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System Simplification using Simplified Routh Approximation Method (SRAM) and Factor Division

Shikha Tyagi [1], C.B. Vishwakarma[2]

[1]*Ginni Devi Modi Girls (PG) College, Modi Nagar (U.P)*

[2]*School of Engineering, Gautam Buddha University, Greater Noida (India)*

ABSTRACT

A mixed simplification method for linear dynamic systems via reducing the order of its transfer function is proposed in this paper. The reduced denominator polynomial of the simplified system is computed using Simplified Routh Approximation Method (SRAM), while the coefficients of numerator are obtained through factor division algorithm. The proposed method is easy to understand and capable to match transient and steady-state value of the high order original system. The proposed algorithm is illustrated through one example.

Keywords: Factor Division, Order Reduction, SRAM, Stability, System Simplification, Transfer Function.

I. INTRODUCTION

In many engineering simulations, one can obtain a complex model of a linear dynamic system for analytic considerations and controller design purposes. The complex systems are not easy to understand as well as controller design is also quite cumbersome. Therefore, it becomes essential to simplify the dynamic models for analysis and controller design purposes. The various simplification methods for system simplification have been suggested in the last few decades. But, still research is going on to identify more effective methods for simplification.

A good number of simplification methods [1-8] in frequency as well as time domain are available in the literature. The order reduction based on Routh approximation method [9] is available in which $'\alpha'$ and $'\beta'$ parameters are computed to get denominator and numerator of the simplified model. But in SRAM, only $'\alpha'$ parameters are required to synthesize the reduced model. The reduced denominator of simplified is determined by only $'\alpha'$ parameters and numerator is computed through factor division algorithm.

II. PROBLEM STATEMENT

Consider alarge-scale systemhaving its transfer function of the order $'n'$ as

$$G(s) = \frac{b_0 + b_1 s + b_2 s^n + ... + b_{n-1} s^{n-1}}{a_0 + a_1 s + a_2 s^2 + ... + a_n s^n} \quad (1)$$

Where a_i and b_i are known scalar constants.

Thecorresponding k^{th} -order simplified model isto be determined as

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{B_0 + B_1 s + B_2 s^2 + ... + B_{k-1} s^{k-1}}{A_0 + A_1 s + A_2 s^2 + ... + A_k s^k} \quad (2)$$

The objective is to find simplified model $R_k(s)$ from the original system $G(s)$ using the proposed method of the system simplification.

III. PROPOSED SIMPLIFICATION ALGORITHM

Thedescription of the proposed methodologyis given as under, which consists of the following two steps.

STEP-1: Computation of the reduced denominator polynomial of the simplified model.

To overcome the problems associated to model order reduction methods based on Routh approximations of the system, a new simplified method SRAM [10] is used here to get reduced denominator polynomial of the reduced simplified model. The $'\alpha'$parameters from denominators polynomial of the original system, are calculated as

$\alpha -$ **Table of SRAM [10]**

$$\left.\begin{array}{ll}
\alpha_1 = \dfrac{a_0}{a_1} & \begin{cases} a_0 & a_1 & a_2 & \cdots\cdots \\ a_1 & a_2 & a_3 & \cdots\cdots \end{cases} \\
\alpha_2 = \dfrac{c_1}{c_2} & \begin{cases} c_1 & c_2 & c_3 & \cdots\cdots \\ c_2 & c_3 & c_4 & \cdots\cdots \end{cases} \\
\alpha_3 = \dfrac{d_1}{d_2} & \begin{cases} d_1 & d_2 & d_3 & \cdots\cdots \\ d_2 & d_3 & d_4 & \cdots\cdots \end{cases} \\
\alpha_4 = \dfrac{e_1}{e_2} & \begin{cases} e_1 & e_2 & e_3 & \cdots\cdots \\ e_2 & e_3 & e_4 & \cdots\cdots \end{cases} \\
\vdots & \\
\vdots &
\end{array}\right\} \quad (3)$$

For 'i 'odd

$$\left.\begin{array}{ll}
c_i = a_i & \\
d_1 = c_2 & \\
d_i = a_{i+1} & (i = 3,5,....) \\
e_1 = d_2 & \\
\vdots &
\end{array}\right\} \quad (4)$$

For 'i' even

$$\left.\begin{aligned} c_i &= a_i - (a_0 a_{i+1})/a_1 \\ d_i &= c_{i+1} - (c_1 c_{i+2})/c_2 \\ e_i &= d_{i+1} - (d_1 d_{i+2})/d_2 \\ &\vdots \end{aligned}\right\} \quad (5)$$

Finally,

$$D_k(s) = s^k + \frac{(\alpha_k.\alpha_{k-1}......\alpha_1)}{a_0}\sum_{j=0}^{k-1} a_j s^j$$

STEP-2: Computation of the numerator of simplified model using factor division algorithm [11].

The $D_k(s)$ is already obtained in Step-1and $N_k(s)$ is determined as

$$G(s) = \frac{N(s)\times D_k(s)}{D(s)\times D_k(s)} = \frac{N(s)\times D_k(s)/D(s)}{D_k(s)} \quad (6)$$

Therefore, $N_k(s)$ of the simplified model will be defined as

$$\frac{N(s)\times D_k(s)}{D(s)} = \frac{\sum_{i=0}^{n+k-1} p_i s^i}{\sum_{j=0}^{n} a_j s^j} \text{ (about } s=0\text{)} \quad (7)$$

This can be done by using Routh recurrence formula given as follows:

$$\left.\begin{aligned} \beta_0 &= \frac{p_0}{a_0} && \left\langle \begin{matrix} p_0 & p_1 & \cdots & p_{k-1} \\ a_0 & a_1 & \cdots & a_{k-1} \end{matrix} \right. \\ \beta_1 &= \frac{q_0}{a_0} && \left\langle \begin{matrix} q_0 & q_1 & \cdots & q_{k-2} \\ a_0 & a_1 & \cdots & a_{k-2} \end{matrix} \right. \\ &\vdots \\ \beta_{k-2} &= \frac{u_0}{a_0} && \left\langle \begin{matrix} u_0 & u_1 \\ a_0 & a_1 \end{matrix} \right. \\ \beta_{k-1} &= \frac{v_0}{a_0} && \left\langle \begin{matrix} v_0 \\ a_0 \end{matrix} \right. \end{aligned}\right\} \quad (8)$$

Where

$$\left.\begin{array}{ll} q_i = p_{i+1} - \beta_0 a_{i+1} & i = 0,1,2,\dots k-2 \\ r_i = q_{i+1} - \beta_1 a_{i+1} & i = 0,1,2,\dots k-3 \\ \vdots & \\ v_0 = u_1 - \beta_{k-2} a_1 & \end{array}\right\} \quad (9)$$

Hence, reduced numerator can be written as

$$N_k(s) = \beta_0 + \beta_1 s + \beta_2 s^2 + \dots + \beta_{k-1} s^{k-1} \quad (10)$$

IV. RESULTS AND COMPARISON

The simplification method is tested on one system taken from literature. The simplified reduced models are obtained and graphically compared with the original system. The performance index known as integral square error (ISE) [12] is computed with the help of MATLAB/ simulink model in order to check the effectiveness of the proposed method.

The ISE is defined as

$$\text{ISE} = \int_0^\infty [x(t) - x_k(t)]^2 \, dt \quad (11)$$

Where the terms $x(t)$ and $x_k(t)$ are known as step responses of the large-scale systemand simplified model respectively. The smaller value of ISE indicates the better simplified model.

Example-1: Consider an8th –order system taken from literature [4].

$$G(s) = \frac{N(s)}{D(s)}$$

$$N(s) = 19.82s^7 + 429.261s^6 + 4843.8098s^5 + 45575.892s^4 + 241544.75s^3$$
$$+905812.05s^2 + 1890443.1s + 842597.95$$
$$D(s) = s^8 + 30.41s^7 + 358.4295s^6 + 2913.8638s^5 + 18110.567s^4 + 67556.98s^3$$
$$+173383.58s^2 + 149172.19s + 37752.826$$

The values of $\alpha-$parameters are calculated using algorithm given as above

$$\alpha_1 = 0.25308$$
$$\alpha_2 = 0.95448$$

Using Step-1, thereduced denominator polynomial of 2nd –order simplified system is computed as

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$$D_2(s) = s^2 + \frac{\alpha_1\alpha_2}{a_0}(a_0 + a_1 s)$$
$$= s^2 + 0.954472s + 0.24156$$

Using Step-2, 'β'-coefficients can be obtained as

$$\beta_0 = 5.3913 \quad \left\langle \begin{array}{ll} 203537.9608 & 1260891.5852 \\ 37752.826 & 149172.19 \end{array} \right.$$
$$\beta_1 = 12.0960 \quad \left\langle \begin{array}{l} 456659.5572 \\ 37752.826 \end{array} \right.$$

Hence, numerator polynomial of the simplified models is obtained as

$$N_2(s) = \beta_0 + \beta_1 s$$

$$= 5.3913 + 12.0960s$$

Finally, 2^{nd} –order simplified model is obtained as

$$R_2(s) = \frac{5.3913 + 12.0960s}{0.24156 + 0.954472s + s^2}$$

The graphical comparison of 2^{nd}-order simplified model with original high order system is shown in Fig-1. The ISE error index is also computed between the time responses using MATLAB/Simulink model and given in Table-1.

From Table-1, it can be noted that proposed method generates 2^{nd} –order model, which is far better than the 4^{th} –order models obtained by other two methods.

From Fig-1, It is clear that simplified model is closely matching the response of the original large-scale system and also it is seen from Table-1 that proposed method is better in performance in comparison to few other methods.

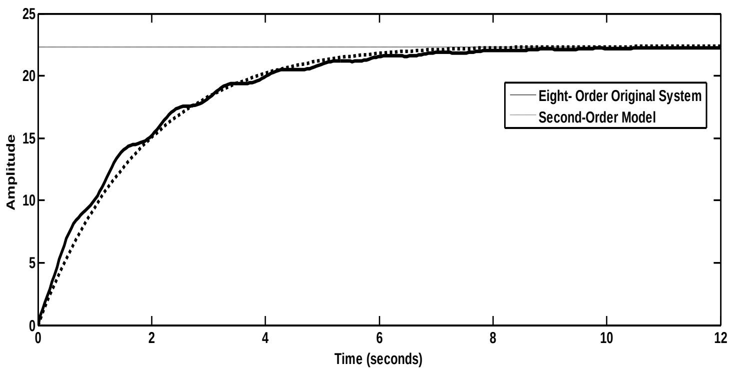

Figure-1 Step Response Comparison

Table-1: Comparison with others methods

Method	Simplified Model	ISE
Proposed Method	$R_2(s) = \dfrac{5.3913 + 12.0960s}{0.24156 + 0.954472s + s^2}$	2.755
Jay Singh [13]	$R_4(s) = \dfrac{19.82s^3 + 18.79s^2 + 724.80s + 1170.5}{s^4 + 9.702s^3 + 23.51s^2 + 122s + 52.45}$	13.29
J. Pal [4]	$R_4(s) = \dfrac{61.27s^3 + 242.8s^2 + 2390s + 3153}{s^4 + 12.78s^3 + 42.77s^2 + 268.2s + 141.3}$	41.54

V. CONCLUSION

The authors suggested a mixed simplification method using SRAM and factor division algorithm. The denominator of the simplified model is derived through simplified Routh approximation method (SRAM), in which only $'\alpha'$-parameters are required and numerator is found using factor division algorithm. The advantage of the algorithm is to retain stability in the simplified models. The algorithm has been illustrated on one high-order system. The step response of simplified model is graphically compared with the original high order system. The graphical comparison in Fig.1shows that simplified model is very near to the original system. The error comparison is given in Table-1 for 2^{nd} –order simplified model and this error is even lesser than 4^{th} –order models obtained by Jay Singh [13] and J. pal [4].

This simplification method can be used on multi-inputs multi-outputs (MIMO)linear time-invariant systems as well.

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