbar(2\pi)^3}\int_{-\infty}^{\infty}\int \exp\{-i(\vec{k}_f - \vec{k}_i)\cdot\vec{r}\}\exp\{i(\vec{k}_f - \vec{k}_i)\cdot\vec{\alpha}(t)\} \times \exp\left\{i(E_{kf} - E_{ki})\frac{t}{\hbar}\right\}V(r)d^3r\,dt$$

$$+\frac{-i}{\hbar(2\pi)^3}\int_{-\infty}^{\infty}\int \exp\{-i(\vec{k}_f - \vec{k}_i)\cdot\vec{r}\}\exp\{i(\vec{k}_f - \vec{k}_i)\cdot\vec{\alpha}(t)\} \times \exp\left\{i\left(E_{kf} - E_{ki}\right)\frac{t}{\hbar}\right)\right\}\left\{\propto_s \frac{\vec{r}\cdot\vec{E}}{r^3}\right\}d^3r\,dt$$

Let the momentum transfer of the scattered electron be $\vec{q} = \vec{k}_i - \vec{k}_f$. Then,

$$S = \frac{-i}{\hbar(2\pi)^3}\int_{-\infty}^{\infty}\int \exp\{i\vec{q}\cdot\vec{r}\}\exp\{-i\vec{q}\cdot\vec{\alpha}(t)\}\exp\left\{i(E_{kf} - E_{ki})\frac{t}{\hbar}\right\} \times V(r)\,d^3r\,dt$$

$$+\frac{-i}{\hbar(2\pi)^3}\int_{-\infty}^{\infty}\int \exp\{i\vec{q}\cdot\vec{r}\}\exp\{-i\vec{q}\cdot\vec{\alpha}(t)\}\exp\left\{i(E_{kf} - E_{ki})\frac{t}{\hbar}\right\} \times \left\{\propto_s \frac{\vec{r}\cdot\vec{E}}{r^3}\right\}d^3r\,dt \tag{11}$$

The first part of equation (11 is

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$$S_{if}^{B1} = \frac{-i}{\hbar(2\pi)^3}\int_{-\infty}^{\infty}\int \exp\{i\vec{q}\cdot\vec{r}\}\exp\{-i\vec{q}\cdot\vec{\alpha}(t)\}\exp\left\{i\left(E_{kf}-E_{ki}\right)\frac{t}{\hbar}\right\}V(r)\,d^3r\,dt$$

$$= f_{el}^{B1}(q)\frac{i}{\hbar(2\pi)^2}\int_{-\infty}^{\infty}\exp\{-i\vec{q}\cdot\vec{\alpha}(t)\}\exp\left\{i\left(E_{kf}-E_{ki}\right)\frac{t}{\hbar}\right\}dt$$

$$= f_{el}^{B1}(q)\frac{i}{\hbar(2\pi)^2}\int_{-\infty}^{t}dt\exp\left\{-i\vec{q}.\frac{e}{\sqrt{2}}\alpha_0\left[\hat{e}_i(\sin\omega t)-\hat{e}_j(\cos\omega t)\right]\right\}\quad\times\exp\left\{i\left(E_{kf}-E_{ki}\right)\frac{t}{\hbar}\right\} \qquad (12)$$

where

$$f_{el}^{B1}(q) = -\frac{1}{2\pi}\int \exp\,(i\vec{q}.\vec{r})V(\vec{r})]d^3r$$

is elastic scattering amplitude. Here,

$$i\vec{q}.\frac{e}{\sqrt{2}}\alpha_0\left[\hat{e}_i(\sin\omega t)-\hat{e}_j(\cos\omega t)\right] = \frac{ie}{\sqrt{2}}\alpha_0\left[\vec{q}.\hat{e}_i(\sin\omega t)-\vec{q}.\hat{e}_j(\cos\omega t)\right]$$

Accordingly, the following notations are introduced

$$\cos\phi_q = \frac{\vec{q}.\hat{e}_i}{\sqrt{(\vec{q}.\hat{e}_i)^2+(\vec{q}.\hat{e}_j)^2}},\quad \sin\phi_q = \frac{\vec{q}.\hat{e}_j}{\sqrt{(\vec{q}.\hat{e}_i)^2+(\vec{q}.\hat{e}_j)^2}},$$

$$R_q = \frac{e}{\sqrt{2}}\alpha_0\sqrt{(\vec{q}.\hat{e}_i)^2+(\vec{q}.\hat{e}_j)^2}\quad\text{and}\quad \phi_q = arctg\left(\frac{(\vec{q}\cdot\hat{e}_j)}{(\vec{q}\cdot\hat{e}_i)}\right)+l\pi \qquad (13)$$

Where ϕ_q is the dynamical phase and l is an integer

so

$$i\vec{q}.\frac{e}{\sqrt{2}}\alpha_0\left[\hat{e}_i(\sin\omega t)-\hat{e}_j(\cos\omega t)\right] = iR_q\left[\cos\,\phi_q(\sin\omega t)-\sin\,\phi_q(\cos\omega t)\right]$$

$$= iR_q\sin(\omega t-\phi_q)$$

So,

$$S_{if}^{B1} = f_{el}^{B1}(q)\frac{i}{\hbar(2\pi)^2}\int_{-\infty}^{t}dt\exp\{-iR_q\sin(\omega t-\phi_q)\}\exp\left\{i\left(E_{kf}-E_{ki}\right)\frac{t}{\hbar}\right\} \qquad (14)$$

We know that the generating function of the Bessel Polynomial is (Watson,)[6]

$$e^{ix\sin\phi} = \sum_{-\infty}^{\infty} J_n(x)\, e^{ni\phi}$$

Therefore,

$$\exp\{-iR_q \sin(\omega t - \phi_q)\} = \sum_{N=-\infty}^{\infty} J_N(\mathcal{R}_q)\, e^{(-iN\omega t)} e^{(iN\phi_q)}$$

So equation (14) becomes

$$S_{if}^{B1} = f_{el}^{B1}(q) \frac{i}{\hbar(2\pi)^2} \sum_{N=-\infty}^{\infty} J_N(\mathcal{R}_q)\, e^{(iN\phi_q)} \int_{-\infty}^{t} dt\, e^{(-iN\omega t)} \exp\left\{i\left(E_{kf} - E_{ki} - N\hbar\omega\right)\frac{t}{\hbar}\right\}$$

In atomic unit, $\hbar = 1$. Therefore,

$$S_{if}^{B1} = f_{el}^{B1}(q) \frac{i}{\hbar(2\pi)^2} \sum_{N=-\infty}^{\infty} J_N(\mathcal{R}_q)\, e^{(iN\phi_q)} \hbar \int_{-\infty}^{t} d\left(\frac{t}{\hbar}\right) \exp\left\{i\left(E_{kf} - E_{ki} - N\hbar\omega\right)\frac{t}{\hbar}\right\}$$

$$= f_{el}^{B1}(q) \frac{i}{2\pi} \sum_{N=-\infty}^{\infty} J_N(\mathcal{R}_q)\, e^{(iN\phi_q)} \delta\left(E_{k_f} - E_{k_i} - N\hbar\omega\right) \qquad (15)$$

Here, we have used

$\delta(x - x') = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ik(x-x')} dk$ [7]

Again, the second part of equation (11) is

$$S_{if}^{B2} = \frac{-i}{\hbar(2\pi)^3} \iint_{-\infty}^{t} \exp\{i\vec{q}.\vec{r}\} \exp\{-i\vec{q}.\vec{\alpha}(t)\} \exp\left\{i\left(E_{kf} - E_{ki}\right)\frac{t}{\hbar}\right\}\left\{\propto_s \frac{\vec{r}\cdot\vec{E}}{r^3}\right\} d^3r\,dt$$

Here,

$$\propto_s \frac{\vec{r}\cdot\vec{E}}{r^3} = \propto_s \frac{E_0}{r^3}\vec{r}\cdot\left[\exp(-i\omega t)\,\vec{\epsilon} - \exp(i\omega t)\,\vec{\epsilon^*}\right]$$

Therefore,

$$S_{if}^{B2} = \frac{-i}{\hbar(2\pi)^3} \propto_s E_0 \iint_{-\infty}^{t} \exp\{i\vec{q}.\vec{r}\}\exp\{-i\vec{q}.\vec{\alpha}(t)\}\exp\left\{i\left(E_{kf} - E_{ki}\right)\frac{t}{\hbar}\right\}$$

IJARSE
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$$\times\left\{\left[\exp(-i\omega t)\frac{\vec{\mathrm{r}}}{\mathrm{r}^3}\cdot\vec{\in} - \exp(i\omega t)\frac{\vec{\mathrm{r}}}{\mathrm{r}^3}\cdot\vec{\in^*}\right]\right\} d^3 r dt \tag{16}$$

The first part of equation (16) is

$$I_1 = \propto_s E_o \iint_{-\infty}^{t} \exp\{i\vec{q}.\vec{r}\}\exp\{-i\vec{q}.\vec{\alpha}(t)\}\exp\left\{i\left(E_{kf} - E_{ki}\right)\frac{t}{\hbar}\right\}\left\{\exp(-i\omega t)\frac{\vec{\mathrm{r}}}{\mathrm{r}^3}\cdot\vec{\in}\right\} d^3 r dt$$

$$= \propto_s E_o \int \exp\{i\vec{q}.\vec{r}\}\frac{\vec{\mathrm{r}}}{\mathrm{r}^3}\cdot\vec{\in}\, d^3 r$$

$$\times \int_{-\infty}^{t} dt \sum_{N=-\infty}^{\infty} J_N(\mathcal{R}_q)\, e^{(-iN\omega t)} e^{(iN\phi_q)} \exp\left\{i\left(E_{kf} - E_{ki}\right)\frac{t}{\hbar}\right\}\exp(-i\omega t)$$

Where

$$\exp\{-i\vec{q}.\vec{\alpha}(t)\} = \sum_{N=-\infty}^{\infty} J_N(\mathcal{R}_q)\, e^{(-iN\omega t)} e^{(iN\phi_q)}$$

Therefore,

$$I_1 = \propto_s E_0 \int \exp\{i\vec{q}.\vec{r}\}\frac{\vec{\mathrm{r}}}{\mathrm{r}^3}\cdot\vec{\in}\, d^3 r \sum_{N=-\infty}^{\infty} J_N(\mathcal{R}_q)\, e^{(iN\phi_q)} \times \int_{-\infty}^{t} dt \exp\left\{i\left(E_{kf} - E_{ki}\right)\frac{t}{\hbar} - i(N+1)\omega t\right\}$$

$$= \propto_s E_0 \int \exp\{i\vec{q}.\vec{r}\}\frac{\vec{\mathrm{r}}}{\mathrm{r}^3}\cdot\vec{\in}\, d^3 r \sum_{N=-\infty}^{\infty} J_N(\mathcal{R}_q)\, e^{(iN\phi_q)}\hbar \times \int_{-\infty}^{t} d\frac{t}{\hbar}\exp\left\{i\{\left(E_{kf} - E_{ki}\right) - (N+1)\hbar\omega\}\frac{t}{\hbar}\right\}$$

$$= \propto_s E_0 \int \exp\{i\vec{q}.\vec{r}\}\frac{\vec{\mathrm{r}}}{\mathrm{r}^3}\cdot\vec{\in}\, d^3 r \sum_{N=-\infty}^{\infty} J_N(\mathcal{R}_q)\, e^{(iN\phi_q)}\hbar\, 2\pi\, \delta(E_{k_f} - E_{k_i} - (N+1)\hbar\omega)$$

Replacing N by $N - 1$,

$$I_1 = \propto_s E_0 \int \exp\{i\vec{q}.\vec{r}\}\frac{\vec{\mathrm{r}}}{\mathrm{r}^3}\cdot\vec{\in}\, d^3 r \sum_{N=-\infty}^{\infty} J_{N-1}(\mathcal{R}_q)\, e^{i(N-1)\phi_q}\hbar\, 2\pi\, \delta(E_{k_f} - E_{k_i} - N\hbar\omega)$$

Here, the integral

$$\int \frac{\vec{r}\cdot\vec{\in}}{\mathrm{r}^3}\exp\{i\vec{q}.\vec{r}\} d^3 r = -\frac{2\pi \in \cos\theta}{iq} = -2\pi\frac{\vec{q}\cdot\vec{\in}}{iq^2}$$

Hence,

$$I_1 = -(2\pi)^2 \hbar \propto_s E_0 \frac{\vec{q}\cdot\hat{\epsilon}}{q^2} \sum_{N=-\infty}^{\infty} J_{N-1}(\mathcal{R}_q)\, e^{i(N-1)\phi_q} \delta(E_{k_f} - E_{k_i} - N\hbar\omega) \tag{17}$$

The second part of the equation (16) is

$$I_2 = -\propto_s E_0 \iint_{-\infty}^{t} \exp\{i\vec{q}.\vec{r}\} \exp\{-i\vec{q}.\vec{\alpha}(t)\} \exp\left\{i(E_{kf} - E_{ki})\frac{t}{\hbar}\right\} \left\{\exp(i\omega t)\frac{\vec{r}}{r^3}\cdot\vec{\epsilon}^*\right\} d^3r dt$$

$$= -\propto_s E_0 \sum_{N=-\infty}^{\infty} J_N(\mathcal{R}_q)\, e^{(iN\phi_q)} \int \frac{\vec{r}}{r^3}\cdot\vec{\epsilon}^* \exp\{i\vec{q}.\vec{r}\}\, d^3r$$

$$\times \int_{-\infty}^{t} \exp\left\{i\left[(E_{kf} - E_{ki})\frac{t}{\hbar} - (N-1)\omega t\right]\right\} dt$$

Here,

$$\int \frac{\vec{r}\cdot\vec{\epsilon}^*}{r^3} \exp\{i\vec{q}.\vec{r}\} d^3r = \iiint \frac{r\,\vec{\epsilon}^* \cos\theta}{r^3} \exp\{iqr\cos\theta\} r^2 dr \sin\theta\, d\theta\, d\phi = -2\pi \frac{\vec{q}\cdot\vec{\epsilon}^*}{iq^2}$$

Therefore,

$$I_2 = -\propto_s E_0 \sum_{N=-\infty}^{\infty} J_N(\mathcal{R}_q)\, e^{(iN\phi_q)} \left(-2\pi \frac{\vec{q}\cdot\vec{\epsilon}^*}{iq^2}\right) \int_{-\infty}^{t} \exp\left\{i\left[(E_{kf} - E_{ki})\frac{t}{\hbar} - (N-1)\omega t\right]\right\} dt$$

$$= (2\pi)^2 \propto_s E_0 \frac{\vec{q}\cdot\vec{\epsilon}^*}{iq^2} \sum_{N=-\infty}^{\infty} J_N(\mathcal{R}_q)\, e^{(iN\phi_q)} \delta(E_{k_f} - E_{k_i} - (N-1)\omega)$$

Replacing N by $N+1$,

$$I_2 = (2\pi)^2 \propto_s E_0 \frac{\vec{q}\cdot\vec{\epsilon}^*}{iq^2} \sum_{N=-\infty}^{\infty} J_{N+1}(\mathcal{R}_q)\, e^{(iN\phi_q)} \delta(E_{k_f} - E_{k_i} - N\hbar\omega) \tag{18}$$

Thus with the help of the equations (17) and (18), equation (16) becomes

$$S_{if}^{B2} = \frac{-i}{2\pi}\, \delta(E_{k_f} - E_{k_i} - N\omega) e^{(iN\phi_q)} \{\propto_s \frac{E_o}{q} [e^{(-i\phi_q)} \frac{\vec{q}\cdot\vec{\epsilon}}{q^2} J_{N-1}(\mathcal{R}_q) - e^{(i\phi_q)} \frac{\vec{q}\cdot\vec{\epsilon}^*}{q^2} J_{N+1}(\mathcal{R}_q) \tag{19}$$

Thus using equations (17) and (18) in equation (11), we get

$$S = \frac{i}{2\pi}\, \delta\left(E_{k_f} - E_{k_i} - N\omega\right) \times e^{(iN\phi_q)} \left\{ f_{el}^{B1}(q) J_N(\mathcal{R}_q) \right.$$

$$\left. - \propto_s \frac{\epsilon_o}{q} [e^{(-i\phi_q)} \frac{\vec{q}\cdot\vec{\epsilon}}{q^2} J_{N-1}(\mathcal{R}_q) - e^{(i\phi_q)} \frac{\vec{q}\cdot\vec{\epsilon}^*}{q^2} J_{N+1}(\mathcal{R}_q)] \right\}$$

$$= \frac{i}{2\pi}\,\delta\left(E_{k_f} - E_{k_i} - N\omega\right) f_N^{B1} \tag{20}$$

where,

$$f_N^{B1} = e^{(iN\phi_q)}\left\{ f_{el}^{B1}(q) J_N(\mathcal{R}_q) - \propto_s \frac{\epsilon_o}{q}\left[e^{(-i\phi_q)}\frac{\vec{q}\cdot\vec{\epsilon}}{q^2} J_{N-1}(\mathcal{R}_q) - e^{(i\phi_q)}\frac{\vec{q}\cdot\vec{\epsilon}^*}{q^2} J_{N+1}(\mathcal{R}_q)\right]\right\} \tag{21}$$

In the presence of radiation field, the scattered electron gain or loose energy equal to $N\omega$, such that $E_f = E_i + N\omega$ where $E_{i(f)}$ is the initial (final) energy of the projectile and N is the net number of photons exchanged (absorbed or emitted) by the colliding system and the CP field. The energy spectrum of the scattered electron therefore consists of the elastic term, corresponding to $N = 0$ and a number of sidebands, each pair of sidebands corresponding to the same value of $|N|$.

In order to calculate differential cross-section

We have

$$S = \frac{i}{2\pi}\,\delta(E_{k_f} - E_{k_i} - N\omega) f_N^{B1}$$

$$= \frac{i}{2\pi} f_N^{B1}\int_{-\infty}^{t} e^{i(E_{k_f} - E_{k_i} - N\omega)t}\,dt$$

and using mathematical relation as [8]

$$\lim_{\eta\to 0} e^{\eta t} = 1$$

$$\lim_{\eta\to 0}\frac{e^{2\eta t}\,\eta}{\omega_{nl}^2 + \eta^2} = \pi\,\hbar\,\delta(E_n - E_l)$$

or

$$\delta(\hbar\omega_n - \hbar\omega_l) = \frac{1}{\hbar}\,\delta(\omega_n - \omega_l)$$

Using this,

$$\frac{dP_{if}}{dt} = \frac{1}{2\pi^2}|f_N^{B1}|^2\,\delta\left(\omega_{k_f} - \omega_{k_i} - N\omega\right)$$

$$= \frac{1}{2\pi^2}|f_N^{B1}|^2\,\delta(E_{k_f} - \gamma)$$

Where $\gamma = E_{k_i} + N\omega$

$$\frac{dP_{if}}{dt} = \frac{-1}{2\pi^2}|f_N^{B1}|^2\delta\left(\frac{k_f^2}{2m} - \frac{k^2}{2m}\right) = \frac{-1}{2\pi^2}|f_N^{B1}|^2\frac{m}{k}\delta(k_f - k)$$

Now, the total transition from an incident momentum state k_i into a solid angle $d\Omega$ is

$$W = \sum_k \frac{dP_{if}}{dt}$$

$$= \frac{m}{(2\pi)^4}|f_N^{B1}|^2 d\Omega \int \delta(k_f - k)kdk$$

$$= \frac{m}{(2\pi)^4}|f_N^{B1}|^2 k_f d\Omega \tag{22}$$

where

$\sum_k \to \frac{1}{(2\pi)^3}\int d^3k$ and $\int \delta(k_f - k)dk = 1$ for $k = k_f$, k is the propagation wave vector and K normalazation is used.

Using $d\sigma = W/j_{\text{inc}}$ with $j_{\text{inc}} = \hbar k_i/m$, we get

$$d\sigma = \frac{m}{(2\pi)^4}|f_N^{B1}|^2 k_f \frac{m}{\hbar k_i} d\Omega = \frac{m^2}{(2\pi)^4}\frac{k_f}{k_i}|f_N^{B1}|^2 d\Omega \tag{23}$$

Then dynamical phase ϕ_k is defined as

$$\exp(i\phi_q) = \frac{\vec{q}\cdot\vec{\epsilon}}{|\vec{q}\cdot\vec{\epsilon}|}$$

so, we can rewrite f_N^{B1} as

$$f_N^{B1} = e^{(iN\phi_q)}\left\{f_{el}^{B1}(q)J_N(\mathcal{R}_q) - \propto_s \frac{\epsilon_0}{q}\left[e^{(-i\phi_q)}\frac{\vec{q}\cdot\vec{\epsilon}}{q}J_{N-1}(\mathcal{R}_q) - e^{(i\phi_q)}\frac{\vec{q}\cdot\vec{\epsilon}^*}{q}J_{N+1}(\mathcal{R}_q)\right]\right\}$$

$$= e^{(iN\phi_q)}\left\{f_{el}^{B1}(q)J_N(\mathcal{R}_q) - \propto_s \frac{\epsilon_0}{q^2}|\vec{q}\cdot\vec{\epsilon}|[J_{N-1}(\mathcal{R}_q) - J_{N+1}(\mathcal{R}_q)]\right\}$$

Now, using

$$J'_N(\mathcal{R}_q) = \frac{[J_{N-1}(\mathcal{R}_q) - J_{N+1}(\mathcal{R}_q)]}{2}$$

we get

$$f_N^{B1} = e^{(iN\phi_q)}\{f_{el}^{B1}(q)J_N(\mathcal{R}_q) - 2 \propto_s \frac{E_0}{q^2}|\vec{q}\cdot\vec{\epsilon}|J'_N(\mathcal{R}_q)\} \quad (24)$$

Thus equation (23) becomes

$$\frac{d\sigma_N^{CP}}{d\Omega} = \frac{m^2}{(2\pi)^4}\frac{k_f}{k_i}\left|\{f_{el}^{B1}(q)J_N(\mathcal{R}_q) - 2 \propto_s \frac{E_0}{q^2}|\vec{q}\cdot\vec{\epsilon}|J'_N(\mathcal{R}_q)\}\right|^2 \quad (25)$$

Where J_n denotes a basal funtion of an order N and F_{el}^{Bl} is the elastic transition amplitude in the first Born approximation for the static potential

Where polarized potential is given by

$$V(r) = -\frac{\alpha_s}{2(r^2+d^2)^2}$$

Therefore,

$$f_{el}^{B1}(q) = \frac{-\alpha_p}{2iq}\frac{\pi iq}{2d}e^{-qd}, \qquad q > 0$$

$$= \frac{-\alpha_p}{4d}e^{-qd} \text{ [9]}$$

III. RESULT AND DISCUSSION

In the present thesis work, we have studied the elastic scattering of an electron-atom interaction by absorbing photons from the Circularly Polarized (CP) laser field. We have considered hydrogen atom and effect of polarized potential in scattering is studied by considering high electron energy of 20 eV and laser field of moderate intensities i.e. $E_0 = 10^7$ V/cm and $\omega = 0.54$ a.u.

We have calculated the differential scattering cross section as;

$$\frac{d\sigma_N^{CP}}{d\Omega} = \frac{m^2}{(2\pi)^4}\frac{k_f}{k_i}\left|\{f_{el}^{B1}(q)J_N(\mathcal{R}_q) - 2 \propto_s \frac{E_o}{q^2}|\vec{q}\cdot\vec{\epsilon}|J'_N(\mathcal{R}_q)\}\right|^2$$

From the relation it is clear that non linear differential scattering cross section depends on number of photons (N) and the momentum transfer of incident electron (q).In atomic unit mass of electron is considered unity.

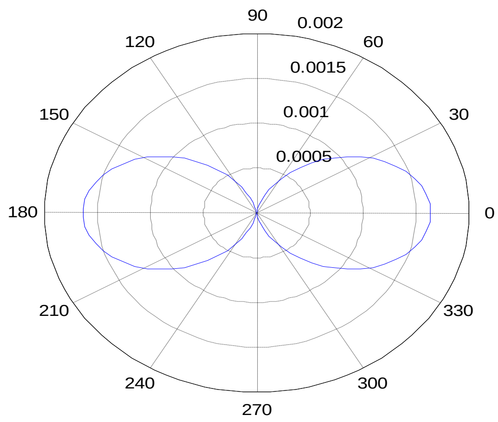

Figure:1 Polar plot

It shows that the area of cross-section becomes maximum at 0^0 and equal to 0.0017 m^2 and minimum at 90^0

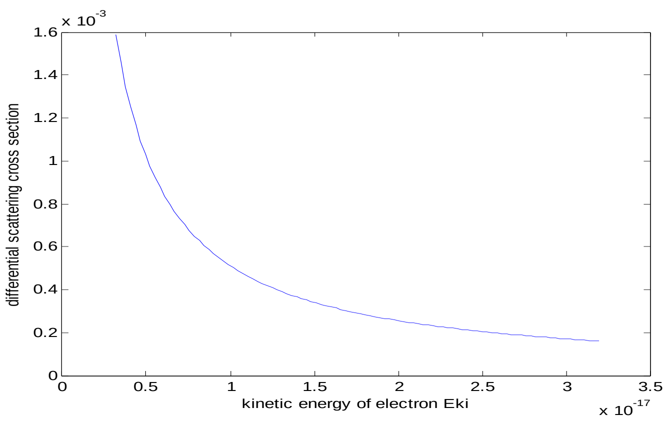

Figure:2 Differential scattering with kinetic energy of the incident electron

From the plot it is clear that when the kinetic energy is equal to one joule differential scattering cross section is equal to 1.5×10^{-3} m^2/Sr is maximum while the kinetic energy equal 0.4 joule, differential cross-section becomes minimum and increases (asymptotically).

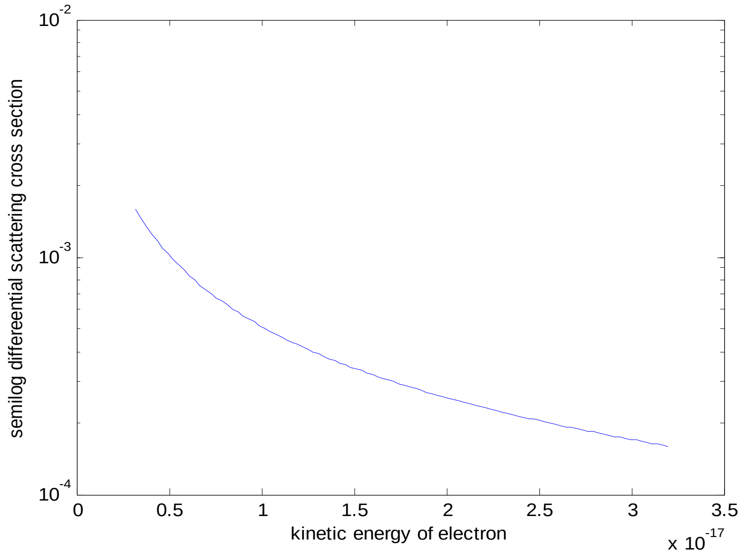

Figure:3 Variation semi log differential cross-section with the kinetic energy of incident electron

From the plot we see that increasing the kinetic energy of the electron implies that decrease the scattering cross section area.

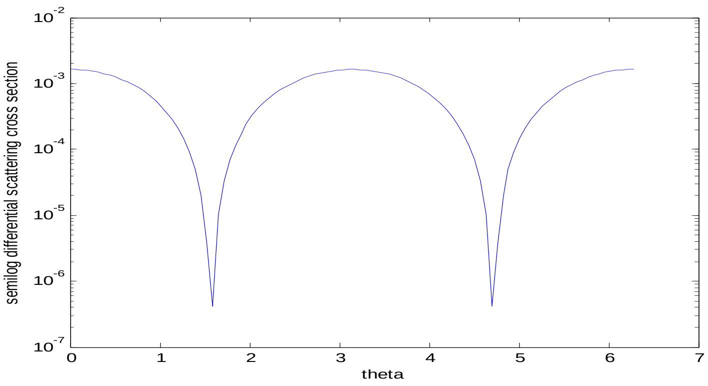

Figure:4 Variation semi log differential with angle theta

From the plot we see that symmetric scattering cross-section area log (10^{-3}) and maximum at angle 3 radian.

IV.CONCLUSION

It generally observed that when electrons are scattered from the atom in the presence of a laser field, a new effect is observed which are not accessible in beings processes in which three subsystem are present (i) the electron (ii) the target atom (iii) the radiation field. The last one provided energy and momentum and is characterized by the polarization of its electric field, which introduces in this collision process a new physical axis.

In this thesis work, we investigated scattering of an electron by hydrogen atom in the presence of circularly polarized (CP) laser field. In our present work we have included the polarization effect of laser field on hydrogen atom and effect of the resulted polarized potential on differential scattering cross-section is studied. Since we assumed the scattered electros to have initially 20 eV kinetic energy, this permitted us to treat the scattering process in the first order Born approximation. The scattering was described by Volkov wave function, during the derivation of differential scattering cross-section.

It shows that the area of cross-section becomes maximum at 0^0 and equal to 0.0017 m^2 and minimum at 90^0,differential cross-section decreases to zero, furthering increase of scattering angle, differential scattering cross-section increases in backward direction up to 180^0 where kinetic energy of electrons is equal to 20 eV and the photon energy is equal to 0.17 eV on the other hand ,the differential cross-section obtained in our result for circular polarization is equal to 1.7×10^{-3} m^2 . It is greater than the result calculated by Gabgriela Buica(2016). [10]

REFERENCES

[1] N. J. Mason, (1993). Laser-assisted electron-atom collisions. *Reports on Progress in Physics*, *56*, 1275.,1993

[2] F. H. M. Faisal, (1987). *Theory of Multiphoton Processes* (Plenum Press, New York,1986

[3] N.M. Kroll, and K.M. Waston,. Phys. Rev. A, 8, 804, 1973

[4] Byron Jr, F. W., Francken, P., & Joachain, C. J. (1987). Laser-assisted elastic electron-atom collisions. *Journal of Physics B: Atomic and Molecular Physics*, *20*(20), 5487.

[5] H. Qui-Bio, and Jing-Feng Sun Laser-Assisted Elastic Electron scattering From Argon, Theor. Phys.(Beijing, China) 51, 2009, pp 9131-134).

[6] G. N. Watson,. *A treatise on the theory of Bessel functions*. Cambridge university press. 1995.

[7] C .Harper. Introduction to Mathematical physics, Department of Physics California State University, Hayward, Prentice Hall of India Pvt. Ltd, New Delhi-110 001 ,2006

[8] E. Merzbacher,. -Quantum Mechanics, Third edition, John Wiley and Sons, Inc, 1998

[9] S. Balasubramian,. Oxford and IBH Publishing Co. New Delhi, Bombay, Calcutta, 1985

[10] G Buica, (2016), Polarization dependence in elastic scattering of electron by hydrogen atom a circularly polarized laser field, Journal of quantities spectroscopy radiative transfer 187, 190 2003