Vibration of C-C-C-C Non-Homogeneous Square Plate with Non-Uniform linearly varying Thickness and Thermal Effect

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ABSTRACT

The present work analyses the vibration behavior of non-homogeneous orthotropic visco-elastic square plate of parabolically varying thickness on the idea of classical plate theory while the all edges are clamped and are subjected to parabolically thermal variation. In this problem, there is a linear variation of thermal effect and thickness i.e. linearly in x- direction and linearly in y -direction. Although, first two modes of vibrations calculated by using Rayleigh- Ritz Technique based on classical plate theory. For calculation purpose latest computer software is used for calculating numerical values of frequency and these results are presented in Graphical form for different values of thermal gradient, taper constant and non-homogeneity constant.

Keywords: vibration, plate, thermal gradient, taper constant, thickness

I. INTRODUCTION

Vibrations can be unwanted or wanted. For example, vibrations in automobiles and aircrafts are undesired because they can result in discomfort to the passengers and structural damage due to fatigue. Free vibration takes place when a system oscillates under the action of forces inherent in the system itself, and when external impressed forces are absent. The system under free vibration will vibrate at one or more of its natural frequencies, which are properties of the dynamic system established by its mass and stiffness distribution.The study in the field of vibration plays an important role in the branch of applied science and engineering. By vibration we mean a movement of the particle of an elastic or rigid body which repeats itself periodically. Vibration is the main cause of collapse in plates with sprung mass. In the modern era, we can not neglect the effect of vibrations as all engineering machines and structures experience vibrations. Since vibrations directly effects the life and work power of the machine, therefore knowledge about the first few modes of vibration is essential and necessary to a mechanical engineer, before finalizing a design.

In previous years, a lot of research has been done in the field of vibration of plates. Vibration of plates having different geometry and behavior such as orthotropic/isotropic, homogeneous/non-homogeneous and either considering or not considering the effect of temperature and thickness variation have been studied by number of

authors. The analysis of temperature dependent vibration of plate is very significant in the design of power plant turbines, nuclear reactors and other structure works at elevated temperature. Thermally induced vibrations of non-uniform plates are also very useful for research workers in aeronautical, chemical and nuclear engineering.
Since modern engineering requires very accurate solutions, therefore the subject matter of the present study has been restrained to a study of vibration problems of non-homogeneous and orthotropic parallelogram plates of uniform and/or inconsistent thickness, having simple and/or mixed boundary conditions taking into account the result of shear deformation under different temperature field. In the industries the materials exposed to high temperature generally deviate from Hooke's law and behaves visco-elastically. The elastic and viscous behavior of material depends mainly on frequency and temperature. Consequently, the vibration analysis has become very important from the point of view of designing a structure to be familiar in advance about its reaction. So that the essential measure to manage the structural vibrations and its amplitudes can be taken.
The objective of the present study is to find the first two modes of vibrations. It is found to be difficult and expensive for an engineer for conducting experiments of plate structure. Although, in accordance with experiment numerical methods yield results. When the dimensions of the plate structure increase they require huge computational capacity. Therefore, development of theoretical and mathematical model for plate structure is an important issue concerning applications of plate structure due to its efficiency and accuracy.

II. MATHEMATICAL FORMULATION OF PLATE

The governing partial differential equation for orthotropic square plate is [1]:

$$D_x \frac{\partial^4 W}{\partial x^4} + 2\,L\,\frac{\partial^4 W}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 W}{\partial y^4} + \rho h \frac{\partial^2 W}{\partial t^2} = 0 \qquad (1)$$

Here $D_x = \frac{E_x l^3}{12(1-\nu_x \nu_y)}, D_y = \frac{E_y l^3}{12(1-\nu_x \nu_y)}$ are flexural rigidity along x and y axis and $D_{xy} = \frac{G_{xy} l^3}{12}$ is torsional rigidity.

$$L = D_1 + 2\,D_{xy}$$

Here, $D_1 = El^3/\,12\,(1-v^2)$ (2)

and corresponding two term deflection is [6]

$$W(x, y) = \eta_1(x, y) + \eta_2(x, y) \qquad (3)$$

where $\eta_1(x, y) = [(x/a)(y/a)(1-x/a)(1-y/a)]^2\, S_1$

$\eta_2(x, y) = [(x/a)(y/a)(1-x/a)(1-y/a)]^3\, S_2$

and S_1, S_2 are constants to satisfy boundary conditions.

(i). Consider two dimensional temperature distribution along x and y-axis as :

$$\Omega = \Omega_0 T(x, y) \qquad (4)$$

where, $T(x,y) = (1 - \frac{x}{a})(1 - \frac{y}{a})$

Where Ω denotes the temperature excess above the reference temperature at any point on the plate and Ω_0 denotes the temperature at any point on the boundary of the plate and 'a' is the length of a side of square plate.

(ii). The temperature dependences of the modulus of elasticity for most of engineering materials can be expressed as:

$$E = E_0 \,(1-\gamma\,\Omega) \qquad (5)$$

Where E is the value of Young's Modulus at reference temperature i.e. $\Omega = 0$ and γ is the slope of the variation of Y with ψ .The modulus variation (5) becomes:

$$E = E_0 \,[\, 1-\alpha(1-\tfrac{x}{a})(1-\tfrac{y}{a}) \,]$$

Modulus of elasticity with time dependence is:

$$E_x(\Omega) = E_1[1-\alpha\Omega],\ E_y(\tau) = E_2[1-\alpha\Omega],\ G_{xy} = G_0[1-\alpha\Omega] \qquad (6)$$

On using temperature distribution along x and y-axis in equation (4) as:

$$\left.\begin{aligned} E_x(\Omega) &= E_1[1-\alpha F(x,y)] \\ E_y(\Omega) &= E_2[1-\alpha F(x,y)] \\ G_{xy}(\Omega) &= G_0[1-\alpha F(x,y)] \end{aligned}\right\} \qquad (7)$$

Where $\alpha = \gamma\Omega_0 (0 \le \alpha < 1)$ is thermal gradient parameter.

(iii). It is assumed that thickness also varies parabolic in x- direction as:.

$$l = l_0 \,[\, 1-\alpha\, H(x,y) \,] \qquad (8)$$

where, $H(x,y) = (1 + \beta_1 \frac{x}{a})\ (1 + \beta_2 \frac{y}{a})$

β_1 and β_2 are taper constants .

Putting the value of E and l from equation (4) and (7) equation (2) , one obtain

$$D_1 = [\, E_0 \,[\, 1-\alpha\, F(x,y) \,]\, l_0^{\,3}\, [H(x,y)]^3 \,/\, 12\,(1-v^2) \qquad (9)$$

(iv). Again assuming the density of non-homogeneous square plate varies linearly in x- direction i.e.

$$\varpi = \varpi_0 \,(1 + \alpha_1 \tfrac{x}{a})$$

III.RAYLEIGH-RITZ METHOD

Rayleigh – Ritz method is apply for an approprate deflection shape is selected and maximum strain and kinetic energy are equated.An equation in the following form is obtained as:

$$\delta(M_E - N_E) = 0 \qquad (10)$$

Limiting condition for the geometry of plate shown as:

$$\left.\begin{aligned} W = W_x = 0 \ \text{at}\ x = 0, a \\ W = W_y = 0 \ \text{at}\ x = 0, a \end{aligned}\right\} \qquad (11)$$

Now, unit less variables having no dimension are using for our convince as:

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$$X = \frac{x}{a}, Y = \frac{y}{a}, \bar{W} = \frac{W}{a}, \bar{l} = \frac{l}{a} \qquad (12)$$

The expression for Kinetic and Strain Energy are :

$$M_E = 0.5 \int_0^1 \int_0^1 \left[D_x \left(\frac{\partial^2 W}{\partial x^2}\right)^2 + D_y \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2D_1 \frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4D_{xy}\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 \right] dx\, dy \qquad (13)$$

By putting the value of D_x, D_y , D_1 and D_{xy} in the equation we get the new equation of strain energy :

$$M_E = 0.5 \int_0^1 \int_0^1 \left[\frac{E_x l^3}{12(1-v^2)} \left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \frac{E_y l^3}{12(1-v^2)} \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2\frac{vE_x l^3}{12(1-v^2)} \frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4\frac{G_{xy} l^3}{12} \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 \right] dx\, dy$$

$$= 0.5 \int_0^1 \int_0^1 \left[\frac{E_x l^3}{12(1-v^2)} \left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \frac{E_y l^3}{12(1-v^2)} \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2\frac{vE_x l^3}{12(1-v^2)} \frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4\frac{E_{xy} l^3}{12(1+v)} \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 \right] dx\, dy$$

$$= 0.5 \frac{l^3}{12(1-v^2)} \int_0^1 \int_0^1 \left[E_x \left(\frac{\partial^2 W}{\partial x^2}\right)^2 + E_y \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2\, v\, E_x \frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4\, E_{xy}\, (1-v)\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 \right] dx\, dy$$

$$= \frac{l_0^3 E_0 a^3}{24(1-v^2)} \int_0^1 \int_0^1 [1-\alpha([T(X,Y)]) \times [H(X,Y)]^3] \left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2v\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4\,(1-v)\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 \right] dx\, dy$$

$$= Q \int_0^1 \int_0^1 [1-\alpha([T(X,Y)]) \times [H(X,Y)]^3] \left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2v\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4\,(1-v)\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 \right] dx\, dy \qquad (14)$$

where $Q = E_0 l_0^3 a^3 / 24(1-v^2)$

and

$$N_E = 0.5\, p^2 \rho\, \bar{l}_0\, a^5 \int_0^1 \int_0^1 T(X,Y)\, \bar{W}^2 dY dX \qquad (15)$$

Substitute the value of M_E and N_E from (13) and (15) in (9), we get:

$$(M_E^* - \lambda^2 N_E^*) = 0 \qquad (16)$$

Where ,

$$M_E^* = \int_0^1 \int_0^1 [1-\alpha([T(X,Y)]) \times [H(X,Y)]^3] \left[\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2v\frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2} + 4\,(1-v)\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 \right] dx\, dy \qquad (17)$$

and

$$N_E^* = \int_0^1 \int_0^1 T(X,Y)\, \bar{W}^2 dY dX \qquad (18)$$

Here , $\lambda^2 = \frac{12 a^4 \rho (1-v_x v_y)}{E_1 l_0^2}$

Eq. (18) contains two unknowns S_1 & S_2 comes after putting equation (11). S_1 & S_2 are to be determined from (16) as:

$$\frac{\partial (M_E^* - \lambda^2 N_E^*)}{\partial S_n} = 0 \qquad \text{for } n = 1,2. \qquad (19)$$

On simplifying (19) ,we get

$$c_{n1} S_1 + c_{n2} S_2 = 0 \qquad \text{for } n = 1,2 \qquad (20)$$

where, c_{n1}, c_{n2} (n = 1,2) involve parametric constant and the frequency parameter. For non-trival solution , the determent of the co-efficient of equation (20) must be zero. So, one gets, the frequency equation as :

$$\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} = 0 \qquad (21)$$

With the help of equation (21), we get a quadratic equation in λ^2 from which the two values of λ^2 can found. These two values represent the first two modes of vibration of frequency i.e. λ_1(first mode) and λ_2(second mode) for different values of taper constant and thermal garedient for a clamped plate.

IV. RESULT AND DISCUSSION

Calculations has been done for frequency of viscoelastic orthotropic square plate for different values of taper constants $\beta_1$and β_2, thermal gradient α and non- homogenous constant α_1 for different points for first two modes of vibrations have been calculated numerically.

In Fig I:- It is clearly seen that value of frequency decreases as value of thermal gradient α increases from 0.0 to 1.0 for $\beta_1 = \beta_2 = \alpha_1 = 0.2$, $\beta_1 = \beta_2 = \alpha_1 = 0.4$ and $\beta_1 = \beta_2 = \alpha_1 = 0.8$ for both modes of vibrations.

In Fig II :- Clearly seen that value of frequency increases as value of taper constant β_1 increases from 0.0 to 1.0 for $\alpha = \beta_2 = \alpha_1 = 0.2$, $\alpha = \beta_2 = \alpha_1 = 0.4$, $\alpha = \beta_2 = \alpha_1 = 0.8$ for both modes of vibrations.

In Fig III :- Increasing value of frequency for both of the modes of vibration is shown for increasing value of taper constant β_2 from 0.0 to 1.0 for $\beta_1 = \alpha_1 = \alpha = 0.2$, $\beta_1 = = \alpha_1 = \alpha = 0.4$ and $\beta_1 = = \alpha_1 = \alpha = 0.8$. Note that value of frequency increased.

In Fig IV :- Sharply, decreasing value of frequency as compared with previous figure for both of the modes of vibration is shown for increasing value of non-homogeneous constant $\alpha_1$from 0.0 to 1.0 for $\beta_1 = \beta_2 = \alpha = 0.2$, $\beta_1 = \beta_2 = \alpha = 0.4$ and $\beta_1 = \beta_2 = \alpha = 0.8$.

Fig .I Frequency vs. Thermal Gradient α

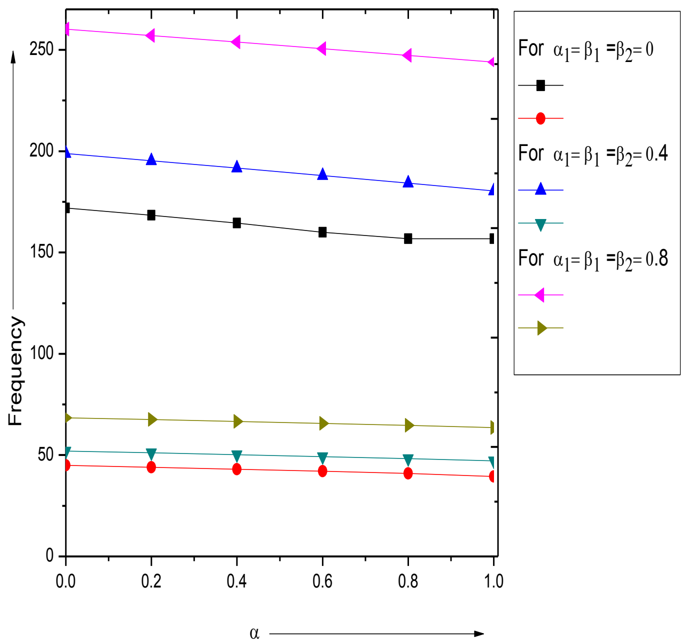

Fig. II Freequency vs. Taper Constant β_1

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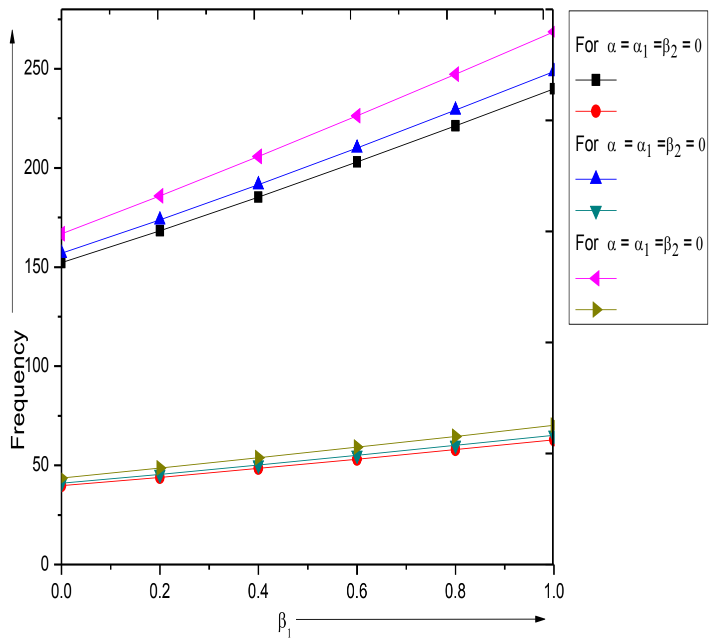


Fig. III Frequency vs. Taper Constant β_2

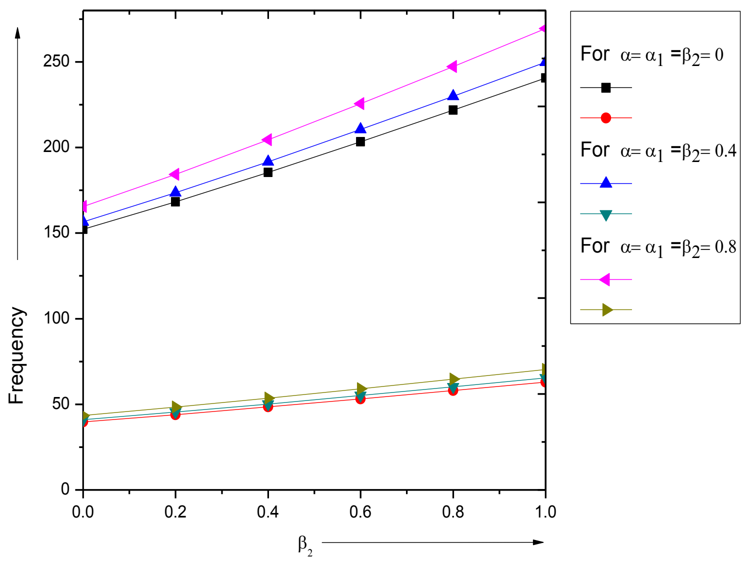
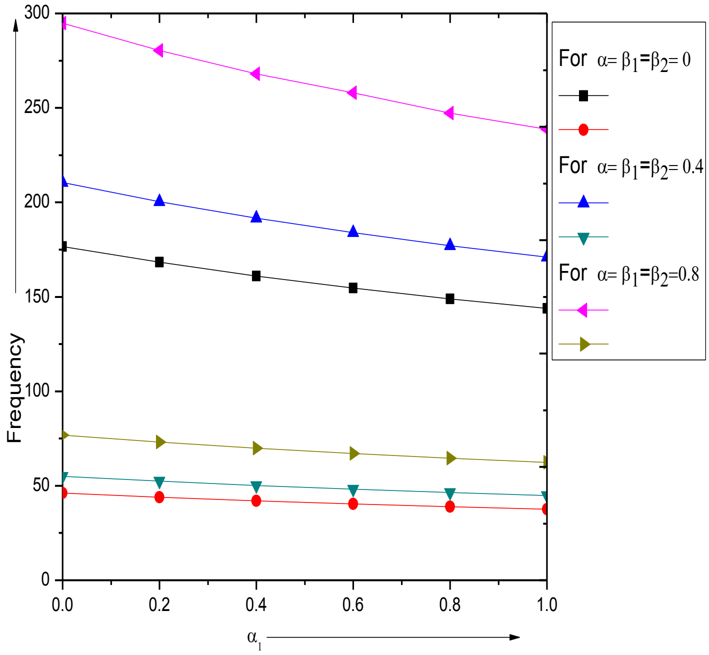


Fig. IV Frequency vs. Non-homogeneous constant α_1



V. CONCLUSION

The effect of non-homogeneity, which is supposed to arise due to variation in Young's moduli and density on natural frequencies of rectangular orthotropic plates of linearly varying thickness resting on Winkler foundation has been studied on the basis of classical plate theory. In present paper the value of frequencies decrease for both the modes of vibrations for the corresponding values of taper parameter as thermal gradient increases. By choosing appropriate values of varying parameters, desired or required values of frequencies can be obtained. Therefore, authors suggest the industrial scientists and design engineers to go through the findings of the present paper in order to provide much better authentic structures and machines with more strength, durability and efficiency.

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