

# ERGODIC CHANNEL CAPACITY OF MULTIPLE INPUT MULTIPLE OUTPUT SYSTEM

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**ABSTRACT**

To design & implement the Ergodic channel with Gaussian for the different number of transmitter and receiver. Irrespective of that the channel capacity is identify without knowing its channel state information for increased number of Transmitter & Receivers. The better SNR provides with better faster data transmission. Channel capacity improves with increased transmitter & Receiver.

**Keywords:** Ergodic, MIMO, channel capacity

**I. INTRODUCTION**

The random variable said to be Ergodic if, the mean of the random variable should be constant. The channel matrix  $H \in \mathbb{C}^{N_r \times N_t}$  has singular value decomposition  $H = U \Sigma P^H$ . Where  $U \in \mathbb{C}^{N_r \times N_r}$  and  $P \in \mathbb{C}^{N_t \times N_t}$  are unitary matrices. The Channel Capacity (CC) of a multiple antennas with  $N_r \times N_t$  increased by factor of  $\min(N_t \text{ and } N_r)$  without using additional transmits power.

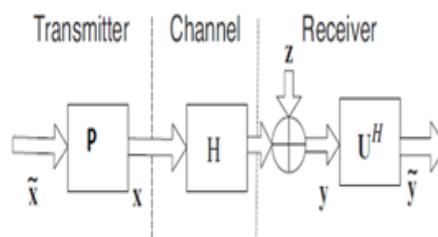
**II. SYSTEM MODEL**

In the deterministic MIMO channel system with [2]  $N_t$  &  $N_r$  receive antennas represented by  $N_r \times N_t$  the channel matrix  $H \in \mathbb{C}^{N_r \times N_t}$ . now consider the transmitter symbol vector  $x \in \mathbb{C}^{N_t \times 1}$ , which is calm of  $N_t$  independent input symbols  $x_1, x_2, x_3, x_4, x_5, \dots, x_{N_t}$ .

$$y = \sqrt{\frac{E_x}{N_t}} Hx + z$$

Where  $z = z_1, z_2, z_3, z_4, \dots, z_{N_r} \in \mathbb{C}^{N_r \times 1}$  is a noise vector which is assumed to be a symmetric Gaussian with zero mean & unit variance (0, 1) [4].

The noise vector is a circular symmetric with the same distribution as z for any  $\theta$ . The auto-correlation of transmitted vector is  $R_{xx}(t, t + \tau) = E\{XX^H\}$



**Fig1. Decomposition data when CSI at TX**

### III. PROPOSED MIMO CHANNEL

The CC of a deterministic channel is defined as  $C = \max I(x;y) \text{ bits/channel use}$

Where  $f_x(x)$  is the probability density function of the transmitted vector  $x$  [3]. The mutual information of the two continuous random vectors  $x$  and  $y$  is given by  $I(X; Y) = H(Y) - H(Y/X)$  (1)

$H\left(\frac{Y}{X}\right) = H(Z)$ . Then,

$$I(X; Y) = H(Y) - H(Z) \tag{2}$$

#### 3.1. Design considerations

The auto correlation matrix of the  $y$  random variable is

$$R_{yy}(\tau) = E\{YY^H\} \tag{3}$$

$$\begin{aligned} &= E\left\{\left(\sqrt{\frac{E_x}{N_t}} Hx + Z\right)\left(\sqrt{\frac{E_x}{N_t}} x^H H^H + Z^H\right)\right\} \\ &= E\left\{\frac{E_x}{N_t} Hxx^H H^H + ZZ^H\right\} \\ &= \frac{E_x}{N_t} E\{Hxx^H H^H + (ZZ^H)\} \\ &= \frac{E_x}{N_t} HE\{xx^H\}H^H + E\{ZZ^H\} \\ &= \frac{E_x}{N_t} HR_{xx}(\tau)H^H + E\{ZZ^H\} \\ &= \frac{E_x}{N_t} HR_{xx}(\tau)H^H + R_{zz}(\tau) \end{aligned}$$

The total transmitted power limited to

$$E\{x^H x\} = \sum_{i=1}^{N_t} E\{|x_i|^2\} = N_t \tag{4}$$

#### 3.1. Channel capacity when CSI is at Transmitter

Now CSI is available at the transmitter side [1] signal decomposition, in which a transmitted signal [7] is pre processed with the  $P$  in the transmitter and then a received signal is post processed with  $U^H$  in the receiver.

Where  $R_{zz}(\tau) = I_N N_0$

$$= \frac{E_x}{N_t} HR_{xx}(\tau)H^H + I_N N_0 \tag{5}$$

$E_x$  is the energy of transmitted signal

$N_0$  is the power spectral density

Then the mutual information of  $Y$  and  $Z$  is

$$H(y) = \log_2 \{\det(\pi e R_{yy}(\tau))\} \tag{6}$$

$$H(z) = \log_2 \{\det(\pi e R_{zz}(\tau))\} \tag{7}$$

$$\text{So, } H(z) = \log_2 \{\det(\pi e I_N N_0)\} \tag{8}$$

$H(y)$  is differential energy. Then,

$$I(X; Y) = \log_2 \det \left( I_{N_r} + \frac{E_x}{N_t N_0} H R_{xx}(\tau) H^H \right) \text{bps/Hz} \quad (9)$$

Now the CC [2] of deterministic MIMO channel is expressed as

$$C = \max \log_2 \det \left( I_N + \frac{E_x}{N_t N_0} H R_{xx}(\tau) H^H \right) \text{bps/Hz} \quad (10)$$

The output signal receiver can be,

$$\tilde{y} = \sqrt{\frac{E_x}{N_t}} U^H H P \tilde{x} + \tilde{z} \quad (11)$$

$$\tilde{y} = \sqrt{\frac{E_x}{N_t}} \Sigma \tilde{x} + \tilde{z} \quad (12)$$

Where,  $\tilde{z} = z U^H$ ,  $U$  is a unitary matrix

### 3.2. Channel capacity when CSI is not at Transmitter

$H$  is not known then, the transmitted vector is  $R_{xx}(\tau) = I_{N_t}$ ,

$$\text{CC is } c = \log_2 \det \left( I_{N_r} + \frac{E_x}{N_t N_0} H H^H \right)$$

$$c = \sum_{i=1}^r \log_2 \left( 1 + \frac{E_x}{N_t N_0} \lambda_i \right) \quad (13)$$

Assume,  $\sum_{i=1}^r \lambda_i = \xi$ , if  $r=N$  then  $\lambda_i = \frac{\xi}{N}$

CC for  $N$  parallel Channel

$$c = N \log_2 \left( 1 + \frac{E_x \xi}{N_t N_0} \right) \quad (14)$$

### 3.3. Channel capacity for MIMO channel

MIMO deterministic randomly channel matrix. In practice channel assume Ergodic channel [9].

In MIMO channel changes randomly.  $H$  is a random matrix, which means that its CC [11] changes randomly. MIMO CC can be given by its time average. The random channel is an Ergodic process.

$$\bar{C} = E\{C(H)\} \quad (15)$$

$$= E \left\{ \max \log_2 \det \left( I_N + \frac{E_x}{N_t N_0} H R_{xx}(\tau) H^H \right) \right\}$$

### 3.4. MIMO channel capacity

The MIMO CC is specified by a sum of the capacities of the virtual single input single output channel [12], i.e.

$$C = \sum_{i=1}^r c_i(\gamma_i) \quad (16)$$

$$= \sum_{i=1}^r \log_2 \left( 1 + \frac{E_x \gamma_i}{N_t N_0} \lambda_i \right)$$

$$= \max \left\{ \sum_{i=1}^r \log_2 \left( 1 + \frac{E_x \gamma_i}{N_t N_0} \lambda_i \right) \right\}$$

$$\gamma_i^{opt} = \left( \mu - \frac{N_t N_0}{E_x \gamma_i} \right) \text{ For } i=1 \dots r-1, r.$$

$$\sum_{i=1}^r \gamma_i^{opt} = N_t$$

Where  $\mu$  is a constant and  $(x)^+$  is defined as

$$(x)^+ = \begin{cases} x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \quad (17)$$

It was solved by water pouring algorithm. The more is power allocation for the mode with higher SNR [5].

### 3.4. Channel capacity for open loop system

Ergodic CC for the open loop system is

$$\bar{C} = E \left\{ \sum_{i=1}^r \log_2 \left( 1 + \frac{E_x}{N_t N_0} \lambda_i \right) \right\} \quad (18)$$

### 3.5. Channel capacity for open loop system

Ergodic CC for the closed loop system is

$$\begin{aligned} \bar{C} &= E \left\{ \max \left\{ \sum_{i=1}^r \log_2 \left( 1 + \frac{E_x}{N_t N_0} \gamma_i \lambda_i \right) \right\} \right\} \\ &= E \left\{ \sum_{i=1}^r \log_2 \left( 1 + \frac{E_x}{N_t N_0} \gamma_i^{opt} \lambda_i \right) \right\} \end{aligned}$$

Where  $\gamma_i^{opt} = \left( \mu - \frac{N_t N_0}{E_x \gamma_i} \right)$  (19)

## IV. SIMULATION RESULTS

The Ergodic CC [6] when CSI is not available at the transmitter side. Initial assumptions are number of iteration 10000, SNR initially at 0db to upto 52db, **min(N<sub>t</sub> and N<sub>r</sub>)**

In Fig2 illustrates that the Ergodic CC with respect to Signal to Noise Ratio (SNR) with five different possible cases. Where observed that these are 1Tx&1Rx, 1Tx. &2Rx, 2Tx&1Rx, 2Tx&2Rx, 4Tx &4Rx. The SNR range is from 0 to up to 52db. Where the SNR at the 26db is 30bps/Hz. Rest of the case it is observed that less capacity 30bps/Hz.

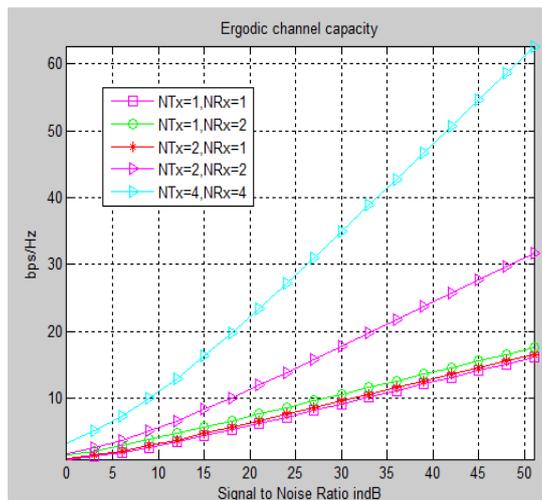


Fig 2.Ergodic CC maximum 4 Tx. & 4 Rx

In Fig3.shows six possible cases simulation response of Ergodic CC. These are 2Tx&2Rx, 2Tx, &4Rx, 4Tx&8Rx, 8Tx, &6Rx, 4Tx&4Rx and 8Tx&8Rx. antennas. It observed that SNR [8] is at 15db 4x8 antennas with capacity  $2.1 \times 10^{-3}$  bps/Hz and 8x8 antenna  $3.41 \times 10^{-3}$  bps/Hz.

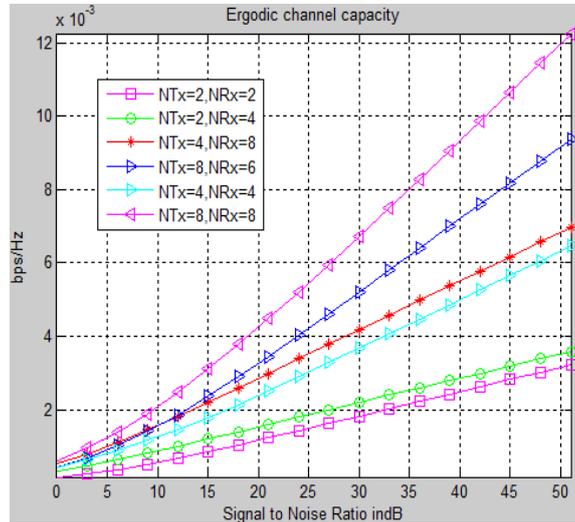


Fig3.Ergodic CC maximum 8Tx. & 8Rx

In Fig4.shows six possible cases simulation response of Ergodic CC. These are 1Tx&1Rx, 1Tx, &2Rx, 2Tx&1Rx, 2Tx&2Rx, 4Tx &4Rx, and 8Tx&8Rx. antennas with  $3 \times 10^{-3}$  bps/Hz and 8Tx. & 8Rx. capacity  $5.9 \times 10^{-3}$  bps/Hz.

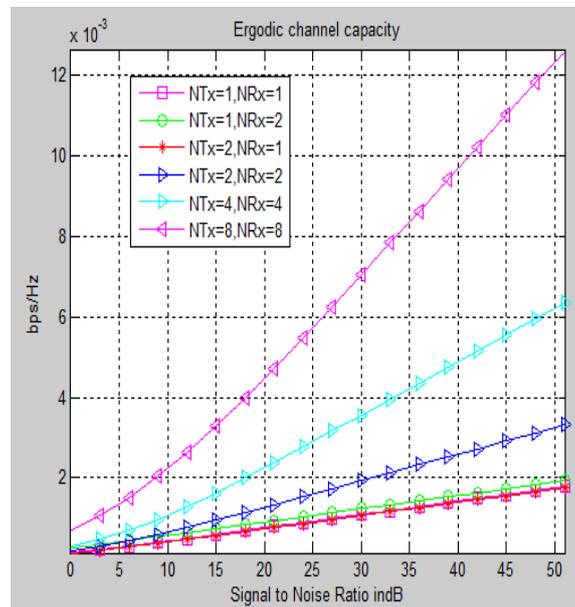


Fig 4.Ergodic CC of maximum 8Tx. & 8Rx

In Fig5.shows six possible cases simulation response of Ergodic CC. These are 2Tx& 8Rx, 2Tx, &16Rx, 4Tx&8Rx, 4Tx, &16Rx, 8Tx &8Rx, and 8Tx&16Rx. antennas. It observed that SNR is at 30db 4x16 antennas with  $4.5 \times 10^{-3}$  bps/Hz and 8x16 antenna  $8.2 \times 10^{-3}$ .

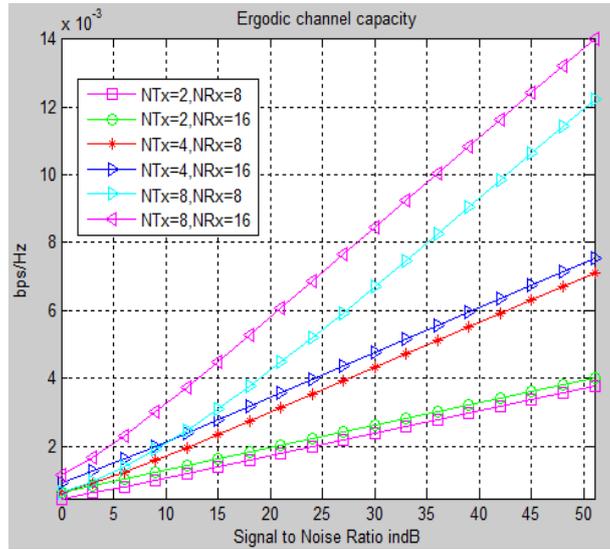


Fig 5 CC maximum 8Tx & 16Rx

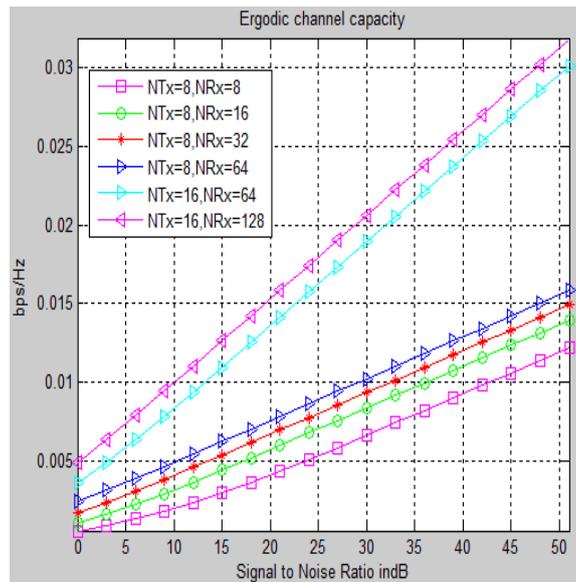


Fig 6. Ergodic CC maximum 16Tx & 128Rx

In Fig6. shows six possible cases simulation response of Ergodic CC. These are 8Tx & 8Rx, 8Tx & 16Rx, 8Tx & 32Rx, 8Tx & 64Rx, 16Tx & 64Rx, and 16Tx & 128Rx. antennas. It observed that SNR is at 20db 8x16 antennas with 0.006bps/Hz and 6x128 antenna 0.015 bps/Hz and 8x8 antennas with  $6.3 \times 10^{-3}$  bps/Hz.

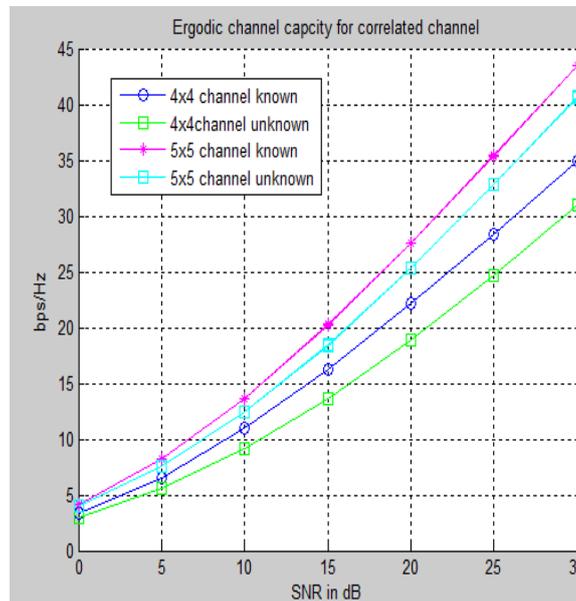


Fig7.channel known vs channel unknown

In illustrates Fig 7.Ergodic CC for correlated channel for case (i) 4x4 channel and case (ii) 5x5 channel for both unknown and known. Where SNR is at 20dB 5x5 unknown channel is at a capacity of 25bps/Hz and for known channel with 28bps/Hz.

## V. CONCLUSION

The number of transmitting & receiving antennas is increased without any increased power levels. The antenna's are improved their channel capacity with respect to the channel matrix.

## REFERENCES

- [1]P.Viswanath,D.N.C.Tse,& R. Laroia, "Opportunistic beam forming using dump antennas," IEEE Trans. Inf. Theory, vol. 48, no. 6, pp. 1277–1294,2002.
- [2]B.Hochwald,T.Marzetta,&V.Tarokh,"Multi-antenna channel-hardening and its implications for rate feedback and scheduling," IEEE Trans. Inf. theory, vol. 50, no. 9, pp. 1893–1909,2004.
- [3]G.Durisi,A.Tarable,C.Camarda,&G.Montorsi,"On the capacity of MIMO Wiener phase-noise channels," in Proc. Inf. Theory Appl, Feb. 2013, pp. 1–7
- [4]N.N.Moghadam,P.Zetterberg,P.Händel,&H. Hjalmarsson, "Correlation of distortion noise between the branches of MIMO transmit antennas," in Proc. IEEE, Sep. 2012, pp. 2079–2084.
- [5]DW.K.Ng, E.S.Lo,&R.Schober,"Energy-efficient resource allocation in OFDMA systems with large numbers of base station antennas," IEEE Trans. Wireless Commun., vol. 11, no. 9, pp. 3292–3304, Sep. 2012.
- [6] H.Q.Ngo,E.G.Larsson,&T.L. Marzetta,"Energy and spectral efficiency of very large multiuser MIMO systems," IEEE Trans. Commun., vol. 61, no. 4, pp. 1436–1449, Apr. 2013.



- [7] W. Zhang, "A general framework for transmission with transceiver distortion and some applications," IEEE Trans. Commun., vol. 60, no. 2, pp. 384–399, Feb. 2012
- [8] X. Gao, O. Edfors, F. Rusek, & F. Tufvesson, "Linear pre-coding performance in measured very-large MIMO channels," in Proc. IEEE VTC Fall, Sep. 2011, pp. 1–5.
- [9] C. Studer & E. G. Larsson, "PAR-aware large-scale multi-user MIMO OFDM downlink," IEEE J. Sel. Areas Commun., vol. 31, no. 2, pp. 303–313, Feb. 2013.
- [10] E. Björnson, M. Kountouris, M. Bengtsson, and B. Ottersten, "Receive combining vs. multi-stream multiplexing in downlink systems with multi-antenna users," IEEE Trans. Signal Process. vol. 61, no. 13, pp. 3431–3446, Jul. 2013.
- [11] N. O'Donoghue and J. M. F. Moura, "On the product of independent complex Gaussians," IEEE Trans. Signal Process., vol. 60, no. 3, pp. 1050–1063, Mar. 2012.
- [12] H. Yin, D. Gesbert, M. Filippou, and Y. Liu, "A coordinated approach to channel estimation in large-scale multiple-antenna systems," IEEE J. Sel. Areas Commun., vol. 31, no. 2, pp. 264–273, Feb. 2013.
- [13] A. M. Tulino and S. Verdú, "Random matrix theory and wireless communications," Found. Trends Commun. Inf. Theory, vol. 1, no. 1, pp. 1–182, 2004.

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