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NON-SEPARABLE REGULARIZATION BASED DE-CONVOLUTION

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ABSTRACT

A standard convex approach for sparse one-dimensional de-convolution improves upon L1-norm regularization. We propose a sparsity -inducing non-separable non-convex bivariate penalty function for this purpose. It is designed to enable the convex formulation of ill- conditioned linear inverse problems with quadratic data fidelity terms. The new penalty overcomes limitations of separable regularization. We show how the penalty parameters should be set to ensure that the objective function is convex, and provide an explicit condition to verify the optimality of a prospective solution. In this project, We present an algorithm (an instance of forward-backward splitting) for sparse de-convolution using the new penalty terms.

***Key Words-* De-Convolution, Convex Functions, Sparse Signal Estimation, Non-Convex Regularization.**

I. INTRODUCTION

In this literature we are going to deal with sparse regularization. This sparse regularization can be categorized into two types. One is convex and other one is non-convex. In this project we are using the term regularization terms is called as penalty functions. In the convex approach, the regularization terms or penalty functions are convex. The convex method consisting of both data fidelity and regularization terms. So that the objective function is convex [1], [2]. By using this convex approach method we have several advantages i.e., the objective function is free of extraneous (irrelevant) local minima, and globally convergent optimization algorithms can be improved [3].

The Non-convex regularization has more advantages [4], [5], [6] comparative to convex regularization . Classical and recent examples of non-convex method is edge preserving tomography [7], [8], [9], [10] and compressed sensing [11], [12], [13], respectively. By using these techniques the non-convex approach performs better than convex one's. In the non-convex approach, penalty functions are non-convex. Because these non-convex functions are designed to induce sparsity more effectively than convex one's. therefore the convexity of the objective function is generally sacrificed. Non-convex regularization is having some complications that is the objective function will generally posses many sub-optimal local minima in which optimization algorithms can become entrapped.

It turns out, without giving up the convexity of the objective function and corresponding benefits. The non-convex penalties can be utilized. This is achieved by carefully specifying the penalty in accordance with the data fidelity term, as described by Blake, Zimmerman, and Nikolova [14], [15], [9], [10]. In recent work, a class of sparcity-inducing non-convex penalties has been developed to formulate convex objective functions and applied to several signal estimation problems [16], [17], [18], [19], [20], [21], [22], [23], [24]. This approach maintains the benefits of the convex framework (absence of spurious local minima, etc.), yet estimates sparse signals more accurately than convex regularization (e.g., the l_1 norm) due to the sparsity –inducing properties of non-convex regularization. However , this previous work considers only *separable* (additive) penalties, which have fundamental limitations.

In this paper, we introduce a parameterized sparsity-inducing non separable non-convex bivariate penalty function. To enable the convex formulation of ill-conditioned linear inverse problems with quadratic data fidelity terms the penalty function is designed. The new penalty overcomes the limitations of separable non-convex regularization. In this paper, now we show how the penalty parameters should be set to ensure the objective function is convex. And we also show how this bivariate penalty can be incorporated into linear inverse problems of N variables $(N \succ 2)$, and then we provide an explicit condition to verify the optimality of a prospective solution. For sparse signal reconstruction using the new penalty we present an iterative algorithm (as instance of forward-backward splitting), and we demonstrate it's effectiveness for one-dimensional sparse de-convolution.

A. *Notation*

We write the vector $x \in R^N$ as $x = \left(x_1, x_{2,....,} x_N\right)$. Given $x \in R^N$, we define $x_n = 0$ for $n \notin \{1, 2, ..., N\}$. (This simplifies expressions involving summations over n.) The l_1 norm of $x \in R^N$ is defined as $\| x \|_1 = \sum_n | x_n |$. If the matrix A is positive semi definite, we write $A \geq 0$. If the $A - B$ is positive semi definite, we write $A \geq B$.

II. SPARSE RECONSTRUCTION

In signal processing the practical problems are involve far more than two variables. Therefore, the proposed bivariate penalty and convexity condition are of little practical use on their own. In this section we show how they can be used to solve an N - point linear inverse problem (with $N \succ 2$). We consider the problem of estimating a signal $x \in R^N$ given y ,

$$y = Hx + w \tag{1}$$

Where H is a known linear operator, x is known to be sparse, and w is additive white Gaussian noise (AWGN). we formulate the estimation of x as an optimization problem with bivariate sparse regularization (BISR),

$$x^{\wedge} = \arg\min_{x \in R^N} \{F(x) = \frac{1}{2} \| y - Hx \|_2^2 + \frac{\lambda}{2} \sum_n \psi((x_{n-1}, x_n); a)\} \tag{2}$$

Where $\lambda \succ 0$, $a = (a_1, a_2)$ and $\psi : R^2 \to R$ is the proposed bivariate penalty. In the penalty term, the first and last signal value pairs, (x_0, x_1) and (x_N, x_{N+1}), straddle the end-points of x. C, we define $x_n = 0$ for $n \notin \{1, 2, ..., N\}$, which simplifies subsequent notation.

If $a_1 = a_2$, then the bivaiate penalty is separable, i.e., $\psi(u; a) = \phi(u_1; a_1) + \phi(u_2; a_1)$, and the N-point penalty term in (2) reduces to $\lambda \sum_n \phi(x_n; a_1)$. Hence, we recover the standard (separable) formulation of sparse regularization. In particular, if $a_1 = a_2 = 0$, then $\psi(u; 0) = |u_1| + |u_2|$ and the N-point penalty term reduces to $\lambda \| x \|_1$, i.e., the classical sparsity-inducing convex penalty.

In order to induce sparsity more effectively, we allow ψ to be non-separable; i.e., $a_1 \neq a_2$. To that end, the following section addresses the problem of how to set a_1 and a_2 in the bivariate penalty ψ to ensure convexity of the N - variate objective function F in (2).

The lemma is proven in appendix D. According to the lemma, it is sufficient to restrict ψ so as to ensure convexity of the bivariate function f in (3). Therefore , the allowed penalty parameters a_i can be determined from the tridiagonal matrix P. using lemma 1, we obtain theorem 1.

A. Optimality Condition

In this section, we derive an explicit condition to verify the optimality of a prospective minimize of the objective function F in (2). The optimality condition is also useful for monitoring the convergence of an optimization algorithm (see the animation in the supplemental material).

The general condition to characterize minimiers of a convex function is expressed in terms of the sub differential. If F is convex, then $x^{opt} \in R^N$ is a minimizer if and only if $0 \in \partial F(x^{opt})$ where ∂F is the sub differential of F.

We seek an expression for the sub differential of the objective function F. The function F in (2) has a regularization term that is non-differentiable, non-convex, and non-separable. But using , we may write the regularization term as

$$\frac{1}{2}\sum_{n}\psi\left(\left(x_{n-1},x_{n}\right);a\right) \tag{3}$$

$$=\frac{1}{2}\sum_{n}[S\left(\left(x_{n-1},x_{n}\right);a\right)+\left\|\left(x_{n-1},x_{n}\right)\right\|_{1}] \tag{4}$$

$$=\left\|x\right\|_{1}+\frac{1}{2}\sum_{n}S\left(\left(x_{n-1},x_{n}\right);a\right) \tag{5}$$

Where $x_n = 0$ for $n \notin \{1,2,...,N\}$. We define $\Theta : R^N \rightarrow R$ as

$$\Theta\left(x,a\right)=\frac{1}{2}\sum_{n}S\left(\left(x_{n-1},x_{n}\right);a\right) \tag{6}$$

The function Θ is twice continuously differentiable because it is the sum of twice differentiable functions. Using (5), we ,may express the objective function F in (2) as

$$F\left(x\right)=\frac{1}{2}\left\|y-Hx\right\|_{2}^{2}+\lambda\Theta\left(x;a\right)+\lambda\left\|x\right\|_{1} \tag{7}$$

The benefit of (7) compared to (2) is that the regularization term (which is non-differentiable, non-convex, and non- separable) is separated into a differentiable part and a convex separable part. The Θ term is differentiable and its gradient is easily evaluated. The l_1 norm is separable and convex and its sub differential is easily evaluated.

The gradient of Θ is given by

$$\left[\nabla\Theta\left(x;a\right)\right]_{n}=\frac{1}{2}S_{1}\left(\left(x_{n},x_{n+1}\right);a\right)+\frac{1}{2}S_{2}\left(\left(x_{n-1},x_{n}\right);a\right) \tag{8}$$

Where S_i is the partial derivative of $S\left(\left(x_1,x_2\right)\right)$ with respect to x_i. The sub differential of the l_1 norm is separable [11],

$$\partial\left\|x\right\|_{1}=sign\left(x_{1}\right)\times\ldots\times sign\left(x_{N}\right) \tag{9}$$

Where sign is the set-valued signum function

$$sign(t) := \begin{cases} \{1\}, & t \succ 0 \\ [-1,1], & t = 0 \\ \{-1\}, & t \prec 0 \end{cases} \tag{10}$$

Since the first two terms of (7) are differentiable, the sub differential of F is

$$\partial F(x) = H^T(Hx - y) + \lambda \nabla \Theta(x;a) + \lambda \partial \| x \|_1 . \tag{11}$$

Hence the condition $0 \in \partial F(x^{opt})$ can be expressed as

$$(1/\lambda) H^T (y - Hx^{opt}) - \nabla \Theta(x^{opt};a) \in \partial \| x^{opt} \|_1 \tag{12}$$

Expressing this condition component-wise, we have the following result.

B. Sparse Deconvolution

We apply theorem 1 to the sparse deconvolution problem. In this case, the linear operator H represents convolution, i.e.,

$$[Hx]_n = \sum_k h_{n-k} x_k \tag{13}$$

That is, H is a Toeplitz matrix. It represents a linear time-invariant (LTI) system with frequency response given by the Fourier transform of h,

$$H(w) = \sum_n h_n e^{-jwn} \tag{14}$$

Similarly, the matrix P in (2a) represents an LTL system with a real-valued frequency response,

$$P(w) = p_1 e^{-jw} + p_0 + p_1 e^{jw} \tag{15}$$

$$= p_0 + 2p_1 \cos(w) \tag{16}$$

Specializing theorem 1 to the problem of de-convolution, we have the following results.

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III. FIGURES

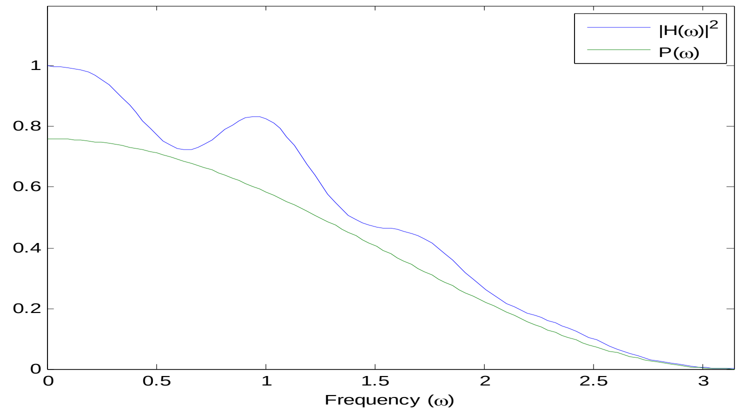

Fig.1. Filters $H(w)$ and $P(w)$ for example 2.

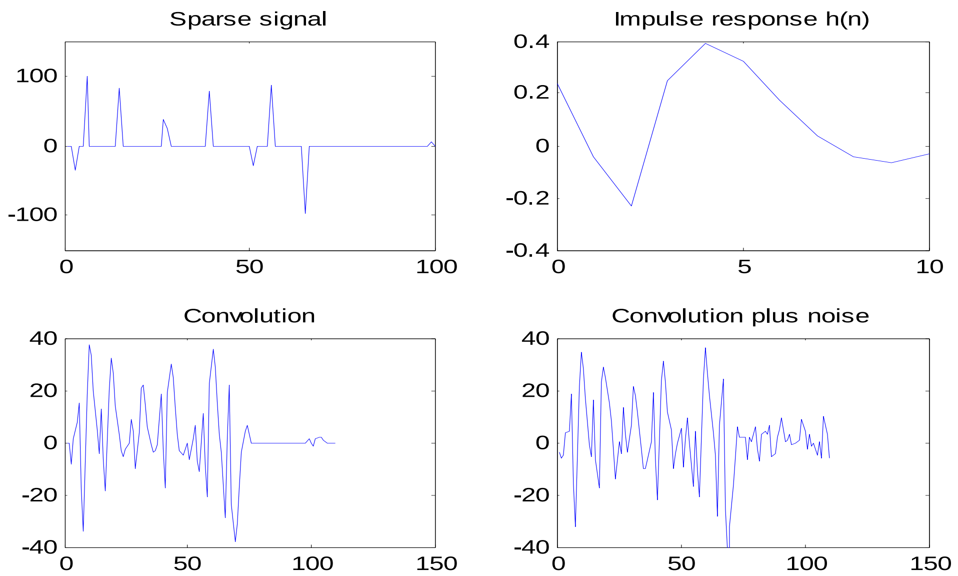

Fig. 2. Example 1 of sparse deconvolution using BISR

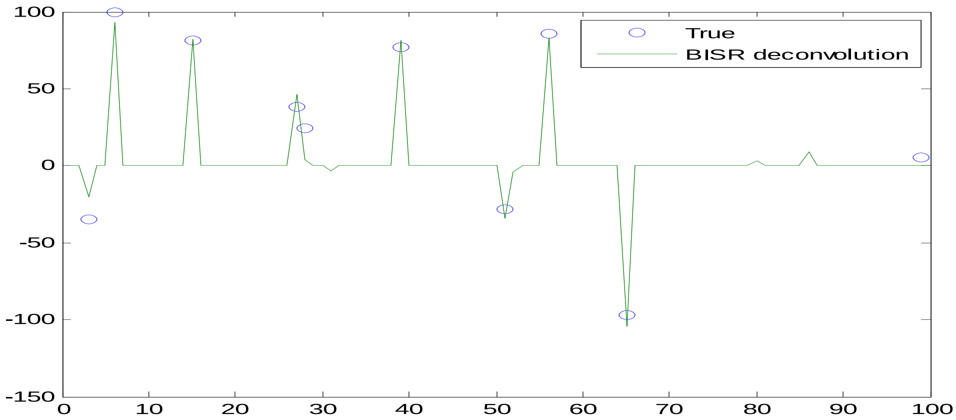

Fig .3. example 2 of sparse deconvolution using BISR.

IV. COMPARISION TABLE

Algorithm	RMSE	No of iterations
l_1 norm De-convolution	4.70	32
Proposed De-convolution	3.11	22

V. CONCLUSION

The results and comparison table show that Proposed De-convolution gives RMSE value better than l_1 norm De-convolution. The signal can be exactly or more approximated estimated using the proposed de-convolution.

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