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OSCILLATORY IMMISCIBLE FLUIDS FLOW AND HEAT TRANSFER BETWEEN TWO PARALLEL PLATES

Abdul Mateen

Department of Mathematics, Government Women's First Grade College,
Gulbarga. 585 102. Karnataka, (India)

ABSTRACT

This paper studies the oscillatory flow and heat transfer of two viscous immiscible fluids between two parallel plates. The partial differential equations governing the flow and heat transfer are solved analytically using two-term harmonic and non-harmonic functions in both fluid regions of the channel. Effects of physical parameters such as height ratio, viscosity ratio, conductivity ratio, Prandtl number, Eckert number, periodic frequency parameter and pressure on the velocity and temperature distributions are given and illustrated graphically.

Keywords: Heat Transfer, Horizontal Channel, Immiscible Fluids, Oscillatory Flow, Viscous Dissipation.

Nomenclature

A	real positive constant
C_P	specific heat at constant pressure
$\vec{g}$	gravitational acceleration
K	thermal conductivity
P	pressure
Ec	Eckert number
Pr	Prandtl number
T	temperature
T_w	wall temperature
t	time
u	velocity components of velocity along the plate.
U_0	average velocity

Greek letters

ρ	fluid density
μ	vis cos ity of fluid
ε	coeffiecie nt of periodic parameter
ω	frequency parameter
ωt	periodic frequency parameter
ν	kinematic viscosity
θ	nondimensional tempeaure

Subscripts

1,2 quantities for region-I and region-II respectively.

I INTRODUCTION

Problems involving immiscible multi-phase flow and heat transfer and multi-component mass transfer arise in a number of scientific and engineering disciplines. Important applications include petroleum industry, geophysics and plasma physics. In modeling such problems, the presence of a second immiscible fluid phase adds a number of complexities as to the nature of interacting transport phenomena and interface conditions between the phases. In general, multi-phase flows are driven by gravitational and viscous forces. There has been some theoretical and experimental work on stratified laminar flow of two immiscible fluids in a horizontal pipe (Packham & Shail 1971, Aliareza and Sahai 1990, and Malashetty and Leela 1992). Loharsbi and Sahai (1988) studied two-phase MHD flow and heat transfer in a parallel plate channel, with one of the fluids being electrically conducting. Two-phase MHD flow and heat transfer in an inclined channel was investigated by Malashetty and Umavathi (1997). Later on convective magnetohydrodynamic two-fluid flow and convective flow and heat transfer in composite porous medium was analysed by Malashetty et al (2001 a, b). Fully developed flow and heat transfer in horizontal channel consisting of an electrically conducting fluid layer sandwiched between two fluids layers is studied analytically by Umavathi et al. (2004).

All the above studies pertain to steady flow. However, most problems of practical interest is unsteady. Umavathi et al. (2005, 2009) have presented analytical solutions for unsteady/oscillatory two-fluid flow and heat transfer in a horizontal channel. Keeping in view the wide area of practical importance of multi-fluid flows as mentioned above, it is the objective of the present study to investigate oscillatory flow and heat transfer of two–fluid model in a horizontal channel.

II MATHEMATICAL FORMULATION

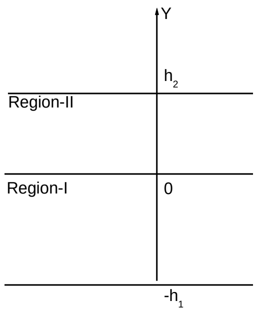

Figure.1 Physical configra

The physical configuration (Fig.1) consists of two infinitely long, parallel plates maintained at different constant temperatures, extending in the Z and X directions. The region $-h_1 \le y \le 0$ (Region-I) is filled with a viscous fluid having density ρ_1, dynamic viscosity μ_1, specific heat constant C_{p1} and thermal conductivity K_1. The region $0 \le y \le h_2$ (Region-II) is filled with a different viscous fluid having density ρ_2, dynamic viscosity μ_2, specific heat constant C_{p2} and thermal conductivity K_2.

It is assumed that the flow is unsteady, fully developed and that all fluid properties are constants. The flow is driven by temperature gradients $T_{w1} - T_{w2}$ and also by pressure gradient $\left(-\frac{\partial p_1}{\partial x}\right)$ in region-I, by a pressure gradient $\left(-\frac{\partial p_2}{\partial x}\right)$ in region-II.

Under these assumptions and taking $C_{p1} = C_{p2} = C_p$, the governing equations of motion (Loharsbi and Sahai, 1988) are given by:

Region-I

$$\rho_1 \frac{\partial u_1}{\partial t} = \mu_1 \frac{\partial^2 u_1}{\partial y^2} - \frac{\partial p_1}{\partial x} \tag{2.1}$$

$$\rho_1 Cp \frac{\partial T_1}{\partial t} = K_1 \frac{\partial^2 T_1}{\partial y^2} + \mu_1 \left(\frac{\partial u_1}{\partial y} \right)^2 \tag{2.2}$$

Region-II

$$\rho_2 \frac{\partial u_2}{\partial t} = \mu_2 \frac{\partial^2 u_2}{\partial y^2} - \frac{\partial p_2}{\partial x} \tag{2.3}$$

$$\rho_2 Cp \frac{\partial T_2}{\partial t} = K_2 \frac{\partial^2 T_2}{\partial y^2} + \mu_2 \left(\frac{\partial u_2}{\partial y} \right)^2 \tag{2.4}$$

where *u* is the x-component of fluid velocity and T is the fluid temperature. The subscripts 1 and 2 correspond to region-I and region-II, respectively. The boundary conditions on velocity are the no-slip boundary condition, which require that the x-component of velocity must vanish at the wall. The boundary conditions on temperature are isothermal conditions. We also assume continuity of velocity, shear stress, temperature and heat flux at the interface between the two fluid layers at y = 0.

The hydrodynamic and thermal boundary and interface conditions for the two fluids can then be written as

$$\begin{aligned} &u_1(-h_1) = 0 \\ &u_2(h_2) = 0 \\ &u_1(0) = u_2(0) \\ &\mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y} \qquad at \quad y = 0 \end{aligned} \tag{2.5}$$

$$\begin{aligned} &T_1(-h_1) = T_{w1} \\ &T_2(h_2) = T_{w2} \\ &T_1(0) = T_2(0) \\ &K_1 \frac{\partial T_1}{\partial y} = K_2 \frac{\partial T_2}{\partial y} \qquad at \quad y = 0 \end{aligned} \tag{2.6}$$

By using the following non-dimensional quantities:

$$\begin{aligned} &u_i = U_0 \;\; u_i^* \qquad y_i = h_i y_i^{\;*} \qquad t = \frac{h_1^2}{\nu_1} t^* \qquad P_i = \frac{h_1^2}{\mu_1 \;\; U_0} \left(\frac{\partial P_i}{\partial x} \right) \\ &\theta = \frac{T - T_{W2}}{T_{W1} - T_{W2}} \qquad Pr = \frac{\mu_1 C_p}{K_1} \qquad Ec = \frac{U_o^2}{C_p (T_{w1} - T_{w2})} \end{aligned} \tag{2.7}$$

and for simplicity dropping the asterisks, equations (3.2.2) to (3.2.3) become

Region-I

$$\frac{\partial u_1}{\partial t}=\frac{\partial^2 u_1}{\partial y^2}-P_1 \tag{2.8}$$

$$\frac{\partial \theta_1}{\partial t}=\frac{1}{\Pr}\frac{\partial^2 \theta_1}{\partial y^2}+Ec\left(\frac{\partial u_1}{\partial y}\right)^2 \tag{2.9}$$

Region-II

$$\frac{\partial u_2}{\partial t}=m\,n\,h^2\frac{\partial^2 u_2}{\partial y^2}-n\,P_2 \tag{2.10}$$

$$\frac{\partial \theta_2}{\partial t}=\frac{bh^2 n}{Pr}\frac{\partial^2 \theta_2}{\partial y^2}+Ec\,mh^2 n\left(\frac{\partial u_2}{\partial y}\right)^2 \tag{2.11}$$

$m=\frac{\mu_2}{\mu_1}$, $n=\frac{\rho_1}{\rho_2}$, $h=\frac{h_1}{h_2}$, $b=\frac{K_2}{K_1}$, are the ratios of viscosity, density , height and conductivity.

The hydrodynamic and thermal boundary and interface conditions for both fluids in non-dimensional form become

$$\begin{aligned} &u_1(-1)=0\\ &u_2(1)=0\\ &u_1(0)=u_2(0)\\ &\frac{\partial u_1}{\partial y}=h\,m\frac{\partial u_2}{\partial y}\quad at\quad y=0 \end{aligned} \tag{2.12}$$

$$\begin{aligned} &\theta_1(-1)=0\\ &\theta_2(1)=0\\ &\theta_1(0)=\theta_2(0)\\ &\frac{\partial \theta_1}{\partial y}=h\,b\frac{\partial \theta_2}{\partial y}\quad at\quad y=0 \end{aligned} \tag{2.13}$$

III SOLUTIONS

The governing momentum equations (2.5) and (2.6) are solved subject to the boundary and interface conditions (2.7) for the velocity distributions in both regions. These equations are partial differential equations that cannot be solved in closed form. However, it can be reduced to ordinary differential equations by assuming

$$u_i(y,t)=u_{i0}(y)+e^{i\omega t}u_{i1}(y) \tag{3.1}$$

$$\theta_i(y,t)=\theta_{i0}(y)+e^{i\omega t}\theta_{i1}(y) \tag{3.2}$$

$$P_i(y,t)=P_{i0}(y)+e^{i\omega t}P_{i1}(y)\qquad for\; i=1,2 \tag{3.3}$$

By substitution of equation (3.1) to (3.3) into equations (2.8) to (2.13), one obtains the following pairs of equations:

Region-I

Non-periodic coefficients

$$\frac{d^2u_{10}}{dy^2} = P_{10} \tag{3.4}$$

$$\frac{d^2\theta_{10}}{dy^2} + \Pr Ec\left(\frac{du_{10}}{dy}\right)^2 = 0 \tag{3.5}$$

Periodic coefficients

$$\frac{d^2u_{11}}{dy^2} - i\,\omega u_{11} = P_{11} \tag{3.6}$$

$$\frac{d^2\theta_{11}}{dy^2} - i\omega \Pr u_{11} + 2Ec\frac{du_{10}}{dy}\frac{du_{11}}{dy} = 0 \tag{3.7}$$

Region -II

Non-periodic coefficients

$$\frac{d^2u_{20}}{dy^2} = \frac{P_{20}}{mh^2} \tag{3.8}$$

$$\frac{d^2\theta_{20}}{dy^2} + \frac{\Pr Ec\, m}{b}\left(\frac{du_{20}}{dy}\right)^2 = 0 \tag{3.9}$$

Periodic coefficients

$$\frac{d^2u_{21}}{dy^2} + i\frac{\omega}{m\,n\,h^2} \quad u_{21} = \frac{P_{21}}{m\,h^2} \tag{3.10}$$

$$\frac{d^2\theta_{21}}{dy^2} + i\frac{\omega \Pr}{bnh^2}\theta_{21} + \frac{2\Pr Ec}{b}\left(\frac{du_{20}}{dy}\frac{du_{21}}{dy}\right) = 0 \tag{3.11}$$

The corresponding boundary and interface conditions become as follows:

Non-periodic coefficients

$$\begin{aligned} &u_{10}(-1) = 0 \\ &u_{20}(1) = 0 \\ &u_{10}(0) = u_{20}(0) \\ &\frac{du_{10}}{dy} = m\,h \quad \frac{du_{20}}{dy} \qquad at \quad y = 0 \end{aligned} \tag{3.12}$$

Periodic coefficients

$$u_{11}(-1)=0$$
$$u_{21}(1)=0$$
$$u_{11}(0)=u_{21}(0) \quad (3.13)$$
$$\frac{du_{11}}{dy}=m\,h\frac{du_{21}}{dy}at \quad y=0$$

Equations (3.4) to (3.11) along with boundary and interface conditions (3.12) to (3.15) represent a system of ordinary differential equations and conditions that can be solved in closed form. Since the solutions can be obtained directly, the expressions are not presented. The results are depicted graphically and are discussed in the next section.

IV RESULTS AND DISCUSSION

In this section representative flow results for oscillatory flow and heat transfer of two immiscible fluids through a horizontal channel are presented and discussed for various parametric conditions. The flow governing equations cannot be solved exactly. However the closed form solutions were found considering the cosine function for frequency parameter on velocity and pressure is assumed. The solutions are depicted graphically in Figs. 2 to 12 for different values of height ratio, viscosity ratio, periodic frequency parameter and pressure on the flow and thermal conductivity ratio, Prandtl number and Eckert number on temperature field. The parameters are fixed as 1 except the varying one, Pr=0.7, Ec=0.5 and ωt =45^0.

Figures 2 and 3 show that velocity profiles are suppressed for large values of height and viscosity ratios. The flow profile is large in region-II compare to region-I for values of viscosity and height ratios less than one. The flow profile is large in region-I compare to region-II for values of height and viscosity ratio greater than one. The flow profiles almost remain the same in both the regions for equal values of height and viscosity ratios and similar effects on the temperature field as shown in Figs. 6 and 7.

Figure 4 shows the effect of periodic frequency parameter ωt on velocity as ωt increases the flow is also increases, since the solutions are approximated by cosine function. The effect of periodic frequency parameter ωt on temperature field is shown in Fig. 8. As the ωt increases temperature profiles is also increases in both the regions. Fig. 10 depicts the effect of thermal conductivity ratio, as the ratio increases the magnitude of suppression is large in region-I compared to region-II. This is obvious because the upper plate is maintained at a low temperature compared to region-I.

Figures 11 and 12 display the effect of Prandtl number and Eckert number respectively on temperature filed. It is seen that temperature is increases with increase in Prandtl number as well as Eckert number. Since the values of Prandtl number are very small for liquid and metals and it is very high for highly viscous fluid.

Keeping in view the physical model of the flow of two immiscible fluids such as water and oil in petroleum industries, a study is made to know the effect of pressure on the flow as shown Fig.5. We have considered different values of pressure for two fluids separately. For positive values of pressure on upper and lower fluids, the flow is promoting. For positive values of pressure in the lower region and negative values of pressure in the upper region display the effect of maximum velocity in region-I. On the other hand if we take negative values of

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pressure in lower region and positive values of pressure in the upper region also show the maximum velocity in region-I itself but the flow direction is opposite. Assigning negative values of pressure also show the similar effect to that for positive values of pressure except in opposite direction. It is observed that controlling the pressure parameter one can also control the direction of flow, which has immense applications in flow reversal problems.

Thus one can conclude that the flow can be controlled by considering different fluids having different viscosities, heights, periodic frequency and applying different pressures.

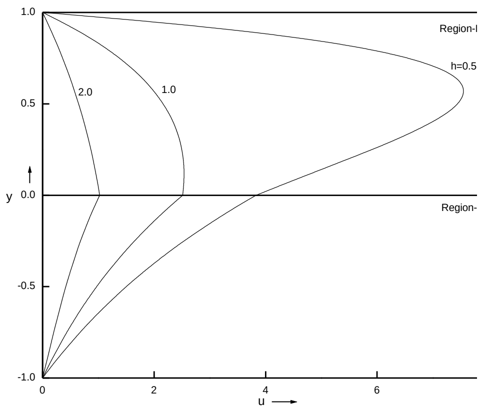

Fig.2 Velocity profiles for different values of height ratio h

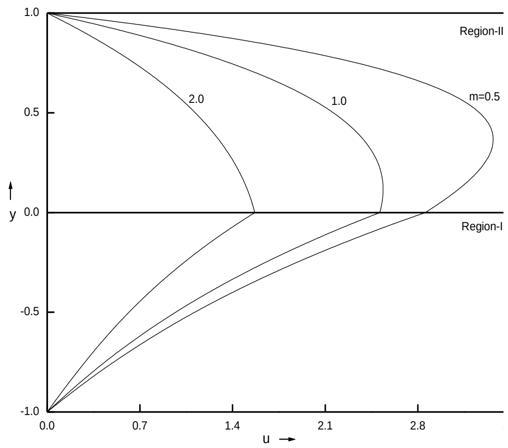

Fig.3 Velocity profiles for different values of viscosity ratio m

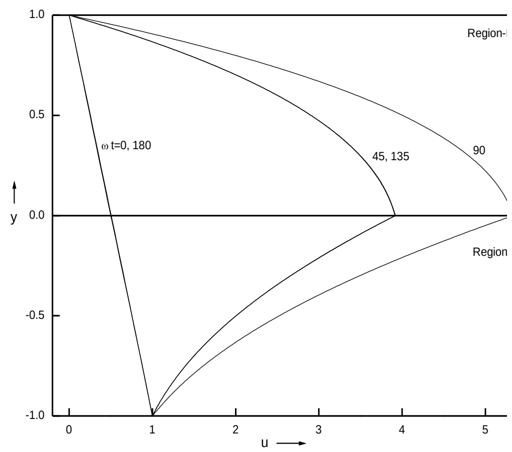

Fig.4 Velocity profiles for different values of periodic frequency parameter ωt

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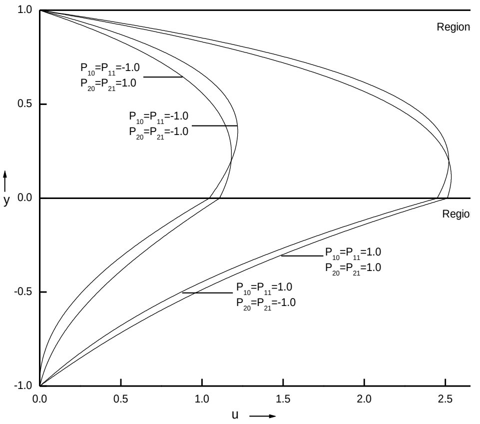

Fig.5 Velocity profiles for different values of pressure P

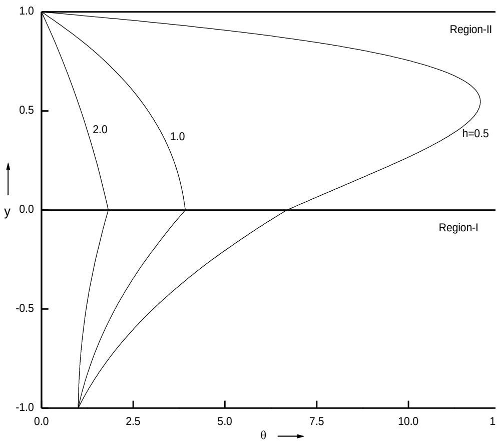

Fig.6 Temperatur profiles for different values of height ratio h

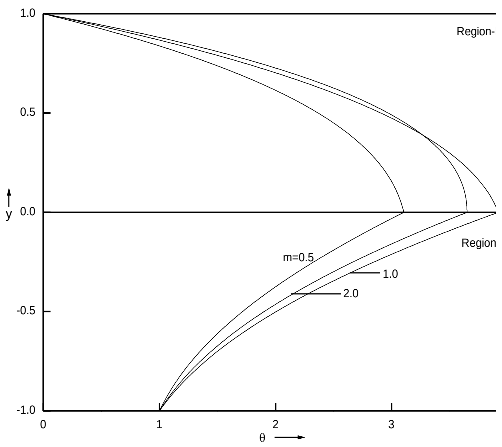

Fig.7 Temperature profiles for different values of viscosity ratio m

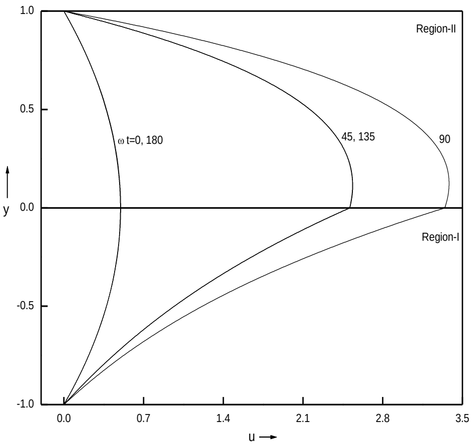

Fig.8 Temperature profiles for different values of periodic frequency parameter ωt

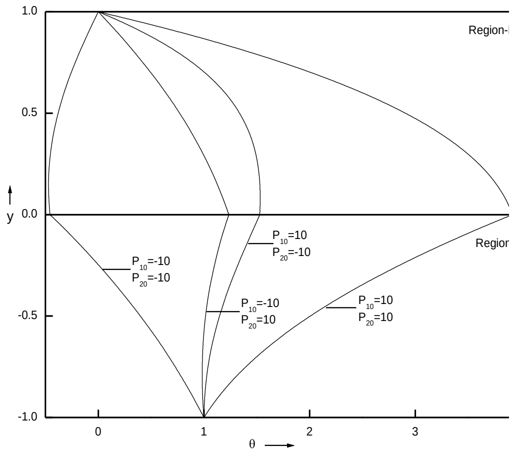

Fig.9 Temperature profiles for different values of pressure P

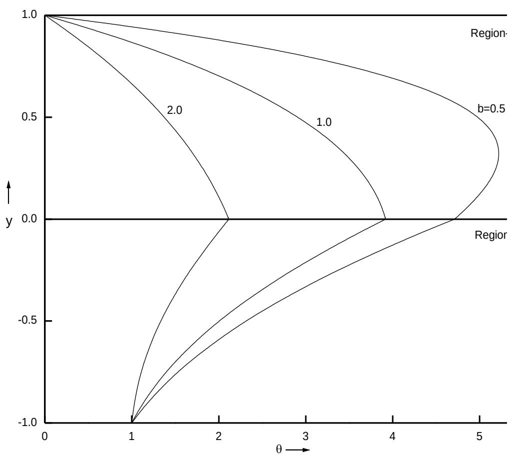

Fig.10 Temperatur profiles for different values of conductivity ratio b

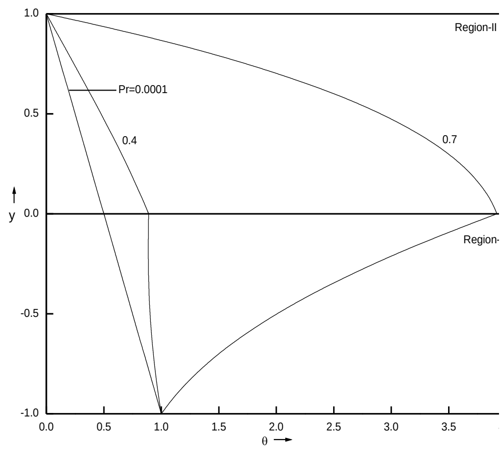

Fig.11 Temperatur profiles for different values of Prandtl number Pr

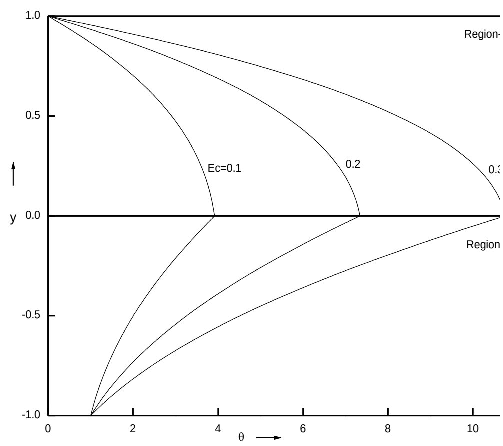

Fig.12 Temperatur profiles for different values of Eckert number Ec

REFERENCE

1. B.A. Packham, and R. Shail, Stratified laminar flow of two immiscible fluids, Proc. Camb. Phil. Soc., **69**, 1971, 443-448.
2. Alireza Setayesh and V. Sahai, Heat transfer in developing magnetohydrodynamic poiseuille flow and variable transport properties, Int. J. Heat Mass Transfer, **33**, 1990, 1711-1720.
3. M.S. Malashetty, and V. Leela, Magnetohydrodynamic heat transfer in two phase flow, Int. J. Engg. Sci., **30**, 1992, 371-377.
4. J. Lohrasbi, and V. Sahai, Magnetohydrodynamic heat transfer in two phase flow between parallel plates, Appl. Sci, Res, **45**, 1988, 53-66.
5. M.S. Malashetty, and J.C. Umavathi, Two phase Magnetohydrodynamic flow and heat transfer in an inclined channel, Int. J. Multiphase flow, **23**, 1997, 545-560.

6. M.S. Malashetty, J.C. Umavathi and J. Prathpkumar , Two fluid magnetoconvection flow in an inclined channel. Int. J. Transport Phenomena, **3**, 2001a, 73-84.
7. Malashetty M.S., Umavathi J.C. and Prathpkumar J., Convective magnetohydrodynamic two fluid flow and heat transfer in an inclined channel. Heat and Mass Transfer, **37**, 2011b, 259-264.
8. J. C. Umavathi, M.S. Malashetty, and Abdul Mateen, Fully developed flow and heat transfer in a horizontal channel containing electrically conducting fluid sandwiched between two fluid layer, Int. J. Applied Mechanical Engineering, **9,** 2004, 781-794.
9. J.C. Umavathi, A.J. Chamkha, Abdul Mateen, A.A. Mudhaf, Unsteady two-fluid flow and heat transfer in a horizontal channel, Heat Mass Transfer, **42**, 2005, 81–90.
10. J.C. Umavathi, A.J. Chamkha, Abdul Mateen, A.A. Mudhaf, Unsteady oscillatory flow and heat transfer in a horizontal composite porous medium channel, Nonlinear Analysis: Modelling and Control, **14**, 2009, 397–415.